Assessing the Effectiveness of E-learning via User Profile Analysis: An AHP-based Dynamic Programming Approach

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\textbf{ABSTRACT:} Electronic learning (E-learning) has been widely adopted as a promising tool by many organizations to offer learning-on-demand opportunities to individual employees (learners) in order to reduce training time and cost. While the success of information system (IS) models has much been investigated by researchers, little work has been conducted to assess the success and/or effectiveness of E-learning systems. The development of information technologies has contributed to the growth of on-line training as an important educational method. The on-line training environment enables learners to undertake customized training at any time and any place. Moreover, information technology allows both the trainers and learners to be decoupled in terms of time, place, and space. Here, we propose an assessment procedure by applying a dynamic programming approach to model the problem of shortest path in the user profile and using AHP (Analytical Hierarchy Process) to turn the qualitative parameters into quantitative values. A dynamic program is used to find the optimal path for the user in the E-learning environment. The validity and effectiveness of the proposed model are illustrated by two examples.


1. Introduction

Modern societies have dramatically changed due to technological changes such as the development of information technology systems. Service industries have become knowledge oriented, production economies have become knowledge economies and production workers have become knowledge workers. Learners need to be flexible and adaptive if they are to function well in today’s complex and global societies.

As part of the larger drive to change the curriculum, assessment needs to be reformed as well. Biggs’ (1996) idea of constructive alignment amongst instruction, learning and assessment implies that these three elements should be based on the same underlying principles. The new assessment methods are not without problems either and some feel that the evidence against traditional tests is not as strong as has been claimed (Hambleton
and Murphy, 1992), and that the claim that newer forms of assessment are better suitable to address learners’ requirements still needs empirical confirmation (Stokking et al., 2004). Studies have shown that no greater impulse for learning exists than assessment (Frederiksen, 1984) and that a strong relationship exists between learning and assessment, implying that what is assessed strongly influences what is learned (e.g., Alderson & Wall, 1993).

Though it is clear that assessment needs re-thinking, but it is not clear what requirements should be used for these new assessments. This is an important question to address, as the quality of assessment is increasingly being regarded as a very important element of the quality of education as a whole. Assessments in education may require new and other quality criteria to evaluate them. These criteria need to be more compatible with the principles and ideas of education (Fazlollahtabar & Sharma, 2008).

Internet has significantly impacted the establishment of Internet-based education, or E-learning. Internet technology evolution has affected all industrial and commercial activities and accelerated E-learning industry growth. It has also fostered the collaboration of education and Internet technology by increasing the volume and speed of information transfer and simplifying knowledge management and exchange tasks. E-learning could become an alternative way to deliver on-the-job training for many companies, saving money, employee transportation time, and other expenditures. An E-learning platform is an emerging tool for corporate training. Employees can acquire competences and problem solving abilities via Internet learning for benefits among business enterprises, employees, and societies while at work (Fazlollahtabar & Yousefpoor, 2009).

Self-regulation requires adequate monitoring strategies and meta-cognitive skills. The created E-learning environments should encourage the application of learners’ meta-cognitive skills by prompting learners to plan, attend to relevant content, and monitor and evaluate their learnings (Fazlollahtabar & Mahdavi, 2009).

Although E-learning has been developing for several years, evaluating E-learning effectiveness is critical as to whether companies will adopt E-learning systems. A considerable number of studies have been conducted emphasizing the factors to be considered for effectiveness evaluation. Several evaluation models are considered with specific aspects. The criteria used for E-learning effectiveness evaluation are numerous and influence one another (Mahdavi, Fazlollahtabar & Yousefpoor, 2008).

User modeling is a fundamental mechanism to achieve individualized interaction between computer systems and humans (Paiva, 1995). It is usually concerned with modeling several user related issues such as goals, plans, preferences, attitudes, knowledge or beliefs. The most difficult task in this context is the process of interpreting the information gathered during interaction in order to generate hypotheses about users.
and students behavior (Paiva, 1995), and involves managing a good deal of uncertainty. Interactive computer systems deal in general with more meager and haphazardly collected users’ data than it usually happens when humans are engaged in a face-to-face interaction (Jameson, 1996). Thus, the gap between the nature of the available evidence and the conclusions that are to be drawn is often much greater (Jameson, 1996). Numerical techniques have been employed in several cases in order to manage uncertainty (Conati, Gertner & Vanlehn, 2002; Herzog, 1994), and neural networks have been used in order to add learning and generalization abilities in user models and draw conclusions from existing user profiles (Chen, Norcio & Wang, 2000; Harp, Samad & Villano, 1995; Stathacopoulou, Magoulas & Grigoriadou, 1999; Yasdi, 2000). Here, we present a dynamic programming approach to identify the shortest path in virtual learning environments, and apply an AHP to change the qualitative parameters to quantitative values.

