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Max-Flow Min-Cost Algorithm for A Supply Chain Network

Shu-Yi Chen[1)], Ching-Chin Chern[2)]

[1)] Dept. of Information Management, National Taiwan University
(d4725005@im.ntu.edu.tw)
[2)] Dept. of Information Management, National Taiwan University
(chern@chern.im.ntu.edu.tw)

Abstract

The management of supply chain is getting more and more attention in a wired world demanding fast response and flexibility with excellent service quality. There are, as there should be, many studies done to aid companies with better knowledge in operating their supply chains. However, these studies are either too conceptual to be directly put into use, or too specific and complex to be applied to an individual case.

The objective of this study, hence, is to find the maximum flow with the minimum cost based on the given supply chain network and product tree structure. Given the original supply chain graph with its product tree graph, this study constructed an algorithm that first transforms the original graph of the supply chain into another form. During the transformation, all the information, such as the cost and capacity of each link, should be kept in the new form of the supply chain.

With the help of the minimum cost flows problems found in network flow models, this study than apply the max-flow min-cost algorithms to the transformed supply chain. In the algorithm developed herein, the capacity and cost relating to each link in the supply chain are taken into account to solve for the maximum possible output with the lowest cost possible in the supply chain.

Since this algorithm is general enough, hence it is suitable for a manager to use to assist his decision regarding maximum flows and minimum cost in a supply chain. Although it may only be a first step toward a more complete model for supply chain management problem, there are some findings worth discussing in the study. An example illustrating the algorithm is also presented and is followed by some managerial implications. A conclusion is also included at the end of this paper..

1. Introduction

In a fast changing world, it is more and more important for a company to respond as quickly as possible so as not to fall behind its competitors or lose customers. To become responsive, however, a company has to work not only on its own performance but also on the management of the supply chains of the products. Since a supply chain is, as the name implies, "a network of facilities that performs the functions of procurement of material, transformation of material to intermediate and finished products, and distribution of finished products to customers" [9, p.835], the management of a supply chain is by no means an easy job. For this reason, the management of supply chains becomes an important issue for any business in the highly globally competitive world today, as it does for any researcher interested in the business domains.

There are quite a few studies focusing on the management of supply chain. Generally speaking, studies can be grouped in two main categories. Studies in one of the two categories lend themselves to the clarifications of important concepts of supply chain management [3, 7, 8, 10], while studies in the other category serve as examples with models, experiences and problems encountered in real cases [2, 6, 9]. Conceptual-oriented research helps a lot in correcting wrong ideas or pitfalls [8], but provides few help when a company tries to improve its own supply chain. On the contrary, case-oriented research can light up the way for a company with similar situation and ready to build its own models, but if the cases were not properly generalized, they would be of little value.

Regardless of what the virtues these two kinds of studies may have, there is still a lack of a general mathematical model for any supply chain to apply. The problem of lacking a general model is more critical when most of the mid-sized or small-sized businesses would probably not have the budget to build their own mathematical models. Moreover, lacking a general model also makes it difficult for researchers to discuss and to discover certain problems in the management of supply chain.

This study tried to answer the above call with a general model of a supply chain, with the help of maximum flow problem seen in network flow models [1]. With the help of the model, a manager can find the maximum flow in his supply chain and makes the most use of the chain. The model is thus designed that with some modifications, it provides a fast and feasible, if not optimal, solution for any supply chain.

After this section, the paper presents the reviews of some important literatures in the following section. The formulation of the model is given in section 3, followed by an illustrative modeling example in section 4 with managerial implications. A conclusion summarizing the findings and the possible research directions appears in the last section.

2. Literature Review

Two categories are used to classify related studies, namely, conceptual-oriented and case-oriented studies. The differentiation line is drawn at how a study presents and generalizes its result. A study is said to be a conceptual-oriented one if the stress of the study is on the instructions of general concepts, while the study can result from a real world case. On the contrary, a case-oriented study focuses more on some special cases and the experiences learnt. Ambiguity exists as it should, but the distinction is still clear.

Lee and Billington [8], Billington [3] and Fisher [7] are examples of conceptual-oriented studies. In this sense, they all introduced many great concepts in managing the supply chain or the inventory in a supply chain. However, the lack of detail description of how to build one's own model may disappoint some managers.

