Dependent Demand Forecasting for Parts in an Automobile

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Abstract

This paper deals with demand forecasting of parts in an automobile of which the model has been extinct. It is important to estimate how many inventories of each part in the extinct model should be stocked because production lines of some parts may be replaced by new ones while demand of the parts still occurs. Furthermore, in some countries, there is a strong regulation that the auto company should provide the auto parts with the customers for several years whenever they are requested.

The major characteristic of automobile parts demand forecasting is that the close correlation between the number of running cars and the demand of each part exists. In this sense, the total demand of each part in a year is determined by two factors, total number of running cars in that year and failure rate of the part. The total number of running cars at year k can be estimated sequentially by the amount of shipped cars and proportion of discarded cars at 0,1,2,3,....,k-1. On the other way, it is very difficult to estimate the failure rate of each part because the available inter-failure time data is not complete. The failure rate is, therefore, determined so as to minimize the mean square error between the estimated demand and the observed demand of a part at 0,1,2,3,....,k-1. In this paper, some parts data of H Motor Co. are used to illustrate our model.

1. Introduction

Life cycles of many products become shorter and shorter as the rate of technological innovation is rapid and the consumer preferences change fast. An automobile industry undergoes the same experience. This means that the model of a car changes fast and it arises a critical problem of the forecasting of many parts of an extinct car because the production lines of some parts may be replaced by new ones while demand of parts still occurs. Furthermore, in some countries, there is a strong regulation that the auto company should provide the auto parts with the customers for several years whenever they are requested.

Forecasting problem of a part in an extinct car has different characteristics from the usual forecasting problem. Demand of a part in time k is dominated by the number of running cars at k and failure of that part. Since we deal with the forecasting problem of a part in an extinct car, the number of running cars at k is determined by the number of shipped cars and the discarded rate of them at $t = 0, 1, 2, 3\Lambda, k-1$. Hence, three factors influence the demand of a part in an extinct car and these factors are the number of shipped cars from the beginning year till the extinct year, the discarded rate of car and the failure rate of the part. Among these factors, the number of shipped cars from the beginning year till the extinct year is the known variable from the given data, and the discarded rate and failure rate are parameters to estimate. The key issue in this forecasting problem is, therefore, how to estimate the two parameters.

Many papers have been presented for the forecasting problem in the different contexts. The researches which address the combined forecasting and estimation problem similar to the issue in this paper are presented in the area of diffusion of new product including the product of short life cycle and the adoption and substitution of high-tech products. [2, 4] But, the models of these papers cannot be directly applied to our problem, because they deal with the independent item (for example, end products such as computer, cellular phone, new telecommunication services, or independent parts such as D-RAM) and our paper deal with the dependent item (demand of a part of a car is basically dependent on the number of that car). Another research area related to our issue is the parameter estimation, especially, the estimation of the failure rate. But most of papers in this area require the failure data ranging over the total life, [1, 3, 5]but field data of failure of part over it's total life is unavailable in our case.

In section 2, a model for the forecasting problem of part is presented. A method of parameter estimation for a model is described in section 3. Numerical analysis is carried out for the field data of a part. Finally, conclusion and further research are presented in section 5

Notations

M : time epoch when the car is no more produced at the factory

N : life span of a car , that is, the amount of demand of part of the car is negligible after N years from the first shipping year

$z_{i,j}$: number of age j cars at the beginning of year i (the first shipping year is counted as 0).

$z_{i,j,k}$: number of age j cars whose part's age is k at the beginning of year i.

x_i : number of shipped cars at the beginning of year i .

y_i : number of enrolled cars at the end of year i.

a_j : discarded rate of which the car's age is j.

f_k : failure rate of which the part's age is k .

$w_{i,j,k}$: amount of demand which is caused from $z_{i,j,k}$ during the period [i, i+1].

w_i : $\sum_j \sum_k w_{i,j,k}$

2. Model

2.1 Type of a part

The key factors in predicting the demand of a part in an extinct car during the periods [M, N] are the discarded rate of age j car and the failure rate of age k part.

We assume that the failure rate of a part is dependent on the age of that part regardless of the car's age with which the part is equipped. Let us consider the failure rate of a part. A part is generally divided into three types according to the major cause of failures. First case is that the failure of a part is caused by the accident. The typical example is bumper. Second case is that the part is replaced periodically before failure and, of course, this replacement brings about the demand of a part. The typical example is oil filter, plug, and so on. Third case is that the failure of a part occurs by the complex causes (accident, aging). Typical examples are muffler, fan belt, crank, etc. Failure rate of this type is the function of age of the part while failure rate of the first type is constant and the failure time of the second type can be considered as the deterministic time. The first and second type are, in a sense, the special cases of the third type and predicting the demand of the part in the first & second type is more easily handled than the third type. So the third type of failure is considered in this paper.