Virtual Learning Environments (VLEs) are computer-based environments that are relatively open systems, enabling interactions and encounters with other people and providing access to a wide range of resources (Wooldridge, 1999). VLEs can supplement face-to-face teaching methods, or totally replace the teaching methods in the case of distance learning. VLEs offer a number of advantages over traditional teaching environments in terms of convenience and flexibility (Carrillo, 2004). There are no geographical boundary limitations for using VLEs. They are capable of reaching potential learners in remote areas around the world at a very low cost. For these reasons, VLE is becoming one of the fastest growing areas in educational technology research and development. Many traditional colleges and universities, individually or in various forms of partnerships, are embracing information technologies to create new learning models that enhance the effectiveness and reach of their programs (Alavi & Leidner, 2001).

Researchers and developers are making rapid improvements in the design and implementation of VLEs, resulting in continuous progress toward successful VLEs. However, on-line learning is not always effective and sometimes fails to meet learning objectives because of the following limitations:

1. **Unstructured learning materials.** On-line learning materials are usually unstructured across different media, without any close associations with the E-learning processes (Zhang & Nunamaker, 2003). Learning material is distributed without consideration for learners’ capacities and prior learnings, and therefore lacks contextual and adaptive supports (Hiltz & Turoff, 2002).

2. **Insufficient flexibility.** In many current VLEs, the material contents and choices have been predefined, regardless of the learning process and learners’ differences. On-line learners have little flexibility to adapt the learning content and process to meet their individual needs (Alavi & Leidner, 2001; Hiltz & Turoff, 2002).
Insufficient interactivity. Studying on-line, by its nature, requires on-line learners to be more actively engaged and interact with their VLEs (Hiltz & Turoff, 2002). However, some current VLEs are not very interactive. There is less opportunity for receiving instant responses and feedbacks from the instructor or VLEs when on-line learners need support.

Nowadays, there is a growing trend of web-based technology applied for distance education. Particularly, web-based educational systems have many advantages because they can adapt the courses to specific needs. Different types of computer based educational systems are proposed as follows:

**Type 1:** Stand-alone learning system, in which direct linking is established with the virtual educational system without the presence of teacher or any collaboration.

**Type 2:** Remote teaching system, in which everyone is conducted with the aid of individual connections between a teacher and multiple students providing a method of teaching more students during a lecture.

**Type 3:** Group learning session, where multiple connections between participants are arranged, and thus a level of support for co-operative work is offered.

The advantages of the stated systems for the students are to experience and to access education and training courses that otherwise wouldn’t have been taken, and besides to participate in a distributed learning environment which they feel to be stronger than the traditional, and also to have the opportunity to discuss their own professional situations with other learners and with their colleagues. From the teacher viewpoint, it is important to experience teaching in a virtual university in order to gain a specialized understanding and to offer the possibility of learning to those learners who would not find it possible to participate in a traditional face to face learning experience. An architecture for a virtual learning environment is presented in Figure 1.

