# A Project Network Protocol Based on Reliability Theory

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**Abstract**

This paper proposes a network protocol that supplements existing protocols employed in project network modeling. After critical (or near critical) paths have been identified employing an existing protocol, the proposed protocol superimposes on these paths additional network elements representing contingency plans to assure that an activity, or set of activities, will actually get done on time. These superimposed elements play the same role as redundant components in a reliability system. Hence, the output from the reliability approach would view project goals in a manner similar to that of product goals typical of airline manufacturers. Namely, the product (or project) must be designed to have a near zero probability for a catastrophic failure (or a near zero probability for a failure to meet project goals).

The reliability approach implies that project goals must be met above all other considerations, including project costs. Justification for this implication follows from empirical evidence suggesting that on the average, companies lose 33% of after tax profit when they ship products six months late, as compared to losses of only 3.5% when they overspend as much as 50% on product development.

The paper develops a methodology based on series, parallel, and combined series-parallel reliability systems, deriving the appropriate mathematical expressions for such systems.

## 1. Introduction

Numerous graphical/network protocols have been developed to provide project managers with appropriate tools for better managing the project management process. These protocols vary from simple Gantt/Bar Charts and Maps of project evolution, to more sophisticated networking approaches which attempt to realistically capture the complexities and uncertainties of real world projects in the business and engineering world. A review of the history and literature of project management research reveals a steady evolution from deterministic approaches based on classic charts and CPM (Critical Path Method) networks, to stochastic approaches based on several network protocols such as PERT (Program Evaluation and Review Technique), GERT (Graphical Evaluation and Review Technique), and VERT (Venture Evaluation and Review Technique) [4][5][9]. With the exception of GERT and VERT, these protocols now appear in the most recent releases of popular PC-based software packages such as *Microsoft Project 98 for Windows* [3], *Primavera Project Planner* [7], and *Time Line 6.5 for Windows* [8]. (As first observed by Wiest and Levy in 1977 [9, pp. 157-158], the complexity GERT and VERT continues to this day to be a significant factor in their poor track record of acceptance by practitioners and software developers [1].)

The purpose of this paper is to propose a network protocol that supplements (or complements) existing protocols which identify critical (or near critical) paths, and then superimposing on these paths additional network elements representing contingency plans to assure that an activity, or set of activities, will actually be done on time. These superimposed elements play the same role as redundant components in a reliability system (e.g., see [6], pp. 551-555). Hence, events (or milestones) that define the beginning and end of critical path activities will have one or more elements emanating from them, with each assigned a reliability probability corresponding to a contingency plan to keep the project on time, or at least to keep the project within some upper time limit so as to a avoid a serious and costly delay. This leads to a very different form of output. For example, PERT output provides an entire *completion time distribution* where time intervals and confidence levels are reviewed by project managers, while reliability theory provides a specific time goal, where the system was designed to achieve that goal with a high level of confidence *expressed by a single probability*. In this context, not only is the output of the reliability approach much simpler, but it would also view project goals in a manner similar to that of the product goals of airline manufacturers (i.e, to design the product with adequate redundancy to assure a near zero probability for project failure).

Despite the simplicity of its output, a key issue to address for employing the reliability approach to project management is the cost of incorporating redundancy and contingency planning in real world projects. After a “smooth” project completion, “hindsight” would likely lead to arguments about wasted resources if many redundant elements failed to materialize during the project. However, some “foresight” does exist to counter these arguments, particularly when project management concepts are applied to product management. House and Price [2, p. 92] have cited some empirical evidence that on the average, companies lose 33% of after tax profit when they ship products six months late, as *compared to losses of only 3.5% when they overspend as much as 50% on product development.*

The following section develops a methodology based on series, parallel, and combined series-parallel reliability systems, deriving the appropriate mathematical expressions for such systems.