For example, Lee and Billington [8] were quite descriptive with many illustrative examples to clarify their point of view. In the article, fourteen often seen pitfalls were discussed, including the basic misunderstanding and misusage of supply chain, the operational problems encountered, and some strategic issues. There were also some opportunities observed by them worth noticing. Fisher [7] provided a good grid to identify the fitness of the product of a company and its supply chain. While it is very helpful, a manager may want to know more about how to find some details in his own situation.

On the other hand, Arntzen et al. [2], Lee and Billington [9] and Cohen and Mallik [6] serve as good examples of case-oriented studies. Experiences were illustrated and can be learnt for those companies who have the capabilities or are ready to start building such models. However, medium to small companies may not benefit much from these studies, since small companies wouldn't afford to build complex mathematical models. Besides, the formulations may not apply to specific condition.

There are still researches trying to provide more general yet mathematical help. Chen and Chern [4, 5] have done some studies utilizing the network flow algorithms such as shortest path algorithm and maximum flow algorithm. In their studies, a general supply chain graph, with the graph of the production tree of the supply chain, is transformed into another form that is ready to apply the shortest path or the maximum flow algorithm. With each algorithm, their algorithms are suitable for easy solution about the lowest cost production path or the maximum amount of production in the supply chain. Hence each of their algorithms provides a simple yet mathematical way for supply chain managers to make decisions regarding one of the above issues. However, their model works for either lowest cost or maximum production output only, but it is usually necessary to consider both objectives in a supply chain decision. Therefore, it is possible to improve their studies by combining the two considerations.

According to the above reviews, it is obvious that there is a need for a general but mathematical model for taking into account of cost and production amount. This study, based on the results achieved by Chen and Chern [4, 5], tried to answer the call with the following algorithm that provides such a solution.

3. Model Formulation

The algorithm of the maximum production output with the lowest cost consideration in a supply chain works as follows. First the graph of the supply chain is transformed to be ready to apply any of the traditional minimum cost flow algorithms, such as those mentioned in [1]. After the maximum flow with the lowest cost "path," i.e. the lowest cost way to produce the final product, is found, the capacity of this path is also known. The most challenging part of this study, hence, is the part of transforming the original supply chain graph, which is not ready for the traditional minimum cost flow algorithm, into the form that is ready to directly apply the well-developed and efficient minimum cost flow algorithms. To apply the algorithm developed in this study, some assumptions, terms and notations have to be defined. Most of them are borrowed from Chen and Chern [4, 5].

For the time being, there can be only one kind of item shipped per arc. That is, although the items would be different on different arcs, it is thought of as the kind of single commodity problems. Moreover, a node in a supply chain can only have production function or distribution function or both. The definitions of production and distribution functions are given below along with other definitions and notations.

$G_y^x\left(N_y^x,\, A_y^x\right)$: A supply chain network or a product tree can be represented by a directed graph G_y^x, in which N_y^x and A_y^x are the set of all nodes and all arcs in G_y^x, respectively. Since the original supply chain graph needs to be converted before applying the maximum flow algorithm, superscript x is used to indicate the number of steps counted while converting the original graph, and subscript y denotes what G stands for. For example, G_{SC}^1 represents a supply chain network after being converted once and G_{PT}^0 a product tree in its original configuration. By definition, x is an integer and $x \geq 0$.

(i, j): The directed arc from node i to node j in A_y^x.

Production function: If a node performs some kind of processing work in a supply chain, it is said to have the production function. By definition, a node with production function transforms components it receives into a different component or components in the product tree item set, N_{PT}^0.

Production node: If a node performs production function only, and all components sent to this node are necessary in producing outward items, it is a production node. A production node can be further categorized as an internal or external production node. An internal production node not only uses the components it receives as materials for production, but also takes the items it produces again as materials to further produce other items. (All items mentioned here could be found in the product tree.) In other words, in the product tree hierarchy, an internal production node produces product items one or more levels higher than those product items it receives. On the other hand, an external production node produces items only one level higher (in the product tree) than what it receives. That is, an external production node uses only what it receives as materials to produce items without using what it produces as materials. A one-item production node is a production node that ships out exactly one kind of item.

Distribution function: If a node ships out what it receives without processing them, it is said to have the distribution function.

Distribution node: If a node performs distribution function only, it is a distribution node. A node is further called a one-item distribution node, if it is a distribution node and it ships out exactly one kind of item.

