Teaching Undergraduates About Dynamic Systems

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This study expands our understanding of how undergraduate business students learn about the behavior of dynamic systems, since the ability to assess and intervene in changing systems is increasingly important to effective business decision-making. A computer simulation is tested in a lower-division social sciences calculus class as a vehicle for improving students' effective understanding of dynamic systems. Pre and post assessments were compared to test the knowledge of the students. Results suggest that the simulation approach did improve student knowledge of dynamic systems.

Introduction

Ample research has indicated accelerating paces of change in business, both at the industry and firm levels (Fine, 2000). With innovation rates averaging nearly 10 percent annually and even faster in technological sectors (Mendelson & Pillai, 1999), business graduates who are capable of assessing, interpreting, and making effective decisions in dynamics situations are more likely to succeed than those who cannot recognize the accumulated implications of varying rates of change. A survey among top-ranked U.S. graduate business programs revealed that three-quarters of faculty view systemic thinking as an "essential" part of business education (Atwater, Kannan & Stephens, 2008). Yet abundant research also suggests that decision-makers perform poorly when facing tasks characterized by complex dynamics such as delays in cause and effect further obscured by separation in space and time (Sterman, 1989; Dörner, 1996; Moxnes, 1998). Consequently, many teachers of business mathematics and other business courses are seeking improved methods of instruction and assessment regarding dynamic systems.

Some researchers of cognition and learning assert that conveying abstract rules, or context-free operations and formulae, help students learn more effectively than concrete instantiations. Kaminski, Sloutsky and Heckler (2008) provide evidence that, when presented with a novel-situated problem, undergraduate students who received mathematical instruction using only generic rules, significantly outperformed students who received instruction on the same principles using concrete examples or even concrete instantiations, plus a generic statement of the rules. It is unclear, however, if abstract mathematical statements describing dynamic systems, such as integration and differential equations, are effective at helping students grasp the consequences of relationships that may change nonlinearly over time. Sweeney and Sterman (2000) contend that students, even those with graduate-level mathematics training at elite institutions, have a poor ability to extrapolate and interpret the situated consequences of dynamic relationships.

Given the pervasiveness of delays, feedback, and nonlinear influences in day-to-day business (Fine, 1998; Sterman, 2000), it becomes critical to understand how to teach students effectively about dynamic systems. The literature providing models and theories of learning are both broad and diverse. One categorization proposed by Fenwick (2000) suggests that educators relinquish the premise that "experiential learning" can be uniquely defined, since every moment is one of experience and thus all learning is experiential. Additionally, Fenwick (2000) argues that cognition cannot be contained by a particular theory, but that multiple perspectives add to the overall understanding of learning and thinking. One of these perspectives, participation, suggests that the learner is an active participant and that learning is situated and all knowledge contextual. According to this idea, effective teaching leverages the contextual knowledge that students bring with them into the classroom setting.

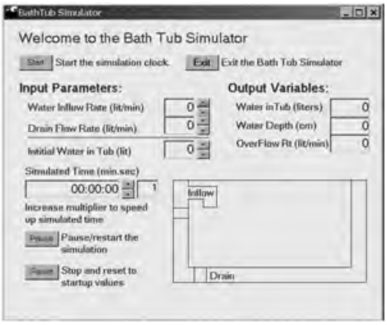
A review of literature defining intelligence, knowledge, and learning (Raymond & Black, 2004) suggested the utility of using computer simulation as a teaching tool to link the academic conceptual development of mathematical abstraction skills to personal prior latent knowledge from common experience. To explore this possibility, the authors developed a simulation tool to link the concepts of system dynamics to the common experience of filling a bathtub. Pre and posttests were conducted to evaluate the effectiveness of the simulation method of supplemental instruction.

The Simulation Exercise

A computer simulation was created with Visual Basic to represent a typical bathtub, as shown in Figure 1, and was used by the students of an introductory calculus course (designed for social science majors, rather than engineering majors) in concert with a structured simulation exercise (shown in Appendix I).

The purpose of the simulation and exercise was to build the student's knowledge of dynamic systems in a step-by-step fashion. This approach was based on the notion that, by "seeing" the mathematical concepts of a familiar situation dynamically simulated, the students would gain a deeper understanding of the mathematical concepts.

Figure 1: *Screen Capture of the Bathtub Simulation*



Experimental Method

To test the hypothesis that the simulation and the structured exercise would improve student understanding of system dynamics, a beginning-of-semester pretest and an end-of-semester posttest were conducted using a set of assessment instruments provided in Sweeney and Sterman (2000). The four assessment problems are reproduced as Appendix II. These four problems assessed student mastery of two concepts of dynamic systems, the accumulation of stocks and rates of change, commonly known as stocks and flows. These two ideas form the conceptual heart of integral (accumulation) and differential (rates of change) calculus.

In each of these four problems, students were provided with the initial value of an accumulated resource, along with a graph delineating the rate at which the resource flowed into and exited from the system. The students were asked to provide a corresponding graph of the resource accumulation over time. These four problems included elementary cash flow and bathtub situations. Each student received only a single version of the problem. The four problems were distributed randomly among the students in the pretest, and again at the end of the course following the completion of the structured learning exercise, as a posttest. No procedures were used to assign the same problem to a particular student on the pretest and posttests, and the assessments were conducted with only the students in attendance on a particular date.

The first set of problems (CashFlow1 and Bathtub1) provided a square wave pattern of inflow (constant inflow with step changes in the rate) with a constant outflow, while the second set of problems (CashFlow2 and Bathtub2) provided a saw-tooth inflow pattern (variable inflow) coupled with a constant outflow. The four problems assumed knowledge of basic arithmetic, Cartesian graphing, and an intuitive understanding of rates/flows and accumulations/stocks. Algebraic abstraction and formulation were not required to solve the problems.

The four problems, CashFlow1, Bathtub1, CashFlow2, and Bathtub2, were assigned randomly across eleven sections of the calculus course. After discarding responses, due to both scoring and response irregularities, 254 instruments were coded from the pretest and 187 were coded from the posttest. Demographics including age, gender, class standing and major were collected from university records to check the randomness of the assignment of the four problems. Demographics were not available for students who added the class after the date of the demographic data collection.

The experiment's null hypotheses stated that the students' performance would be equal when comparing the pretest results with the results of the posttest. Performance was measured as the fraction, p, of students who answered each of the problems correctly. It was assumed that an increase in performance on the posttest could be explained at least in part by the application of the structured simulation exercise. This assumption was verified by comparing the pretest and posttest performance with the performance of an equivalent group of students from the same course in a prior semester that completed the system dynamics assessments without doing the simulation and the structured exercise. This comparison is provided in the following section.