## 2. Methodology

Assume that a project network can be described by the pair (A,B) where A and B represent non-empty sets of arcs (activities) and nodes (events), respectively, with $a_i \in A$ and $b_i \in B$. Associated with each $a_i$ is the time duration $d_i$ with $i=1,2,...I$. If $s,f \in B$ are the source and sink nodes of the network, and $CP_k$ is the kth critical or near critical path from s to f with $k=1,2,...K$; the duration of $CP_k$ for a realization of (A,B) is given by

$$D_k = \sum_{i|a_i \in CP_k} d_i \quad . \tag{1}$$

We assume that near critical paths are determined by a preset criteria to have insignificant differences in total duration and significant probabilities to be critical, and are determined by using one or more of the established protocols which provide such information in their output.

Now assume that the activity time of $a_I$ is $t_i$ with $f(t_i)$ representing the activity time distribution. If tmax is the maximum time permitted for the activity to be completed, then the probability that the activity will completed by tmax is

$$r(tmax_i) = f(t_i \leq tmax) \ , \tag{2}$$

where $r(tmax_i)$ is the reliability probability, or simply the “reliability,” that the activity will be completed by time tmax. If we define $TMAX_k$ as the maximum time duration for path k, then it follows that

$$TMAX_k = \sum_{i|a_i \in CP_k} tmax_i \ . \tag{3}$$

The reliability that the kth path (as a series system) will be completed by $TMAX_k$ is given by (e.g., [6], pp. 551-555)

$$R(TMAX_k) = \prod_{i|a_i \in CP_k} r(tmax_i) \ . \tag{4}$$

This expression applies to a series reliability system, as illustrated in Figure 1.

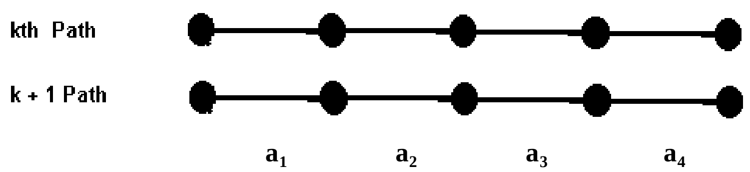


Fig. 1 Series System

Now consider contingency planning for the single activity $a_i$, where such planning assumes the commitment of additional resources (e.g., additional teams, equipment, and facilities) with the intent that $a_i$ does not miss its goal of $tmax_i$. We assume several levels of planning, each with probability $e_l(tmax_i)$ for achieving the activity by time tmax, with l=0,1,2, . . . L. We then obtain for a parallel reliability system ([6], pp. 551-555)

$$r(tmax_i) = 1 - \prod_{l=0}^{L} [\, 1- e_l(tmax_i) \,] \ , \tag{5}$$

where $e_0(tmax_i) = f(t_i \leq tmax)$ for the completion time of the activity on the original network. A schematic of the reliability system represented by (5) is illustrated in Figure 2 for any single activity $a_i$, with contingency plans represented by redundant components. Referring to expressions (4) and (5), it now follows that the reliability for the kth path to meet the completion time goal of TMAX is given by

$$R(TMAX_k) = \prod_{i|a_i \in CP_k} \{ \ 1 \ - \ \prod_{l=0}^{L} [\, 1- e_l(tmax_i) \,] \, \} \ , \tag{6}$$

which is a series-parallel reliability system for all the activities on the kth path, as illustrated in Figure 2.