CO_i: The product component (item) produced/shipped by node i.

$\vec{MinCost}^x$: The maximum flow path vector with minimum cost generated in $G_y^x\left(N_y^x,\, A_y^x\right)$ from node s to t. $\vec{f}\left(\vec{MinCost}^x\right)$ is used to denote the amount of the flow of this path, and $\vec{c}\left(\vec{MinCost}^x\right)$ is the cost.

U^x: The capacity of the path $\vec{S}^x$ in $G_y^x\left(N_y^x,\, A_y^x\right)$.

c_{ij}^x: The cost incurred for i to process an item and/or deliver the item between (i, j) in $G_y^x\left(N_y^x,\, A_y^x\right)$. c_{ij}^x is thus defined that it includes only the added costs but not the material cost except for the raw material providers.

$n_{i,j}$: The number of CO_i needed to produce one CO_j.

$u_{i,j}^x$: The capacity limit of flow on (i, j) in G_y^x. The unit of $u_{i,j}^x$ is the number of CO_i.

Before modeling a supply chain, two extra virtual nodes and some arcs have to be added in the supply chain. Node s and t represents the source and sink node respectively. There should also be some arcs added to link s and t to other nodes. A node i without arcs linked into it, that is, a starting node without suppliers in a supply chain, should add *(s, i)* with $c_{si} = 0$ and $u_{s,i} = \infty$. A node j without arcs linked to other node, that is, an ending node without demand nodes in a supply chain, should add *(j, t)* with $c_{jt} = 0$ and $u_{j,t} = \infty$. $G_{SC}^0\left(N_{SC}^0, A_{SC}^0\right)$ is formed from the original configuration of the supply chain and these added nodes and arcs.

With the above definitions, a supply chain $G_{SC}^0\left(N_{SC}^0, A_{SC}^0\right)$ can be modified and solved by the algorithm for the maximum flow with the lowest cost consideration.

Step 1. Differentiate the production function from the one-item distribution function by splitting node i into sub-nodes. That is, separate the one-item distribution function, if any, performed by any node in $G_{SC}^0\left(N_{SC}^0,\, A_{SC}^0\right)$ with sub-nodes managing these one-item distribution functions. Add and move necessary arcs and set a new capacity limitation for each arc, at the same time record all the information about cost and capacity. Create $G_{SC}^1\left(N_{SC}^1, A_{SC}^1\right)$ after step 1.

Step 2. Make all production nodes become one-item external production nodes. That is to split all production nodes into one or more one-item external production nodes. Furthermore, a production node after the splitting should not receive the same component from different vendors. Add and move necessary arcs, and also calculate for the correct capacity limit for each arc. Create $G_{SC}^2\left(N_{SC}^2, A_{SC}^2\right)$ to denote the changes and include new sub-nodes and arcs added or moved after step 2. For every j in N_{SC}^1 that *(j, i)* is split into *(j, i_1)* to *(j, i_k)*, add a constraint that $\sum_k u_{j,i\_k}^2 = u_{j,i}^1$.

Step 3. Combine the arcs carried different items into a production node in G_{SC}^{2} to create G_{SC}^{3} which is suitable for the maximum flow algorithm, so that these arcs and nodes form a new arc *(Q, i)* and a new node *Q*. In which *Q* is the set of these combined vendor nodes. Begin from the original starting nodes in N_{SC}^{2}, i.e. nodes have no upstream arcs other than those to *s*, calculate for $u_{Q,i}^{3} = \min\left\{u_{j,i}^{2} / n_{j,i}\right\}$, in which *j* is any supplier node of *i*. And if $u_{i,k}^{2} > u_{Q,i}^{3}$, set $u_{i,k}^{3} = u_{Q,i}^{3}$. Also calculate for $c_{Q,i}^{3} = \sum_{j} c_{j,i}^{2}$. At last create $G_{SC}^{3}\left(N_{SC}^{3}, A_{SC}^{3}\right)$ for these changes.

Step 4. Apply any of the minimum cost flow algorithms and search to find $\overrightarrow{MinCost}^{3}$.