Null Hypotheses: Student performance was equal on the pretest and the posttest:

H_o: $p_{PRE} = p_{POST}$

Alternative Hypothesis: Student performance was lower on the pretest than on the posttest:

H_a: $p_{PRE} < p_{POST}$

Results

A summary of the demographics is provided in Appendix III. There appeared to be no biased assignment of the four problems based on these demographic variables. The random method of distributing the four problems to students attending class on a particular day prevented ensuring that the same problem was administered to the same student on both the pretest and the posttest and that the same sample of students performed both the pretest and the posttest (not all of the students completed the simulation exercise). Because of this, we limited the hypothesis tests to a subset of students. Specifically, we focused only on students who completed the online simulation exercise and the posttest. Table 1 summarizes the results.

Table 1: *Results for students who completed the simulation exercise and took the posttest*

	PreTest			PostTest			Hypotheses Tests		
	Num	Correct	%	Num	Correct	%	χ^2	p	Sig?
Bathtub1	30	3	10.00%	37	10	27.03%	3.07	0.04	Yes
CashFlow1	26	0	0.00%	31	7	22.58%	6.69	0.01	Yes
Bathtub2	24	0	0.00%	31	0	0.00%	0.00	1.00	No
CashFlow2	23	0	0.00%	35	0	0.00%	0.00	1.00	No

Hypothesis Test: H_o: $p_{PRE} = p_{POST}$, H_a: $p_{PRE} < p_{POST}$

The results suggest that the performance difference between the pretest and the posttest for students who completed the simulation exercise was significant at the $\alpha = 0.05$ level on the Bathtub1 and CashFlow1 problems. Student performance did not improve on the Bathtub2 and CashFlow2 problems. To ascertain that the performance improvements were due, at least in part, to the completion of the simulation exercise rather than to other factors (including the completion of the basic calculus course), the pre and posttest results were also compared to the performance results collected at the end of a prior semester. This group consisted of 165 similar calculus students who completed the Bathtub and CashFlow problems but did not complete the simulation exercise (Table 2). We assumed that completion of a basic integral and differential calculus course would improve student understanding of dynamic systems.

Table 2: *Comparison of Pretest Results to Prior Semester Performance*

	Pretest			Prior Semester			Hypotheses Tests		
	Num	Correct	%	Num	Correct	%	χ^2	p	Sig?
Bathtub1	30	3	10.00%	43	9	20.93%	1.54	0.11	No
CashFlow1	26	0	0.00%	44	4	10.00%	2.51	0.06	No
Bathtub2	24	0	0.00%	38	1	2.63%	0.64	0.21	No
CashFlow2	23	0	0.00%	38	1	2.63%	0.62	0.22	No

Hypothesis Test: H_o: $p_{PRE} = p_{Prior}$; H_a: $p_{PRE} < p_{Prior}$

These results indicate that completion of the basic calculus class alone did lead to improvements in knowledge regarding dynamic systems on the Bathtub1 and CashFlow1 problems, but the performance increases were not significant at α = 0.05. Table 3 shows the posttest results for students completing the basic calculus course and the simulation exercise, compared to the performance results for students completing the basic calculus course in the prior semester (with no simulation exercise).

Table 3: *Comparison of Posttest Results to Prior Semester Performance*

	Prior Semester			Posttest			Hypotheses Tests		
	Num	Correct	%	Num	Correct	%	χ^2	p	Sig?
Bathtub1	43	9	20.93%	37	10	27.03%	0.41	0.26	No
CashFlow1	44	4	10.00%	31	7	22.58%	2.64	0.05	Yes
Bathtub2	38	1	2.63%	31	0	0.00%	0.83	0.18	No
CashFlow2	38	1	2.63%	35	0	0.00%	0.93	0.17	No

Hypothesis Test: H_o: $p_{Prior} = p_{POST}$; H_a: $p_{Prior} < p_{Post}$

For students completing the Bathtub1 problem, posttest performance was higher than in the prior semester, but the difference was not significant at the α =0.05 level. Student performance on the posttest was significantly higher on the CashFlow1 problem when compared to students from the prior semester.

Discussion and Conclusion

Introductory calculus texts often introduce integration by having students calculate the area under a graphical function on a Cartesian graph and also introduce differentiation by calculating the slope of tangential lines to a graphical function.

Apparently this common approach did make some difference in student understanding of dynamic systems, as noted in Bathtub1 and CashFlow1 results. However, the concepts, exercises, and exams provided in an introductory social sciences calculus course did not significantly improve student knowledge of dynamic systems in our study. What prevented these students from connecting their knowledge of elementary calculus to observations of state variables and the rates at which they change?

Furthermore, more students completed the Bathtub1 problem accurately than completed the CashFlow1 problem accurately, even though the mathematical structure underlying the exercises is identical. Why is this? It may be that in traditional calculus education, integration and differentiation are often not explicitly related to physical activities and states until students reach more advanced engineering-focused courses. But, if students were unable to relate the abstract language of mathematics to any real-world scenarios, we would expect that the data would suggest equivalent struggles with the bathtub and cash flow scenarios.

In the learning literature, we see evidence of situational knowledge that may help us interpret these results. Schliemann (1998) described the mathematical capabilities of street vendors and cooks and the use of scalar arithmetic in their every day activities. Their mathematical capabilities were sophisticated within the context of their work, but not generalizable without additional training. Another contextual anecdote involved the voting behavior and perceptions of a Brazilian woman. In this situation, the woman's interpretation of graphical information was influenced by her voting preference. Lave (1988) studied the mathematical sophistication of ordinary people living their day-to-day lives and found that in many instances, mathematical capabilities were uniquely situated. Lave described individuals who performed poorly on basic arithmetic tests given in a school environment but performed the same math calculations accurately and without apparent difficulty in the bowling alley. Shoppers were also found to demonstrate mastery of basic arithmetic in stores but they performed poorly in school settings on the same arithmetic operations.

We speculate that more students enrolled in first-semester calculus (usually undertaken during the first three semesters of college) have tacit, or automatic and situated knowledge of the dynamics of filling a bathtub with water than do have tacit knowledge of cash flows through a bank account. Probably very few students have translated their tacit understanding of stock-and-flow bathtub dynamics into the abstract language of mathematics. But those students who have developed an explicit understanding of first-derivative calculus may, with the tips offered in the simulation exercise, be able to relate their explicit understanding of first-derivative dynamics to their implicit understanding of bathtub dynamics.