2. Adaptive learning systems

Here, we assume a decentralized paradigm of instruction where educational resources and services are made available beyond the boundaries of any single institution or discipline. Information technology can play the pivotal role of making resources more malleable and reusable, linking instructors and learners frequently and intensely, while adjusting to their requirements in novel ways, and compensating for limitations imposed by time and geography. Ultimately, information technology tools resulting from this program must foster a demand-oriented market for instruction (Mahdavi et al., 2008). If this occurs, the market for educational technologies can be expected to expand far beyond
the current bounds. Learners in particular and the economy as a whole will benefit greatly. The term *adaptive learning system* best depicts the role that technology can play in correcting problems that have stymied educational technology markets in the past (Specht, 1998). This program is designed to facilitate the development of technology that will adapt knowledge to the needs of learners. Experience proves that the best way to avoid a mismatch between supply and demand is to deploy technology in response to clearly articulated needs. Unlike any other tool applied to instruction in the past, many of the latest information technologies can comprise the platform of a future *learning economy* where learning experiences are pulled by demand rather than driven by any supply. Thus, if deployed within an open-market paradigm, information technologies can have positive impacts on the accessibility, affordability, and quality challenges that now confront education and training. The term *adaptive* also relates to the flexibility and scalability of the envisioned learning system. In the future, content and courseware must be reusable, interoperable and easily organized at many different levels of complexity throughout the on-line instructional environment. Tools for developing instructional content and courseware will need to operate across different platforms and communicate with other tools used to build and manage learning systems (Mahdavi et al., 2008). The systems themselves must accommodate numerous and varied learner requirements, needs and objectives. The needs of instructors and instructing organizations must also be addressed. Achieving this level of adaptability will require advances in a wide range of technologies that support diverse training and educational tasks. They include but are not limited to authoring systems, multi-sensory interfaces, search technology, and network middleware. Continued progress will also be required in software reusability and interoperability, specially for high-bandwidth applications. Some of the advantages of the adaptive learning systems are presented in Table 1.
Key technical challenges that now confront net-centric, web-based learning systems fall into four categories: content, delivery, search, and quality of service (using technologies such as tele-presence, virtual reality, and multi-sensory interfaces). These comprise the scope of the adaptive learning systems and are represented in Figure 2, illustrating the envisioned system. The four research thrusts constitute only a portion of the research challenges now at hand in on-line learning technologies:

- **Content.** Research will concentrate on efficient production of instructional components, interoperability and reusability of components, as well as the wrapping, retrofitting, and transformation of legacy data to fit training purposes.

- **Modes of Delivery.** Research will focus on technical solutions for synchronous and asynchronous collaboration, as well as providing full access to and mobility of data in a complex system of distributed repositories. Innovation may address

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**Table 1** Advantages of Adaptive Learning System

- To make high-quality instructional content more affordable.
- To adapt instruction to the end-user requirements of both learners and educators.
- To make high-quality instructional content more available.
- To increase the capacity and usability of instructional content production technology.
- To improve upon the precision and context-sensitivity of search and retrieval technology.
- To improve upon quality of service levels in distributed instructional systems.

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**Figure 2** Basic Components of Adaptive Learning Systems

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needs arising from new architectures and linking structures for repositories and may focus on one or more layers of metadata, i.e., new technologies in data labeling, authentication, and filtering.

- **Search and Retrieval.** Research will concentrate on the infrastructure to support educators, students, and workers with complex information acquisitions and management requirements. Search and retrieval systems should be highly interactive and must extend the current limits of artificial intelligence and expert systems in order to produce knowledge outputs that are precisely tailored to end-user needs and constraints.

- **Quality of Service.** Research will focus on middleware applications to ensure low latency and robustness for high-performance applications (synchronous and asynchronous interactions or simulated environments) in the midst of scale-up and extension. In addition, certain transactions specific to education and training will be researched, some of which may pose the need for special components or large-scale modules (i.e., translation, evaluation, accreditation, collaboration around learning tasks, and education transaction processing) and others of which present systems integration challenges.

Currently, E-learning is based on complex virtual collaborative environments where the learners can interact with other learners and with the tutors or the teacher. It is possible to provide different synchronous and asynchronous services for the learners. The synchronous services include virtual classrooms and individual sessions with the teachers or tutors. The asynchronous services include the classic didactic materials as well as web-based seminars or simulations always on-line. These functions can be usually accessed by the means of software platforms called Learning Management Systems (LMSs). Among the other functions, the LMS manages learners, keeping track of their progress and performance across all types of training activities. It also manages tasks and allocates learning resources such as registration, classroom and instructor availability, monitors instructional material fulfillment, and provides the on-line delivery of learning resources (Tajdin et al., 2008).