Step 5. Use $\overrightarrow{MinCost}^{3}$, $\overline{n}\left(\overrightarrow{MinCost}^{3}\right)$, $G_{SC}^{3}\left(N_{SC}^{3}, A_{SC}^{3}\right)$, $G_{SC}^{2}\left(N_{SC}^{2}, A_{SC}^{2}\right)$ and $G_{SC}^{1}\left(N_{SC}^{1}, A_{SC}^{1}\right)$ to expand $\overrightarrow{MinCost}^{3}$ and $\overline{n}\left(\overrightarrow{MinCost}^{3}\right)$ back into the path $\overrightarrow{MinCost}^{0}$ and $\overline{n}\left(\overrightarrow{MinCost}^{0}\right)$ along with the minimum cost in $G_{SC}^{0}\left(N_{SC}^{0}, A_{SC}^{0}\right)$.

4. Example and Managerial Implications

This section provides a simple example for all the steps in the previous section to show the feasibility of the formulation. There are also some managerial implications after the example.

An example supply chain $G_{SC}^{\upsilon}\left(N_{SC}^{\upsilon}, A_{SC}^{\upsilon}\right)$ of a company is shown in Fig. 1. The "CO_x: *a* / *b*" notation above or at the right side of each arc (*i*, *j*) are used to denote the component and its quantity shipped from node *i* to *j*, in which CO_x is the component shipped, and *a* is the capacity $u_{i,j}$ of each (*i*, *j*), and *b* is the largest possible quantity of the final product made from these CO_x in the product tree. Furthermore, the number under or at the left side of each arc is the c_{ij} of each (*i*, *j*). Moreover, the product tree $G_{PT}\left(N_{PT}, A_{PT}\right)$ of G_{SC}^{υ} is shown in Fig. 2, in which the number after the slash is $n_{i,j}$.