The learning literature also documents the importance of expert/apprentice mentoring particularly in regards to "tips" or "rules of thumb" that are shared within a discipline or a knowledge community (Henning, 1998; Gick & Holyoak, 1980). Individuals do not readily transfer knowledge to new scenarios unless guided or tipped in advance. Reed, Ernst and Banerji (1974) examined the possibility that individuals use applicable learning and knowledge from a solved problem to solve an additional problem. They examined the transferability of the solution of the missionary-cannibal problem to the solution of the jealous-husband problem. The two problems are similar.

They found variable support for the hypothesis of learning transfer. In a similar study, Gick and Holyoak (1980) studied the learning transfer between two analogous problems, the radiation problem and the attack-dispersion problem. Once again they found that learning was not transferred from one problem to the next unless the subjects were prompted to make the connection between the two problems. However, once prompted, nearly all subjects were able to make the connections.

We propose that completion of the bathtub simulation exercise not only led to improvement of student performance on the Bathtub1 problem but also led to improvement on the Cashflow1 problem. Since the simulation exercise used the same graphical and structural cues, the students were “tipped off” regarding the structure of the cash flow problem even though the simulation exercise did not mention cash flow examples. We hypothesize that a wide range of experiences, and many translations among various levels of abstractions, are required for students to develop a robust understanding of generalized structures, such as the consequences of accumulating a rate that changes over time. Students may understand Cartesian coordinates and may be able to interpret graphs within the context of the graph itself, but translating that understanding, both into the concrete and physical realm as well as into the pure abstractions of math equations, apparently poses significant challenges.

Since many students do not appear to make connections readily between the abstractions of their math courses to their real-world experiences, we propose that simulations can offer intermediate abstractions. By providing interactive and visual experiences with rapid feedback, simulations can help students translate their tacit understanding of the dynamics in various circumstances into more general (and mathematically representable) structures that facilitate analytical understanding to complement intuitions that may or may not prove accurate. The learning literature supports this approach of “dual coding” provided through visual and interactive experiences (James & Galbraith, 1985; Carnevale et al., 1990; Gagne, 1977; Johari, 1998; Shu-Ling, 1998).

Student performance on the Bathtub2 and CashFlow2 problems, with linearly changing rates of inflow, suggests that even very simple dynamics systems are not well understood. It is uncommon in businesses that outflows will remain constant while only inflows change, or vice-versa. To be effective, business decision-makers must understand the difference between flows (such as annual profits) and stocks (such as retained earnings). In our experience, many undergraduate upper-division business students do not understand that income statement entries (sales, profits, expenses) are measured per time period (monthly, quarterly, or annually) and so are akin to the flow of water into and out of a bathtub. Similarly, they do not comprehend that balance sheet entries are stocks (snapshots of the accumulated business resources at a particular point in time) analogous to a bathtub’s accumulated water. A resource-based view of business requires that decision-makers distinguish between stocks and activities measured per time period. Focusing on accumulated customers for a word-of-mouth marketing campaign will yield different and better results than focusing only on the rate of customer acquisition. Likewise, a business manager who does not understand that the accumulated customer base, rather than the new-customer acquisition rate, drives demands for after-sales service and maintenance risks losing customers dissatisfied by staffing inadequate to meet customers' needs.

This study provides some support for the assertion that targeted simulation exercises can improve student understanding of stocks and flows in dynamic systems. The results were tempered, however, by concerns regarding the student sampling procedures. Future studies are needed to refine the simulation exercises and the experimental methods. Even so, the experiment's results raise provocative questions about how students learn (and do not learn) about dynamic systems:

- Why did students who completed a basic integral and differential calculus course perform poorly on tests of understanding regarding dynamic systems?
- How did the simulation exercise administered in this study improve student performance on tests of understanding regarding dynamic systems?
- Why did the performance results vary between the mathematically equivalent Bathtub and CashFlow problems?
- Why did performance not improve on the Bathtub2 and CashFlow2 problems (the ones with variable input flows)?

It has been demonstrated (Sterman, 1989; Dörner, 1996) that people's decision-making abilities deteriorate as system complexity (as indicated by the number of relationships among variables, and the delays between action and consequence) increases. Since the pace of change in businesses and markets is increasing over time, our educational endeavors will benefit from further research on how students learn, and fail to learn, to assess dynamic situations, so that we can improve the effectiveness of the instructional activities we provide.

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Appendix I

Bathtub Stocks and Flows Structured Learning Assignment

Consider the bathtub shown below. Water flows in at a certain rate, and exits through the drain at another rate. If the water in the tub exceeds 252 liters, then the excess inflow runs out through an overflow drain.

The purpose of this exercise is to build your conceptual understanding of rates of change (flows) and accumulations (stocks). You will use a computerized simulation of a bathtub to assist you in this exercise.

Familiarity – To start you need to become familiar with the controls and data displays of the simulator.

- Start — Start the simulation clock, water inflow must be >= zero
- Pause — Pause the simulation clock, resume after pausing
- Exit — Exit the simulation
- Reset — Reset the simulation to the beginning zero values

Input Parameters – are used to set initial values before starting the simulation clock. Each of the input parameters is increased/decreased by clicking the left button of the mouse on the up arrow or down arrow of the spinner control.

Water Inflow Rate (liters/min) – Use the spinner control to increase/decrease the rate at which water will flow into the bathtub; this control is a real-time control that can be used anytime during the simulation to change the rate at which water is flowing into the bathtub. The inflow rate must be greater than zero to start the simulation clock.

Initial Water in Tub (liters) – Use this control to start the simulation with up to 200 liters of water in the tub.

Drain Flow Rate (liters/min) – Set the initial drain rate of water from the tub.

Output Variables – display the current state of various aspects of the simulation. These values provide feedback about what is happening in the bathtub.

Water In Tub (liters) – Displays the amount of water in the bathtub in liters.

Water Depth (cm) – Displays the depth of the water in the bathtub in centimeters.