The adoption of Course Management Systems (CMSs) for web-based instruction continues to increase in today’s higher education. A CMS is a software program or integrated platform containing a series of web-based tools to support a number of activities and course management procedures (Severson, 2004). Examples of CMSs are Blackboard, e-WebCT, e-College, Moodle, Desire2Learn, Angel, etc. A supportive argument for the adoption of E-learning environments using CMSs is the flexibility of such environments when reaching out to potential learners in remote areas where brick and mortar institutions are non-existent. It is also believed that E-learning environments can have potential added learning benefits and can improve students’ and educators’ self-regulation skills, and in
particular their meta-cognitive skills (Akkerman et al., 2007). In spite of this potential to improve learning by means of using a CMS for the delivery of E-learning, the features and functionalities built into these systems are often underutilized. As a consequence, the created learning environments in CMSs do not adequately scaffold learners to improve their self-regulation skills. In order to support the improvement of both the learners’ subject matter knowledge and learning strategy application, the E-learning environments within CMSs should be designed to address learners’ diversity in terms of learning styles, prior knowledge, cultures, and self-regulation skills. Self-regulative learners are learners who can demonstrate ‘personal initiative, perseverance and adaptive skill in pursuing learning’ (Zimmerman, 2002).

The goal of adaptive presentation is to adapt the content of a hypermedia page to the user’s goals, knowledge, and other information stored in the user model. In a system with adaptive presentation, the pages are not static but adaptively generated or assembled from different pieces for each user. For example, with several adaptive presentation techniques, expert users may receive more detailed and deep information, while novices receive additional explanations.

The goal of curriculum sequencing (also referred to as instructional planning technology) is to provide the student with the most suitable, individually planned sequence of knowledge units to learn and sequence of learning tasks (examples, questions, problems, etc.) to work with. In other words, it helps the student to find an “optimal path” through the learning material (Kay, 1995).

The goal of an adaptive navigation support is to support the student in hyperspace orientation and navigation by changing the appearance of visible links. In particular, the system can adaptively sort, annotate, or partly hide the links of the current page to simplify the choice of the next link. Adaptive navigation support can be considered as an extension of curriculum sequencing technology into a hypermedia context. It shares the same goal -- to help students to find an “optimal path” through the learning material. At the same time, an adaptive navigation support has fewer directives than traditional sequencing: It guides students implicitly and leaves them with the choice of the next knowledge item to be learned and next problem to be solved.

Intelligent analysis of student solutions deals with students’ final answers to educational problems (which can range from a simple question to a complex programming problem), no matter how these answers are obtained. Unlike non-intelligent checkers, which can only detect whether an answer is correct, intelligent analysers can tell exactly what is wrong or incomplete and which missing or incorrect piece of knowledge may be responsible for the mistake. Intelligent analysers can provide the student with extensive feedbacks for a mistake.
The goal of the interactive problem solving support is to provide the student with intelligent help on each step of problem solving -- from giving a hint to proceeding with the next step for the student. The systems implementing this technology can monitor the actions of the student, understand them, and use the understanding to provide help.

3. The proposed model

Here, we propose an adaptive learning system based on an individual student’s profile. The aim is to choose the best path associated with student’s interests and qualitative attributes such as capability, attitude, knowledge level, motivation, and learning style. Of course, other criteria could also be added based on the decision maker’s choice. In this system, a user is encountered with some service providers that support VLE. Each service provider submits courses that are being taught by some instructors. The layers are identified in a network as Figure 3. Initially, a user should choose a service provider which suffices his preferences. Thus, the user systematically weights the service providers by measuring each service provider using the stated criterion. This way, the qualitative knowledge of the user is turned into numerical values and recorded in the learning management system. The same process is repeated for selecting courses and instructors. Finally, the user finds his optimal path applying the aforementioned assessment procedure. At the final stage, his path is recorded in the data base and then a next user is allowed to enter the system and choose his learning items. For each user the system refreshes itself and records the results in the data base of the LMS.
The selection of the user is based on the qualitative criterion on each arc. To apply dynamic programming, we need quantitative values for the arcs. To achieve this, we have to transform the qualitative criteria to numerical values based on the user preferences. A suggested arc preferences associated with their numerical values are given in Table 2.