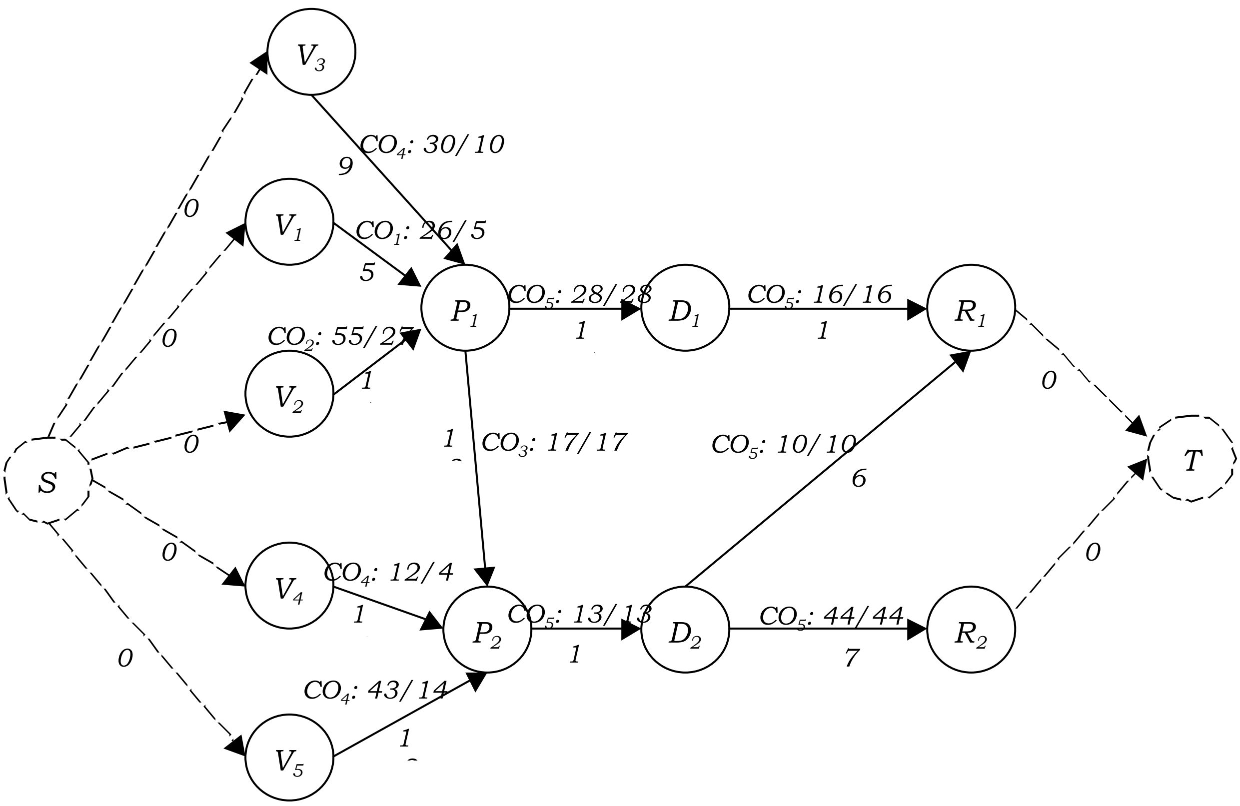

Fig. 1 $G_{SC}^{\upsilon}\left(N_{SC}^{\upsilon}, A_{SC}^{\upsilon}\right)$ **of a company**

By carrying out step 1 to step 2, $G_{SC}^{2}\left(N_{SC}^{2}, A_{SC}^{2}\right)$ is created as shown in Fig. 3. Dotted circles and arcs are sub-nodes or arcs being added or split. Since node P_2 is split into $P_{2\text{-}1}$ and $P_{2\text{-}2}$, u_{P_1,P_2}^{2} should also split to $u_{P_{1-1},P_{2-1}}^{3}$ and $u_{P_{1-1},P_{2-2}}^{3}$ that $u_{P_{1-1},P_{2-1}}^{3} + u_{P_{1-1},P_{2-2}}^{3} = u_{P_1,P_2}^{2}$. If it is assigned that $u_{P_{1-1},P_{2-1}}^{3} = x_1$, then $u_{P_{1-1},P_{2-2}}^{3} = u_{P_1,P_2}^{2} - x_1$. In this example, $u_{P_1,P_2}^{2} = 17$, so $u_{P_{1-1},P_{2-2}}^{3} = 17 - x_1$, in which $0 \le x_1 \le 17$. The same logic can be applied on u_{P_{2-1},D_2}^{3} and u_{P_{2-2},D_2}^{3}, and $0 \le x_2 \le 13$.

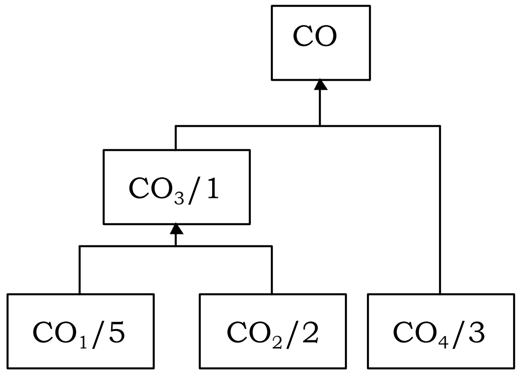

Fig. 2 $G_{PT}(N_{PT}, A_{PT})$ **of the supply chain in Fig. 4**

After step 3, some of the nodes are combined and $G^3_{SC}(N^3_{SC}, A^3_{SC})$ is now shown in Fig. 4. By executing step 4 and search for the maximum flow with lowest cost, $\breve{M}inCost^3$ is now found as $\breve{M}inCost^3 = \{S, V_1V_2, P_{1-1}V_4, P_{2-1}, D_2, R_1, T\}$, with $\tilde{f}(\breve{M}inCost^3)$ = 5 and $\breve{c}(\breve{M}inCost^3)$ = 62. Backtrack from $\breve{M}inCost^3$, $\breve{M}inCost^0$ is shown in Fig. 5 with bold arcs.