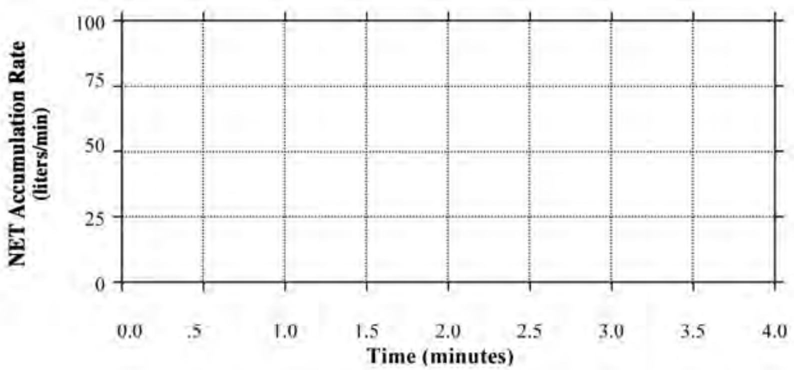
Overflow Rate (liters/min) – Displays the rate at which water is flowing through the overflow of the bathtub in liters per minute.

Activity I: Constant rates flowing into and out of the tub

- Before starting the simulation, use the spinner controls to set the inflow rate at 50 liters/min; the water in the tub to 75 liters, and the drain flow rate to 25 liters/min.

- Given the information above, at what rate will water be accumulating in the tub? ________

- Plot the constant net accumulation rate on the graph below over a 4 minute time period.



- Calculate the amount of water in the bathtub at the times shown below and enter your calculated results in the indicated row of the table. The first value is provided for you.

Time	1 min: 00 sec	1 min:30 sec	2 min: 45 sec	3 min: 00 sec
Calculated Water in Tub	100 liters			

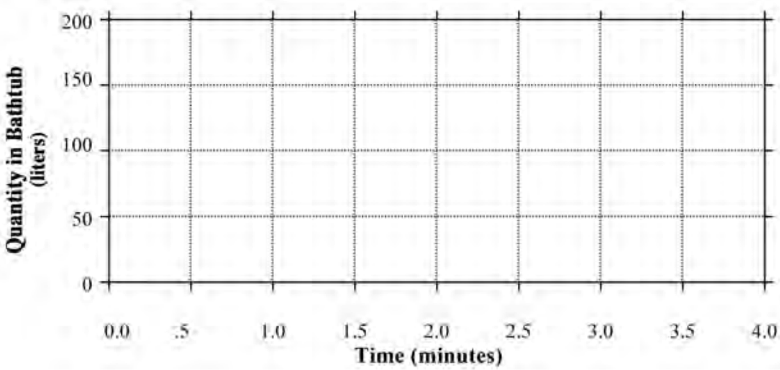
Hint: accumulation at TimeNew = initial accumulation in *liters* + [(rate of inflow in *liters/minute*) * (TimeNew – TimeStart in *minutes*)] y = f(x) = 75 + 25x where y = the liters of water in the tub and x = the elapsed time in minutes.

- Now start the simulation and pause the simulation at the times shown in the table to verify your calculations. Enter the values for the liters of water in the tub in the row for actual water in the tub. Are the actual values the same as the values you calculated? Why or why not?

Time	1 min: 00 sec	1 min:30 sec	2 min: 45 sec	3 min: 00 sec
Calculated Water in Tub	100 liters			

- If the Actual Water in Tub differed from your calculated amounts at any of the 4 times above, why is that? Re do your calculations and the simulations as often as you need until you understand how to calculate the effects of a constant rate on the accumulated water in the tub. You can experiment with changing the values of the inflow and drain rate to check your understanding.

- Using the graph below plot the four values for the actual amount of water in the tub at the four times noted above and the changes between those four points, along with the beginning value at time zero.



Is the graph linear or curved? Why?

Is it increasing or decreasing? Over what time periods?

If your graph is linear, calculate the slope of each line segment by considering how much the y value changes for each unit of x (i.e., use the traditional "rise/run" calculation). The x and y values for the first segment along with the required calculations are provided below as an example. Fill in the rest of the table.

Time Period	X1	X2	Y1	Y2	Slope = (Y2 – Y1)/(X2 – X1) =	Result
1. 00:00 – 01:00	0.0	1.0	75	100	(100 – 75)/(1.0 – 0.0) =	25
2. 01:00 – 01:30	1.0	1.5				
3. 01:30 – 02:45	1.5	2.75				
4. 02:45 – 03:00	2.75	3.0				

Note that the slope indicates the rate of change in the water in the tub. If water is accumulating, the slope is positive (and the line on the graph is increasing). If water is draining from the tub, the slope is negative (and the line you graphed is decreasing).
Are the slopes of the line segments you calculated the same value as the rate at which water is accumulating in the tub when you ran the simulation? You can run the simulation again, to make sure.

If the simulation continued to run would the bathtub eventually be full i.e. excess water would run out the overflow drain? If so, calculate the time at which the tub would be full. Run the simulation to verify your calculated answer.

Hint: Total Volume in liters = depth * length * width = (30 cm * 140 cm * 60 cm)/1000 = 252 liters

Activity II: Changing accumulation of water in the tub

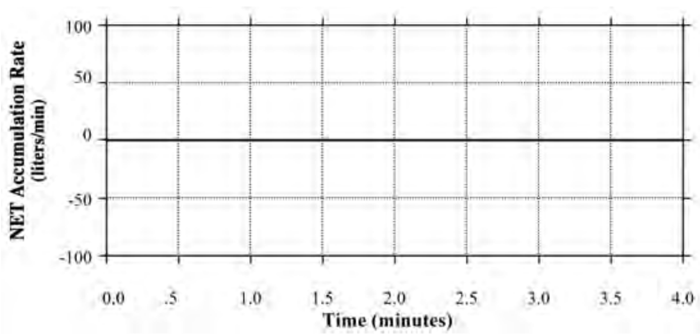
In the previous exercise, the inflow and outflow from the tub were constant. This time you will be changing the flows of water into, and draining from, the tub as time passes.

- The table below lists the initial values (0 min: 00 sec) and the changing inflow and outflow values. Before running the simulation calculate the net flow of water accumulating in or dispersing from the tub as well as the water in the tub.

Calculated Values (do these calculations prior to starting the simulation)

Time	Inflow Rate	Drain Rate	Net Accumulation Rate	Ending Water in Tub
0 min: 00 sec (initial values)	75 lit/min	25 lit/min		100 liters
0 min: 00 sec - 1 min: 00 sec	75 lit/min	25 lit/min	50 liters/minute	150 liters
1 min: 00 sec - 2 min: 00 sec	25 lit/min	50 lit/min		
2 min: 00 sec - 3 min: 00 sec	100 lit/min	0 lit/min		
3 min: 00 sec - 4 min: 00 sec	0 lit/min	50 lit/min		

- Plot the net accumulation rate on the graph below over the 4 minute time period.