<table>
<thead>
<tr>
<th>Preferences</th>
<th>Numerical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extremely Preferred</td>
<td>9</td>
</tr>
<tr>
<td>Very Strongly Preferred</td>
<td>7</td>
</tr>
<tr>
<td>Strongly Preferred</td>
<td>5</td>
</tr>
<tr>
<td>Moderately Preferred</td>
<td>3</td>
</tr>
<tr>
<td>Equally Preferred</td>
<td>1</td>
</tr>
<tr>
<td>Preferences among the Above Preferences</td>
<td>2, 4, 6, 8</td>
</tr>
</tbody>
</table>

3.1 A dynamic programming approach for optimal path

Dynamic programming (DP) was introduced by Bellman (1957). Toth (1980) presented the early DP-based approaches and reported numerical experiments with a limited success. More recently, Pisinger proposed a DP algorithm, which constructing a core problem of a minimal size, to minimize the sorting and reduction efforts. Hybrid methods, combining dynamic programming and implicit enumeration, were developed. The first approach was developed by Plateau and Elkihel (1985). A recent approach, the so-called combo algorithm, is able to solve very large instances of up to 10,000 variables within less than one second, with basically no difference in the required solution times for “easy” and “hard” instances (Martello, Pisinger & Toth, 2000). Marsten and Morin (1978) proposed the first hybrid method, which combines heuristic algorithms, dynamic programming and branch-and-bound approaches. More sophisticated methods can be found in Ibaraki (1987).

Dynamic programming is a technique to tackle multistage decision processes. A given problem is subdivided into smaller subproblems, which are sequentially solved until the initial problem is solved by the aggregation of the subproblem solutions. In each stage, a set of states is defined. The states would describe all possible conditions of the process in the current decision stage, which corresponds to every feasible partial solution. The set of all possible states is known as the state space. The states of a stage \( u \) can be transformed to states of a stage \( u + 1 \) by using a transition. A transition indicates the decisions adopted in a stage, and a sequence of transitions taken to reach a state starting from another state is known as a policy. Dynamic programming approaches can be seen as transformations
of the original problem to one associated with the exploration of a multistage graph $G(S, T)$, where the vertices in $S$ correspond to the state space and the arcs in $T$ to the set of transitions, leading to an optimal policy.

The basis of dynamic programming can be traced to the optimality principle of Bellman (2003). The optimality principle states that an optimal policy should be constituted by optimal policies from every state of the decision chain to the final state. Here, we make use of a dynamic programming approach to our proposed network to identify the optimal path for the user. This model helps the users to determine their curriculum profiles in an adaptive learning system. The advantages of such a model are simplicity, the ability to determine the exact optimal value, and implementability on more detailed networks. The dynamic model would be defined as:

Indices:

- $i$ Number of layers $i = 0,1,2,3$
- $j'$ Start node number corresponding to layer $j' = 1,2, \ldots, H$ (in layer 1); $1,2, \ldots, K$ (in layer 2); $j' = 0$ (for the start node)
- $j$ End node number corresponding to layer $j = 1,2, \ldots, H$ (in layer 1); $1,2, \ldots, K$ (in layer 2); $1,2, \ldots, L$ (in layer 3)
- $H$ Number of service providers
- $K$ Number of courses
- $L$ Number of instructors

Notations:

- $S_i(j')$ The maximum value of moving from node $j'$ in layer $i$ to an end node in layer 3
- $F_{j'j}$ Numerical value of an arc from node $j'$ to node $j$

Objective function: Optimal path

$$S_i(j') = \max_{j \text{ in layer } i+1} \{S_{i+1}(j) + F_{j'j}\}, \quad i = 0,1,2, \forall j' \text{ in layer } i,$$

$$S_0(j) = 0, \quad j = 1, \ldots, L,$$

$$S^* = S_0(0).$$
Note that $S^*$ identifies the optimal path. An approach is needed to turn qualitative criteria into numerical values. We propose the approach of Analytical Hierarchy Process (AHP) as illustrated in the next section.

### 3.2 Determining weights by AHP

The analytical hierarchy process (AHP) was proposed by Saaty (1980). AHP was originally applied to uncertain decision problems with multiple criteria, and has been widely used in solving problems of ranking, selection, evaluation, optimization, and prediction decisions. Harker and Vargas (1987) stated that “AHP is a comprehensive framework designed to cope with the intuitive, rational, and the irrational when we make multi-objective, multi-criteria, and multi-factor decisions with and without certainty for any number of alternatives.”

The AHP is expressed by a unidirectional hierarchical relationship amongst decision levels. The top element of the hierarchy is the overall goal for the decision model. The hierarchy decomposes to a more specific criterion on a level and each criterion may be related to some subcriteria (Meade & Presley, 2002). The AHP separates complex decision problems into elements within a simplified hierarchical system (Shee, Tzeng & Tang, 2003).