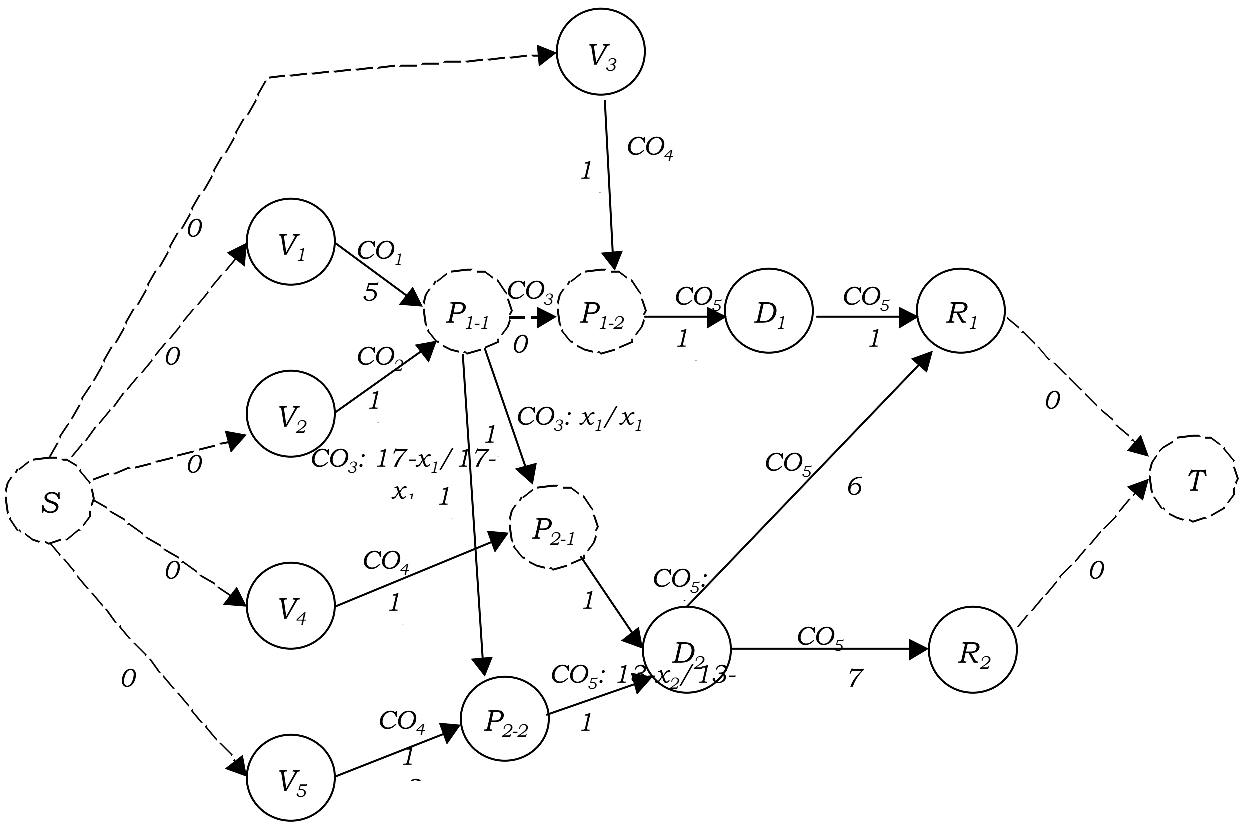

Fig. 3 $G^2_{SC}(N^2_{SC}, A^2_{SC})$ **after step 3**

The solution found here can be used as a way to find the lowest cost of manufacturing the final product from the supply chain without performing complex optimization calculations. A manager can formulate the supply chain with the algorithm established here, so that he can find the low cost choice to produce the products needed.

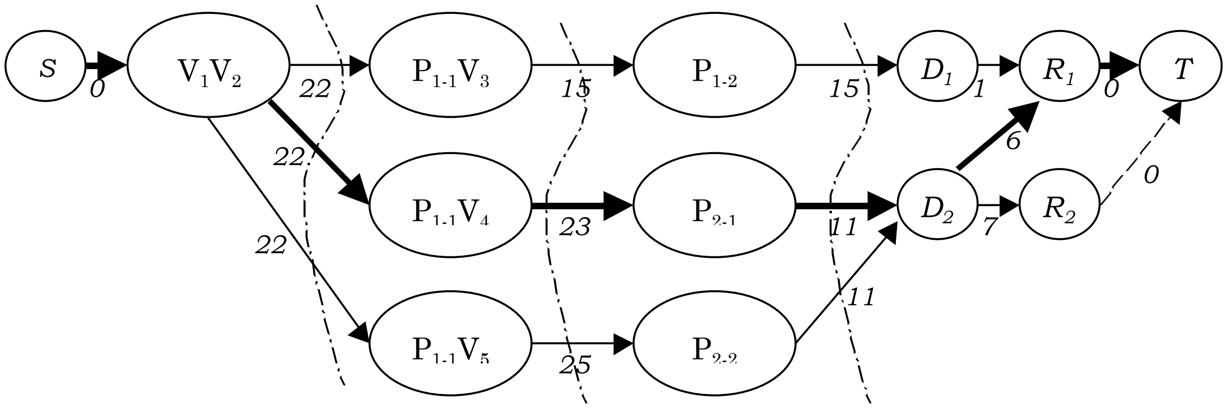

Figure 4 $G_{SC}^4\left(N_{SC}^4, A_{SC}^4\right)$ **and** $\overset{\omega}{M}inCost^3$ **(in bold arc) after step 4**