- Now start the simulation and pause the simulation at the times shown in the table to adjust the inflow and drain rates; enter the simulation actual values at each time to verify your calculations.

Actual Values (read from simulation)

Time	Inflow Rate	Drain Rate	Ending Water in Tub
0 min: 00 sec (initial values)	75 lit/min	25 lit/min	100 liters
0 min: 00 sec - 1 min: 00 sec	75 lit/min	25 lit/min	
1 min: 00 sec - 2 min: 00 sec	25 lit/min	50 lit/min	
2 min: 00 sec - 3 min: 00 sec	100 lit/min	0 lit/min	
3 min: 00 sec - 4 min: 00 sec	0 lit/min	50 lit/min	

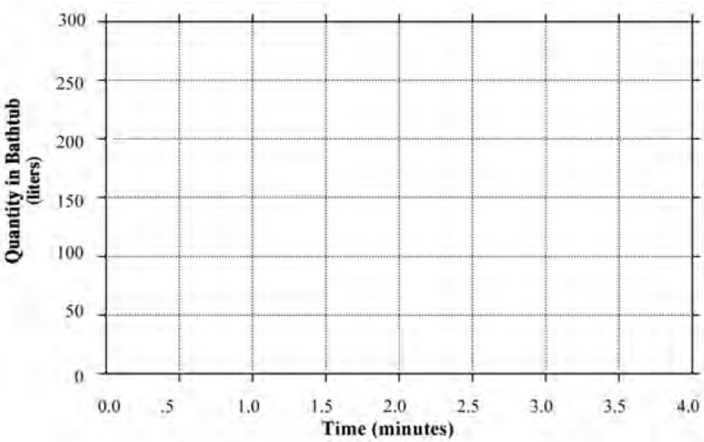
Did you calculate the volume of water in the tub at each point in time correctly? If not, continue experimenting with the controls on the simulator until you have an understanding of how

Inflow Rate – Drain Rate = Net Accumulation Rate,

and how, for each segment of time,

Net Accumulation Rate * Duration + Previous Volume in Tub = New Volume in Tub

- Using the graph below, plot the actual amount of water in the tub.



Calculate the slope of each line segment by considering how much the y value changes for each unit of x (i.e., use the traditional "rise/run" calculation). In each time segment (0 – 1 min, 1 – 2 min, 2 – 3 min, 3 – 4 min) verify that the slope of your graph is equal to the Net Accumulation rates noted in the table above.

Note that the slope indicates the rate of change in the total volume of the water in the tub. If water is accumulating, the slope is positive (and the line on the graph is increasing). If water is draining from the tub, the slope is negative (and the line you graphed is decreasing). Are the slopes of the line segments you calculated the same value as the rate at which water is accumulating in the tub when you ran the simulation? You can run the simulation again, to make sure.

Time Period	X1	X2	Y1	Y2	Slope = (Y2 – Y1)/(X2 – X1) =	Result
1. 00:00 – 01:00	0.0	1.0	100	150	(150 – 100)/(1.0 – 0.0) =	50
2. 01:00 – 02:00	1.0	2.0				
3. 02:00 – 03:00	2.0	3.0				
4. 03:00 – 04:00	3.0	4.0				

In the last time period if the simulation continued to run would the bathtub eventually be empty? Is so, calculate the time at which the bathtub would be empty. Run the simulation to verify your calculated answer.

Activity III: Constantly changing accumulation of water in the tub

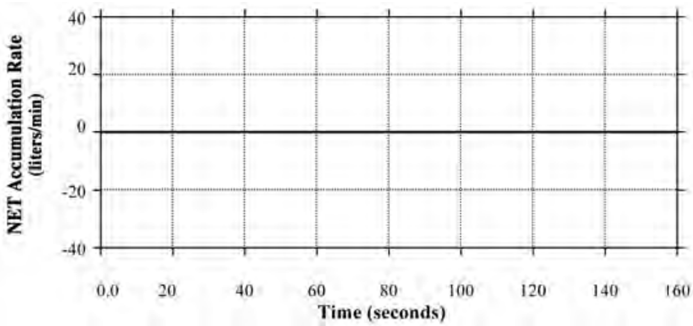
This time you will be changing the flows of water into, and draining from, the tub in very short time segments (every ten seconds). After working through a few time periods, you should be able to predict the shape of the curve for additional time periods.

- The table below lists the initial values (0 min: 00 sec) and the changing inflow and outflow values. Before running the simulation, calculate the net flow of water accumulating in or dispersing from the tub as well as the water in the tub. Remember to convert liters/min to liters/sec when calculating the water volume.

Calculated Values

Time	Inflow Rate	Drain Rate	Accumulation Rate (liters/minute)	Accumulation Rate (liters/second)	Water in Tub
0 min: 00 sec (initial values)	50 lit/min	50 lit/min			100 liters
0 min: 00 sec - 0 min: 10 sec	50 lit/min	50 lit/min			
0 min: 10 sec - 0 min: 20 sec	60 lit/min	50 lit/min			
0 min: 20 sec - 0 min: 30 sec	70 lit/min	50 lit/min			
0 min: 30 sec - 0 min: 40 sec	80 lit/min	50 lit/min			
0 min: 40 sec - 0 min: 50 sec	90 lit/min	50 lit/min			
0 min: 50 sec - 1 min: 00 sec	80 lit/min	50 lit/min			
1 min: 00 sec - 1 min: 10 sec	70 lit/min	50 lit/min			
1 min: 10 sec - 1 min: 20 sec	60 lit/min	50 lit/min			
1 min: 20 sec - 1 min: 30 sec	50 lit/min	50 lit/min			
1 min: 30 sec - 1 min: 40 sec	40 lit/min	50 lit/min			
1 min: 40 sec - 1 min: 50 sec	30 lit/min	50 lit/min			
1 min: 50 sec - 2 min: 00 sec	20 lit/min	50 lit/min			
2 min: 00 sec - 2 min: 10 sec	10 lit/min	50 lit/min			
2 min: 10 sec - 2 min: 20 sec	20 lit/min	50 lit/min			
2 min: 20 sec - 2 min: 30 sec	30 lit/min	50 lit/min			
2 min: 30 sec - 2 min: 40 sec	40 lit/min	50 lit/min			

• Plot the net accumulation rate in liters/minute on the graph below over the 160 second time period.