Our enquiry in the AHP has the purpose to construct a hierarchical evaluation system based on the independent factors as capabilities, attitudes, knowledge level, motivation and learning style. The AHP could gain factor weights and criteria, and then obtain the final effectiveness of each arc. The AHP usually consists of three stages of problem solving: decomposition, comparative judgments, and synthesis of priority. The decomposition stage aims at the construction of a hierarchical network to represent a decision problem, with the top level representing the overall objectives and the lower levels representing the criteria, subcriteria, and alternatives. With comparative judgments, users are requested to set up a comparison matrix at each hierarchy by comparing pairs of criteria or subcriteria. A scale of values ranging from 1 (Equally Preferred) to 9 (Extremely Preferred) as given in Table 2, is used to express the users preferences. Finally, in the synthesis of priority stage, each comparison matrix is then solved by an eigenvector method for determining the importance of the criteria and alternative performance.

One major advantage of AHP is its applicability to the problems of group decision-making. In a group decision setting, each participant is required to set up the preference of each alternative by the AHP and the collective views of the participants are used to obtain an average weighting of each alternative.

Here, for the stated criteria, the following hierarchy is proposed. The aim is to obtain the numerical value for each arc to be used in the objective function of shortest
path. The hierarchy is presented in Figure 4 (where $N = H$ or $K$ or $L$, respectively for the corresponding layer).

![Figure 4](image)

**Figure 4** The Hierarchy of the Proposed Model

For each layer (1, 2, or 3) in Figure 3, a corresponding pairwise comparison table is used to calculate the numerical value ratio of arcs determined by capabilities, attitudes, knowledge levels, motivations and learning styles of a user based on their preference numbers ($A_{bc}$) as stated in Table 2 ($N$ is $H$ or $K$ or $L$ depending on the layer).

<table>
<thead>
<tr>
<th>Capability</th>
<th>Arc 1</th>
<th>Arc 2</th>
<th>.</th>
<th>Arc $H$, $K$, $L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arc 1</td>
<td>1</td>
<td>$A_{12}$</td>
<td>.</td>
<td>$A_{1N}$</td>
</tr>
<tr>
<td>Arc 2</td>
<td>$1/A_{12}$</td>
<td>1</td>
<td>.</td>
<td>$A_{2N}$</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>Arc $N$</td>
<td>$A_{N1} = 1/A_{1N}$</td>
<td>$A_{N2} = 1/A_{2N}$</td>
<td>.</td>
<td>1</td>
</tr>
</tbody>
</table>

The same table is used for each one of the other criteria (capability, attitude, knowledge level, motivation and learning style). After calculating the above tables, a table that indicates the weights ($W_{bc}$) of the arc for the considered criteria is formed as Table 4.
Table 4  The Weight of the Arcs

<table>
<thead>
<tr>
<th>Arc</th>
<th>Capability</th>
<th>Attitude</th>
<th>Knowledge level</th>
<th>Motivation</th>
<th>Learning style</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arc 1</td>
<td>$W_{11}$</td>
<td>$W_{12}$</td>
<td>$W_{13}$</td>
<td>$W_{14}$</td>
<td>$W_{15}$</td>
</tr>
<tr>
<td>Arc 2</td>
<td>$W_{21}$</td>
<td>$W_{22}$</td>
<td>$W_{23}$</td>
<td>$W_{24}$</td>
<td>$W_{25}$</td>
</tr>
<tr>
<td>. . . . . .</td>
<td>. . . . . .</td>
<td>. . . . . .</td>
<td>. . . . . .</td>
<td>. . . . . .</td>
<td>. . . . . .</td>
</tr>
<tr>
<td>Arc N</td>
<td>$W_{N1}$</td>
<td>$W_{N2}$</td>
<td>$W_{N3}$</td>
<td>$W_{N4}$</td>
<td>$W_{N5}$</td>
</tr>
</tbody>
</table>

Next, the criteria pair-wise comparison matrix is configured in Table 5.