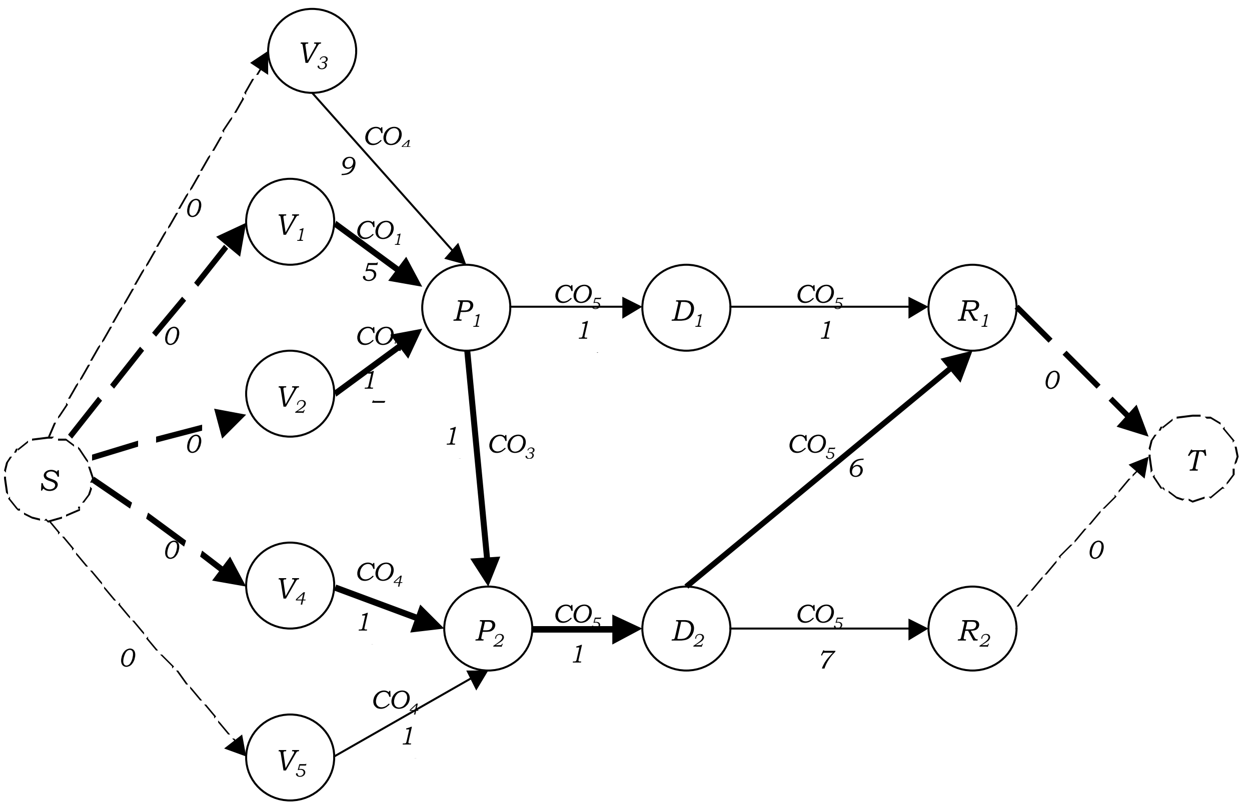

Figure 5 $\overset{\omega}{M}inCost^0$ **in** $G_{SC}^0\left(N_{SC}^0, A_{SC}^0\right)$

The max flow-min cost "path" found in a network is not really a "path" in its traditional meaning. It is a route which including necessary entities to produce the final product. Furthermore, the minimum "cost" is not necessarily the financial cost incurred, but it can represent time, actual distance or other relating factors from a source node to a sink node. Hence it is suitable to be used in a supply chain to find the lowest cost, whatever it represents, production path with maximum output in the supply chain. In a sense, it is like the critical path in the supply chain, and a manager can arrange the allocation of resources/orders in his supply chain according to the critical path.