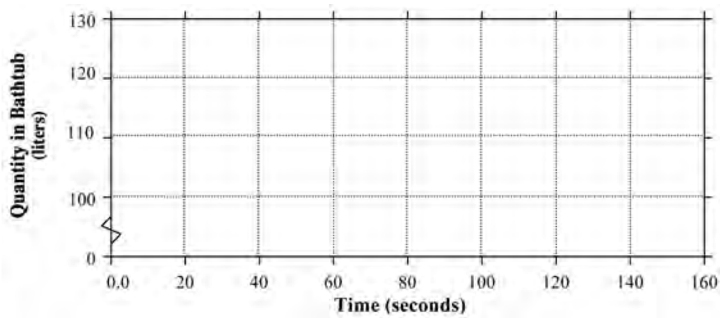
• Now run the simulation pausing at the times shown in the table to verify your calculations.

Actual Values (read from simulation)

Time	Inflow Rate	Drain Rate	Water in Tub
0 min: 00 sec (initial values)	50 lit/min	50 lit/min	100 liters
0 min: 00 sec - 0 min: 10 sec	50 lit/min	50 lit/min	
0 min: 10 sec - 0 min: 20 sec	60 lit/min	50 lit/min	
0 min: 20 sec - 0 min: 30 sec	70 lit/min	50 lit/min	
0 min: 30 sec - 0 min: 40 sec	80 lit/min	50 lit/min	
0 min: 40 sec - 0 min: 50 sec	90 lit/min	50 lit/min	
0 min: 50 sec - 1 min: 00 sec	80 lit/min	50 lit/min	
1 min: 00 sec - 1 min: 10 sec	70 lit/min	50 lit/min	
1 min: 10 sec - 1 min: 20 sec	60 lit/min	50 lit/min	
1 min: 20 sec - 1 min: 30 sec	50 lit/min	50 lit/min	
1 min: 30 sec - 1 min: 40 sec	40 lit/min	50 lit/min	
1 min: 40 sec - 1 min: 50 sec	30 lit/min	50 lit/min	
1 min: 50 sec - 2 min: 00 sec	20 lit/min	50 lit/min	
2 min: 00 sec - 2 min: 10 sec	10 lit/min	50 lit/min	
2 min: 10 sec - 2 min: 20 sec	20 lit/min	50 lit/min	
2 min: 20 sec - 2 min: 30 sec	30 lit/min	50 lit/min	
2 min: 30 sec - 2 min: 40 sec	40 lit/min	50 lit/min	

Were the amounts you calculated for each cell validated by the simulation? For any cells that differ, please make sure you understand the source of discrepancy and make sure you can calculate the correct value for each cell.

- Using the graph below plot the amount of water in the tub (time in seconds). Note the broken vertical axis.



In this final exercise the rate of change of water in the tub varies between each time segment. We can therefore make some observations about the rate of change in the rate of change. When the Net Accumulation Rate is increasing, but by less each time segment (we call that "increasing at a decreasing rate," the graph approaches a peak, or local maximum. Similarly, when the Net Accumulation Rate is decreasing, but by less each time segment (decreasing at a decreasing rate), the graph approaches a local minimum.

Appendix II

Four problems for assessing dynamic system knowledge.

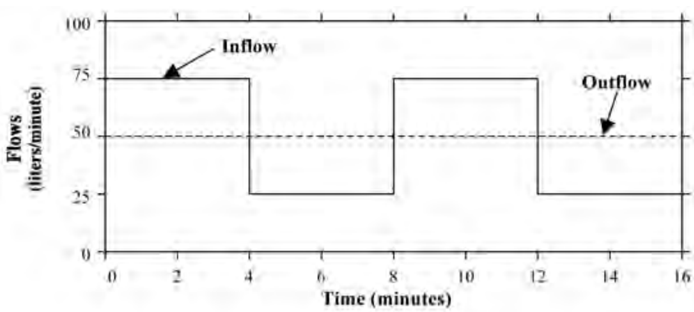
(Used by permission of the author.)

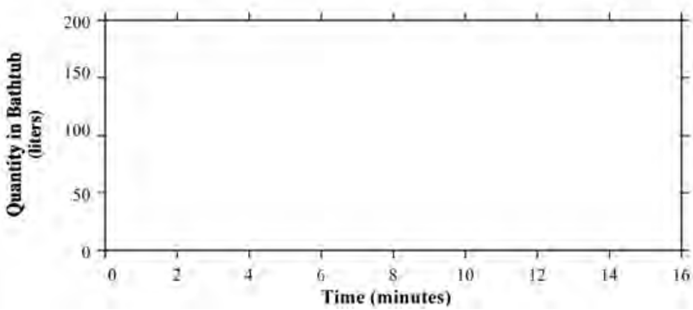
Bathtub 1: *Square Wave Pattern*[1]

Consider the bathtub shown below. Water flows in at a certain rate, and exits through the drain at another rate:

The graph below shows the hypothetical behavior of the inflow and outflow rates for the bathtub. From that information, draw the behavior of the quantity of water in the tub on the second graph below.

Assume the initial quantity in the tub (at time zero) is 100 liters.

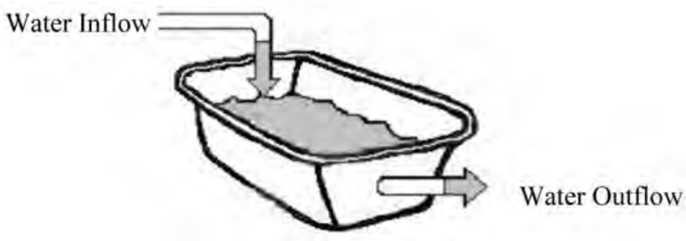




[1]Adapted from Sweeney, L.B. and Sterman, J.D. See references.

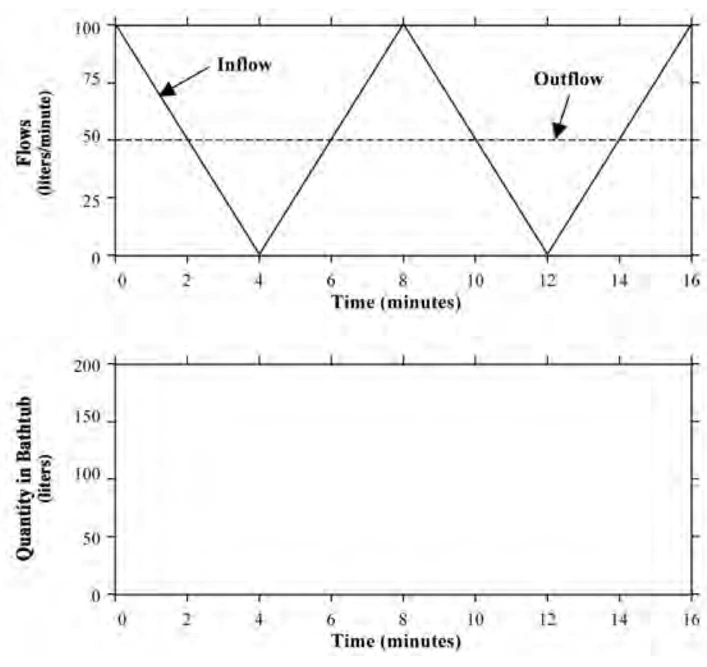
Bathtub 2: *Sawtooth Pattern*[2]

Consider the bathtub shown below. Water flows in at a certain rate, and exits through the drain at another rate:



The graph below shows the hypothetical behavior of the inflow and outflow rates for the bathtub. From that information, draw the behavior of the quantity of water in the tub on the second graph below.