Table 5  The Criteria Pairwise Comparison

<table>
<thead>
<tr>
<th>Capability</th>
<th>Attitude</th>
<th>Knowledge level</th>
<th>Motivation</th>
<th>Learning style</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capability 1</td>
<td>$A_{12}$</td>
<td>$A_{13}$</td>
<td>$A_{14}$</td>
<td>$A_{15}$</td>
</tr>
<tr>
<td>Attitude</td>
<td>$1/A_{12}$</td>
<td>1</td>
<td>$A_{23}$</td>
<td>$A_{24}$</td>
</tr>
<tr>
<td>Knowledge level</td>
<td>$1/A_{13}$</td>
<td>$1/A_{23}$</td>
<td>1</td>
<td>$A_{34}$</td>
</tr>
<tr>
<td>Motivation</td>
<td>$1/A_{14}$</td>
<td>$1/A_{24}$</td>
<td>$1/A_{34}$</td>
<td>1</td>
</tr>
<tr>
<td>Learning style</td>
<td>$1/A_{15}$</td>
<td>$1/A_{25}$</td>
<td>$1/A_{35}$</td>
<td>$1/A_{45}$</td>
</tr>
</tbody>
</table>

At this point, the weight of each criterion is at hand using the above matrix. Therefore, the weight for each arc considering the criteria is determined as follows ($N$ is $H$ or $K$ or $L$ depending on the layer):

Total weight for arc 1 = \( W_{11} \times W_C + W_{12} \times W_A + W_{13} \times W_K + W_{14} \times W_M + W_{15} \times W_L \)

Total weight for arc 2 = \( W_{21} \times W_C + W_{22} \times W_A + W_{23} \times W_K + W_{24} \times W_M + W_{25} \times W_L \)

. . . .

Total weight for arc $N$ = \( W_{N1} \times W_C + W_{N2} \times W_A + W_{N3} \times W_K + W_{N4} \times W_M + W_{N5} \times W_L \)

where $W_C$ = capability’s weight, $W_A$ = attitude’s weight, $W_K$ = knowledge level’s weight, $W_M$ = motivation’s weight, $W_L$ = learning style’s weight, that are obtained by Table 5. This way, the weights of the arcs are calculated. These weights are used as the numerical values for the proposed network arcs. As a result, the optimal path for the user will be determined after solving the proposed dynamic programming problem. In Section 4 below, we provide examples for illustration of the proposed model.
4. Numerical examples

To illustrate the effectiveness of the proposed model, two comprehensive examples are described. In the following examples, the number of service providers, courses, and instructors are determined for layers. We apply the LINGO software package for the necessary computations. The encoding of LINGO is illustrated next.

4.1 LINGO encoding

The following is the LINGO encoding for our proposed model in a virtual learning environment.

MODEL:
SETS:
Nodes /1..j/: P;
Arc (Nodes, Nodes)/
1,2 1,3 ... 1,j
2,5 ... 2,j
j-1,j`/: Pj;
ENDSETS
DATA:
Pj =
a ... b
z;
ENDDATA
P (@SIZE( Nodes)) = 0;
@FOR (Nodes (j)| j #LT# @SIZE (Nodes):
P(j) = @Max( Arcs( j, j'): D( j, j') + P(j)));
END

In the above encoding, $P$ is the numerical value of the optimal path. Two comprehensive examples have been considered for the testing.

4.2 Example 1

Here, the number of service providers is 3, the number of offered courses is 3, and the number of instructors for the courses is 2. Initially, based on the user preferences the numerical value for each arc is determined by the AHP. The arcs and their related numerical values are shown in Table 3. Note that the value of each arc $i, j$ (from node $i$ to node $j$) is set after determination of the weights in the AHP using the stated attributes. Then, the data are input to the LINGO encoding and the numerical value for each node is obtained. The results are shown in Tables 6 and 7.
Table 6  Arcs and Their Related Values