Besides, the model is itself general and easy to build without a large amount of money or professional knowledge. The point is important because many companies, especially in Asian regions, are small to middle size businesses. For this reason, it is not possible for them to hire or introduce complex modeling staff or technique to enhance their management in supply chains. The model developed here is helpful for them to adopt and modify into their own business considerations.

Second, with the help of this algorithm, the model can be used to generate an initial result of the possible path to produce the products required in a supply chain. It is especially important when there is no time to develop complex models while the market are demanding fast response, or while the managers want to get an approximate yet quantitative idea about the shortest production cycle or the lowest cost possible to produce a product in their supply chains.

At last, because the model is split in a “product tree” fashion to apply the shortest path algorithm, it can be seen that there seems to be different “sections” or “cuts” existing in the supply chain. The observation is furthered by examine the different levels in a product tree that these levels have some relationships with the cuts in the supply chain. These seemingly hierarchical levels do exist in the supply chain networks. It is thus possible to find a way to optimize in each cut to still approach some degree of optimal solution for the whole supply chain. It is worth studying for the researchers or managers. By examining their own supply chain models, companies may get some insight to better understand and manage their supply chains.

5. Conclusion

The algorithm developed in this study is another step toward a general model of a supply chain management problem. There are some points worth considering in the study.

First of all, the result yielded from the algorithm serves as an initial basis for a manager to make a decision about saving the most from his supply chain so as to fulfill the demand of the market. Since the market is greatly fluctuant, sometimes a manager needs to make a quick decision. The algorithm developed herein is simple enough to assist the job.

Second, such an algorithm is general enough to be put into practice with a few modifications. It is especially suitable for those medium to small size companies, who may not have enough resources to build their own specific models, yet still need to evaluate their supply chain based on a mathematical way.

Third, the algorithm, along with the formulation closely connected to the product tree, also serves as a ground for researchers to discuss the issue about the management of a supply chain. More research should be done in this direction, and a more detailed yet general enough model can be expected.

At last, since the formulation honoring the single commodity max flow-min cost problem, there are some future directions of research lying here. The most important one is the consideration of uncertainty. If uncertainty can be formulated into the model, it would be of great help. Multi-commodities maximum flow is another interesting topic.

The supply chains have become so complex that it's impossible for a single company to handle. Therefore, closer cooperation and coordination are in urgent need. It is also true for the development of a better model for supply chain management, no matter in practice or in academic research. A good model is surely helpful in this direction, and that's the reason why this study was set out.

References

[1] Ahuja, Ravindra K., Thomas L. Magnanti, and James B. Orlin, Network Flows: Theory, Algorithms, and Applications, Prentice-Hall, 1993.

[2] Arntzen, Bruce C., Gerald G. Brown, Terry P. Harrison, and Linda L. Trafton, “Global Supply Chain Management at Digital Equipment Corporation,” Interface, Vol. 25, No. 1, January-February 1995, pp.69-93.

[3] Billington, Corey, “Strategic Supply Chain Management,” OR/MS Today, April 1994, pp.20-27.

[4] Chen, Shu-Yi, and Ching-Chin Chern, “Shortest Path for A Supply Chain Network,” Proceedings of the 4th Asia Pacific Decision Sciences Institute Conference, Shanghai, China, June 9-12, 1999, pp.579-582.

[5] ________, and ________, “Maximum Flow for a Supply Chain Network,” Proceedings of the 30th Annual Meeting of the Decision Sciences Institute, New Orleans, Louisiana, USA, November 20-23, 1999, pp.1304-1306.

[6] Cohen, Morris A. and Suman Mallik, “Global Supply Chains: Research and Applications,” Working paper, Operations and Information Management Dept., Univ. of Penn., 1996.

[7] Fisher, Marshall L., “What Is the Right Supply Chain for your Product?” Harvard Business Review, March-April, 1997, pp.105-116.

[8] Lee, Hau L., and Corey Billington, "Managing Supply Chain Inventory: Pitfalls and Opportunities," Sloan Management Review, Spring 1992, pp.65-73.

[9] ________, and ________, "Material Managing in Decentralized Supply Chains," Operations Research, Vol. 4, No. 5, September-October 1993, pp.835-847.

[10] Lee, Hau L., V. Padmanabhan, and Seungjin Whang, "The Bullwhip Effect in Supply Chains," Sloan Management Review, Spring 1997, pp.93-102.