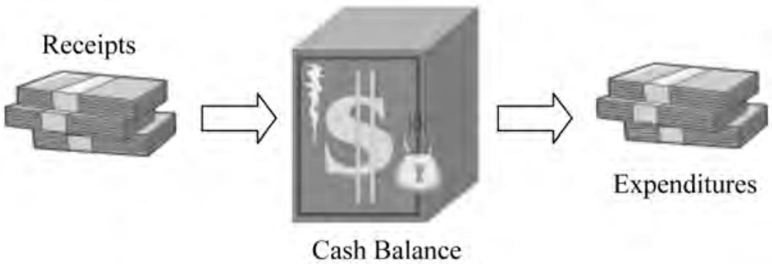
Assume the initial quantity in the tub (at time zero) is 100 liters.



[2]Adapted from Sweeney, L.B. and Sterman, J.D. See references.

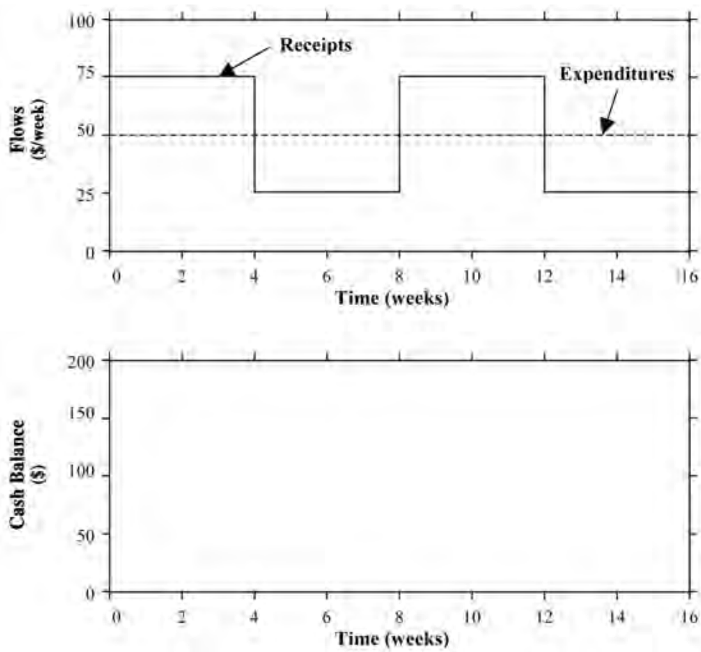
Cash Flow 1: *Square Wave Pattern*[3]

Consider the cash balance of a company. Receipts flow into the balance at a certain rate, and expenditures flow out at another rate:



The graph below shows the hypothetical behavior of receipts and expenditures. From that information, draw the behavior of the firm's cash balance on the second graph below.

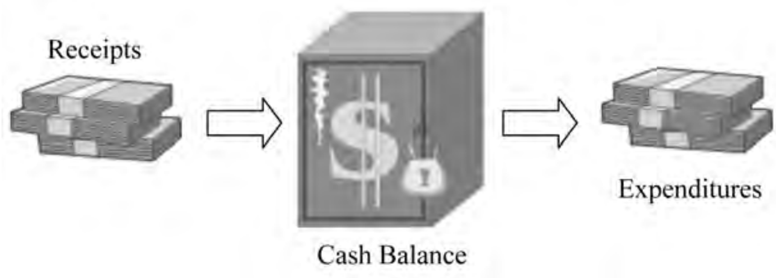
Assume the initial cash balance (at time zero) is $100.



[3]Adapted from Sweeney, L.B. and Sterman, J.D. See references.

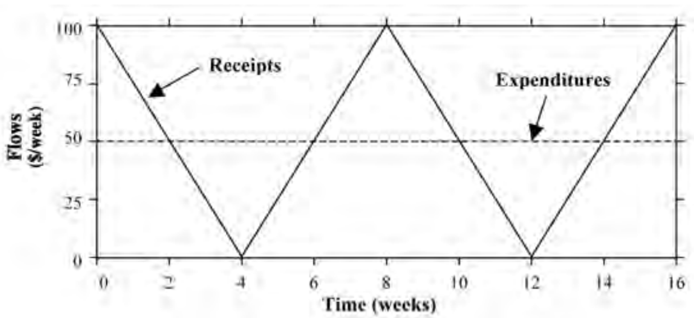
Cash Flow 2: *Sawtooth Pattern*[4]

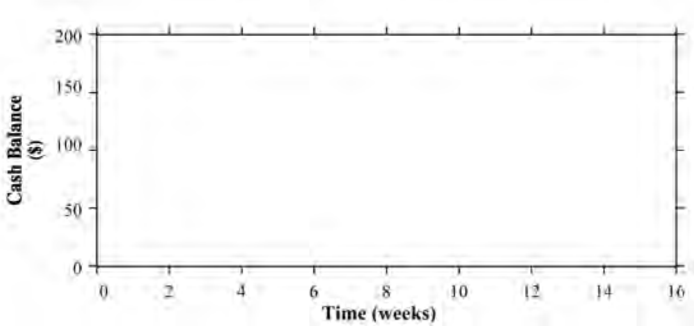
Consider the cash balance of a company. Receipts flow into the balance at a certain rate, and expenditures flow out at another rate:



The graph below shows the hypothetical behavior of receipts and expenditures. From that information, draw the behavior of the firm's cash balance on the second graph below.

Assume the initial cash balance (at time zero) is $100.





[4]Adapted from Sweeney, L.B. and Sterman, J.D. See references.