<table>
<thead>
<tr>
<th>Arc</th>
<th>Value</th>
<th>Arc</th>
<th>Value</th>
<th>Arc</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,1</td>
<td>1</td>
<td>0,2</td>
<td>5</td>
<td>0,3</td>
<td>2</td>
</tr>
<tr>
<td>1,4</td>
<td>13</td>
<td>1,5</td>
<td>12</td>
<td>1,6</td>
<td>11</td>
</tr>
<tr>
<td>2,4</td>
<td>6</td>
<td>2,5</td>
<td>10</td>
<td>2,6</td>
<td>4</td>
</tr>
<tr>
<td>3,4</td>
<td>12</td>
<td>3,5</td>
<td>14</td>
<td>3,6</td>
<td>10</td>
</tr>
<tr>
<td>4,7</td>
<td>3</td>
<td>4,8</td>
<td>9</td>
<td>5,7</td>
<td>6</td>
</tr>
<tr>
<td>5,7</td>
<td>6</td>
<td>5,8</td>
<td>5</td>
<td>6,7</td>
<td>8</td>
</tr>
<tr>
<td>7,9</td>
<td>5</td>
<td>8,9</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7  Variables and Their Related Values

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>P (0)</td>
<td>27</td>
</tr>
<tr>
<td>P (1)</td>
<td>24</td>
</tr>
<tr>
<td>P (2)</td>
<td>21</td>
</tr>
<tr>
<td>P (3)</td>
<td>25</td>
</tr>
<tr>
<td>P (4)</td>
<td>11</td>
</tr>
<tr>
<td>P (5)</td>
<td>11</td>
</tr>
<tr>
<td>P (6)</td>
<td>13</td>
</tr>
<tr>
<td>P (7)</td>
<td>5</td>
</tr>
<tr>
<td>P (8)</td>
<td>2</td>
</tr>
<tr>
<td>P (9)</td>
<td>0</td>
</tr>
</tbody>
</table>

Consequently the optimal path for a user based on his preferences is identified. As shown in Figure 5, the optimal path is 0-3-6-7-9, i.e., the set comprised of the third service provider, the third course, and the first instructor is determined to be optimal.

4.3 Example 2

Here, the number of service providers is 5, the number of offered courses is 5, and the number of instructors for the courses is 5. Initially, based on the user preferences the numerical value for each arc is determined by the AHP. The arcs and their related numerical values are shown in Table 8. Then, the data are input to the LINGO encoding and the numerical value for each node is obtained. The results are shown in Table 9.
Consequently, the optimal path for a user based on his preferences is identified. As shown in Figure 6, the optimal path is 0-3-9-12-16, i.e., the set comprised of the third service provider, the fourth course, and the second instructor is determined to be optimal.

5. Conclusions

We presented a model to find the optimal learning path for a user in an adaptive learning system. Considering the network as a model, a dynamic programming approach
### Table 9  Variables and Their Related Values

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(0)$</td>
<td>27.1</td>
</tr>
<tr>
<td>$P(1)$</td>
<td>19.7</td>
</tr>
<tr>
<td>$P(2)$</td>
<td>19.3</td>
</tr>
<tr>
<td>$P(3)$</td>
<td>24.4</td>
</tr>
<tr>
<td>$P(4)$</td>
<td>21.9</td>
</tr>
<tr>
<td>$P(5)$</td>
<td>22.2</td>
</tr>
<tr>
<td>$P(6)$</td>
<td>13.6</td>
</tr>
<tr>
<td>$P(7)$</td>
<td>13.6</td>
</tr>
<tr>
<td>$P(8)$</td>
<td>13.2</td>
</tr>
<tr>
<td>$P(9)$</td>
<td>15.6</td>
</tr>
<tr>
<td>$P(10)$</td>
<td>14.9</td>
</tr>
<tr>
<td>$P(11)$</td>
<td>7</td>
</tr>
<tr>
<td>$P(12)$</td>
<td>8</td>
</tr>
<tr>
<td>$P(13)$</td>
<td>4.2</td>
</tr>
<tr>
<td>$P(14)$</td>
<td>5</td>
</tr>
<tr>
<td>$P(15)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$P(16)$</td>
<td>0</td>
</tr>
</tbody>
</table>

**Figure 6**  Optimal Path for Example 2
is applied. To allow for assessing quality in the educational system, an environment providing several qualitative criteria for the user is designed. The system is capable to help the user in identification of the optimal path. Using the AHP, the qualitative criteria of a user for selecting the arcs are turned into numerical values. User can select from amongst the provided facilities in accordance with his preferences. Thus, a user is able to reach both a better quality and an optimal learning path simultaneously. The cleared path is mainly concerned with the personal profile of a user and therefore the user is the decision maker. The validity and effectiveness of the proposed model are illustrated by working out two comprehensive examples using a LINGO encoding for the computations.

References


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