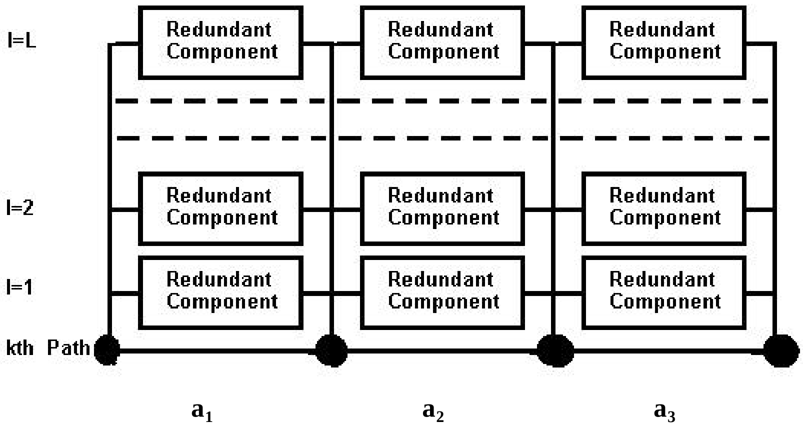


Fig. 2 Series-Parallel System

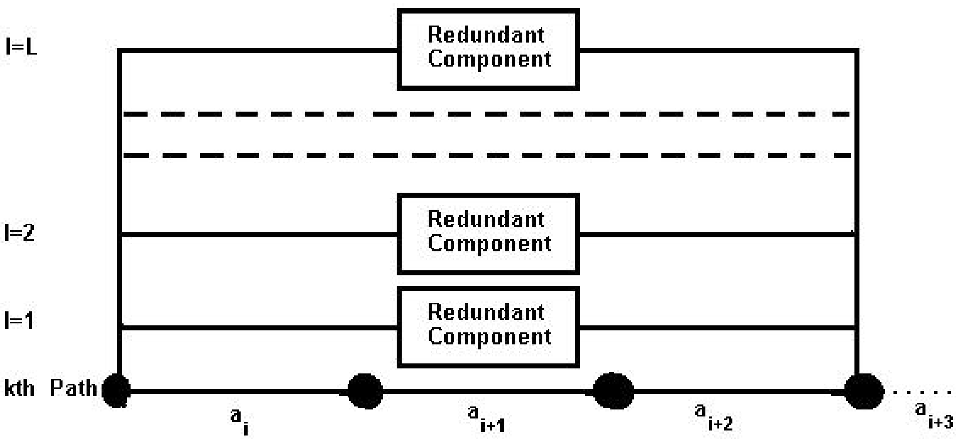


Fig. 3 Variation of a Series-Parallel System

Obviously, numerous combinations of series-parallel systems are possible if redundant components cover one or more activities on the kth path. For example, consider a series of consecutive activities covered by redundant components as shown in Figure 3. Let $a_i \in S$ represent the consecutive activities. If we assume several levels of planning, we can assign the probability of the lth level as $E_l(tmax_s)$ for achieving the time goal of $tmax_s$, where

$$tmax_s = \sum_{i|a_i \in S} tmax_i \ . \tag{7}$$

For a series parallel system such as that in Figure 3, we obtain

$$r(tmax_s) = 1 - [\ 1 - \prod_{i|a_i \in S} r(tmax_i)\ ] \bullet \prod_{l=0}^{L} [\ 1 - E_l(tmax_s)\ ] \ . \tag{8}$$

It now follows that the reliability of the kth path becomes

$$R(TMAX_k) = [\ \prod_{i|a_i \notin S} r(tmax_i)\ ] \bullet [\ r(tmax_s)\ ] \ . \tag{9}$$

If we assume TMAX is the completion time goal which must be met for the entire project, then

$$TMAX = TMAX_{k'} = TMAX_k \ , \tag{10}$$

for all $k,k' \in CP_k$ with $k \neq k'$. Hence, the reliability that the TMAX goal will be met is given by

$$R(TMAX) = \prod_{k=1}^{K} R(TMAX_k) \ , \tag{11}$$

where $R(TMAX_k)$ follows from expressions (6) or (9).

## 3. Conclusion

While the methodology developed in this study provides an alternative network-based approach to project management, it must be noted that several assumptions were made about statistical independence. The first involves independence of the redundant components which were placed in parallel with network arcs (activities), and the second assumes independence of the K near critical and critical paths. Such assumptions are not always true given the possibility of shared activities on critical and near critical paths, and shared resources either among activities or even within single activities. However, the independence requirement can be relaxed somewhat for large networks where no single activity on a near critical or critical path has a completion time significantly larger than the other activities, and where contingency plans corresponding to redundant components can be decided upon just prior to the activity's realization in the real world. A refinement of the methodology to accommodate dependence provides an opportunity for further research.

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