Appendix III

Demographics of the Student Population:

Demographics are provided for the entire course population, along with the demographics for the four problem assignment groups on both the pretest and the posttest. The course population was defined as the students enrolled on the 15th class day, and the sample of students who actually took the pretest and the posttest depended on attendance on that particular class day.

Self-reported ethnicity of the respondents was obtained from university records; as shown below the majority of the population was Caucasian.

- Asian: 5
- Hispanic: 7
- Native American: 4
- Other: 3
- Missing Data: 24
- Caucasian: 340

Table AIII-1: *Age Distribution*

	Population		**Pretest**		Bathtub1		CashFlow1		Bathtub2		CashFlow2	
Age	#	%	#	%	#	%	#	%	#	%	#	%
<=18	92	24.02%	70	26.79%	21	33.33%	16	28.57%	22	26.44%	11	18.64%
<=20	125	32.64%	74	27.92%	17	26.98%	21	37.50%	17	19.54%	19	32.20%
<=22	82	21.41%	55	20.75%	12	19.05%	10	17.86%	15	17.24%	18	30.51%
<=24	35	9.14%	17	6.42%	5	7.94%	4	7.14%	5	5.75%	3	5.08%
<=26	17	4.44%	10	10.94%	3	4.76%	0	0.00%	2	24.14%	5	8.47%
<=28	11	2.87%	6	2.26%	2	3.17%	1	1.79%	3	3.45%	0	0.00%
>28	21	5.48%	13	4.91%	3	4.76%	4	7.14%	3	3.45%	3	5.08%
Total	383		245		63		56		67		59	

	Population		**Posttest**		Bathtub1		CashFlow1		Bathtub2		CashFlow2	
Age	#	%	#	%	#	%	#	%	#	%	#	%
<=18	92	0.2402%	40	0.2286%	12	0.2727%	9	0.2093%	8	0.1905%	11	0.2391%
<=20	125	0.3264%	60	0.3429%	14	0.3182%	17	0.3953%	10	0.2381%	19	0.413%
<=22	82	0.2141%	35	0.2%	8	0.1818%	9	0.2093%	10	0.2381%	8	0.1739%
<=24	35	0.0914%	16	0.0914%	2	0.0455%	4	0.093%	6	0.1429%	4	0.087%
<=26	17	0.0444%	7	0.04%	2	0.0455%	1	0.0233%	1	0.0238%	3	0.0652%
<=28	11	0.0287%	5	0.0286%	1	0.0227%	0	0%	4	0.0952%	0	0%
>28	21	0.0548%	12	0.0686%	5	0.1136%	3	0.0698%	3	0.0714%	1	0.0217%
Total	383		175		44		43		42		46	

Table AIII-2: *Gender of Respondents*

	Population		**Pretest**		Bathtub1		CashFlow1		Bathtub2		CashFlow2	
Gender	#	%	#	%	#	%	#	%	#	%	#	%
Female	178	46.48%	107	43.67%	28	44.44%	25	44.64%	27	40.30%	27	45.76%
Male	205	53.52%	138	56.33%	35	55.56%	31	55.36%	40	59.70%	32	54.24%
Total	383		245		63		56		67		59	

	Population		**Posttest**		Bathtub1		CashFlow1		Bathtub2		CashFlow2	
Gender	#	%	#	%	#	%	#	%	#	%	#	%
Female	178	46.48%	81	46.29%	23	52.27%	26	60.47%	14	33.33%	18	39.13%
Male	205	53.52%	94	53.71%	21	47.73%	17	39.53%	28	66.67%	28	60.87%
Total	383		175		44		43		42		46	

Table AIII-3: *Class standing of respondents*

	Population		**Pretest**		Bathtub1		CashFlow1		Bathtub2		CashFlow2	
Class	#	%	#	%	#	%	#	%	#	%	#	%
Freshman	190	49.61%	126	51.22%	34	53.97%	32	56.14%	36	53.73%	24	40.68%
Sophomore	98	25.59%	62	25.20%	16	25.40%	14	24.56%	15	22.39%	17	28.81%
Junior	56	14.62%	36	14.63%	10	15.87%	4	7.02%	10	14.93%	12	20.34%
Senior	32	8.36%	16	6.50%	3	4.76%	5	8.77%	3	4.48%	5	8.47%
*Other	7	1.83%	6	2.44%	0	0.00%	1	3.51%	3	4.48%	1	1.69%
Total	383		245		63		56		67		59	

	Population		**Posttest**		Bathtub1		CashFlow1		Bathtub2		CashFlow2	
Class	#	%	#	%	#	%	#	%	#	%	#	%
Freshman	190	49.61%	87	49.71%	21	47.73%	22	51.16%	17	40.48%	27	58.70%
Sophomore	98	25.59%	47	26.86%	15	34.09%	9	20.93%	13	30.95%	10	21.74%
Junior	56	14.62%	23	13.14%	4	9.09%	11	25.58%	4	9.52%	4	8.70%
Senior	32	8.36%	15	8.57%	4	9.09%	1	2.33%	5	11.90%	5	10.87%
*Other	7	1.83%	3	1.71%	0	0.00%	0	0.00%	3	7.14%	0	0.00%
Total	383		175		44		43		42		46	

Other includes international and non-degree students

Table AIII-4: *Academic major of the respondents*

	Population		**Pretest**		Bathtub1		CashFlow1		Bathtub2		CashFlow2	
Major	#	%	#	%	#	%	#	%	#	%	#	%
Ag	42	10.97%	97	39.59%	5	7.94%	32	57.14%	36	53.73%	24	40.68%
Bus	131	34.20%	64	26.12%	18	28.57%	14	25.00%	15	22.39%	17	28.81%
Sci/Tech	102	26.63%	46	18.78%	20	31.75%	4	7.14%	10	14.93%	12	20.34%
Other	108	28.20%	38	15.51%	20	31.75%	6	10.71%	6	8.96%	6	10.17%
Total	383		245		63		56		67		59	

	Population		**Posttest**		Bathtub1		CashFlow1		Bathtub2		CashFlow2	
Major	#	%	#	%	#	%	#	%	#	%	#	%
Ag	42	10.97%	17	9.71%	7	15.91%	3	6.98%	4	9.52%	3	6.52%
Bus	131	34.20%	60	34.29%	16	36.36%	15	34.88%	12	28.57%	17	36.96%
Sci/Tech	102	26.63%	48	27.43%	11	25.00%	16	37.21%	10	23.81%	11	23.91%
Other	108	28.20%	50	28.57%	10	22.73%	9	20.93%	16	38.10%	15	32.61%
Total	383		175		44		43		42		46	