2.2 Derivation of the relationship between $z_{i,j,k}$ and x_i .

Since the amount of demand of a part depends on $z_{i,j,k}$, we have to derive the relationship between $z_{i,j,k}$ and x_i . At first, we consider the variations of the age j running cars. Since $z_{i+1,j+1}$ is the number of car which is not discarded from $z_{i,j}$ after one year, it is easy to know that $z_{i,j}$ can be recursively expressed as follows,

$$z_{i,o} = x_i \tag{1}$$

$$z_{i,j} = (1 - a_{j-1}) \cdot z_{i-1,j-1} \qquad j \neq 0 \tag{2}$$

At year i, the age j car which contains the age k part results into three cases after 1 year. First, the car is discarded. Secondly the car survives and the part fails. Thirdly, the car survives and the part doesn't fail. Considering these cases, the recursive equation between $z_{i,j,k}$ and x_i can be expressed as follows;

$$z_{i,j,0} = \sum_r z_{i-1,j-1,r} \cdot (1 - a_{j-1}) \cdot f_r \qquad j \neq 0 \tag{3}$$

$$z_{i,j,k} = z_{i-1,j-1,k-1} \cdot (1 - a_{j-1}) \cdot (1 - f_{k-1}) \qquad k \neq 0, j \neq 0 \tag{4}$$

$$z_{i,0,0} = x_i \tag{5}$$

2.3 Derivation of the total demand of a part

As mentioned above, the age j car results into three cases. The first and second cases bring about the demand of the part. We assume that the part fails at most once in a year and in case the car is discarded at year i, the probability that the demand of a part is caused by the failure of that part before discard is 1/2. If the failure of a part is caused by aging, the assumption that the part fails at most once in a year is reasonable. If the failure of a part is caused by accident, this assumption is unreasonable. But, this case is easily handled by considering the average failure number of parts in a year.

Under these assumptions, the average number of the failed part from $z_{i,j,k}$ is,

$$w_{i,j,k} = z_{i,j,k} \cdot (1 - a_j) \cdot f_k + \tfrac{1}{2} z_{i,j,k} \cdot a_j \cdot f_k \tag{6}$$

And so, the total number of demand for the part at period [i, i+1]

$$w_i = \sum_j \sum_k w_{i,j,k} \tag{7}$$

3. Parameter Estimation

The key parameters in our model are the discarded rate of age j car and the failure rate of age k part. The input variable is the number of shipped cars at t = 0,1,2,….,M. The observed variables is the amount of demand for the part at t = 0,1,2,…,M. The relationships between the observed variable and the input variable including parameters are as follows;

From equation (6)&(7),

$$w_i = \sum_j \sum_k z_{x,i,j,k} \cdot (1 - a_j) \cdot f_k + \frac{1}{2} \sum_j \sum_k z_{i,j,k} \cdot a_j \cdot f_k \tag{8}$$

From (8), w_i is generally expressed as the function of **z**, **a**, and **f**.

Let this function be denoted as $g_{1,i}$, then,

$$w_i = g_{1,i}(\mathbf{z}, \mathbf{a}, \mathbf{f}) \tag{9}$$

Similarly, **z** can be expressed by the function of **z**, **a**, and **f** from (3), (4)&(5).

Let this function be denoted as $g_{2,i}$, then ,

$$z = g_{2,i}(\mathbf{x}, \mathbf{a}, \mathbf{f}) \tag{10}$$

where, $\mathbf{x} = (x_1, \Lambda, x_i)$, $\mathbf{a} = (a_1, \Lambda, a_j)$, $\mathbf{f} = (f_1, \Lambda, f_k)$

The form of the function g_2 is determined by the recursive equation of (3),(4), &(5).

Integrating (8)&(9) yields,

$$w_i = g_{1,i}(g_{2,i}(\mathbf{x}, \mathbf{a}, \mathbf{f}), \mathbf{a}, \mathbf{f}) \tag{11}$$

Equation (11) shows that the parameter estimation in this paper is basically the type of the estimation in non-linear regression analysis composed of the set of the equations. But, the problem is that the amount of data is too small to estimate **a**, **f** simultaneously. We solve this problem by estimating **a**, **f** separately.

At first, **a** is estimated by the data of x_i & $\mathbf{y_i}$ for the similar case. That is, a_i is equal to the ratio of the number of discarded cars at [i, i+1] over the number of age i cars. Secondly, f is generated by the Weibull probability density function. Then, the number of parameter to estimate in non-linear regression equations reduces to 2. The MSE method is applied to estimate the two parameters of Weibull pdf by two-dimensional search.

4. An example

Muffler in a 'A' model car of 'H' motor company is adopted as a case example. Muffler fails by accumulated aging and this aging can be accelerated by shock resulted from a light accident. Hence, failure rate of a muffler is time-dependent.

Table 1 The amount of shipped cars and enrolled cars

Year	0	1	2	3	4	5
Sale of cars	95394	93125	100092	76816	22633	24093
Enrolled cars	91985	183530	279382	352937	369548	381671

The period at which the car is no more produced is 6.

Table 2 The amount of shipped parts

Year	0	1	2	3	4	5
Shipped parts	2325	5578	1795	6885	16676	21041

At first, let us try to estimate the discarded rate of A model car. **Table 1** shows that we have to use the another data w.r.t shipped cars & enrolled car to estimate the discarded rate because the life span of a car is usually 15 years but we do have only 6 production periods data.

Table 3 The general discarded rate of car

Year \ Life year	6	7	8	9
`91	8.3	12.1	17.6	8.0
`92	9.8	11.6	17.5	11.0
`93	10.3	13.4	16.5	9.4
Average	9.5	13.4	16.5	9.4

At this time, the data which is available to us w.r.t the discarded rate of a car is only **Table 3**. It is possible to estimate a_i for $i = 6, 7, 8, 9$ based on the data of Table 3. It may be reasonable to assume that all of a_i is the same for $i \le 5$ because the discard of a car is usually caused by the accident during these periods.

The data of Table 1 is used to estimate a_i for $i \le 5$. The $\hat{a}_i = \dfrac{(\sum_i x_i - y_6)}{\sum_i x_i}$

We assume that the a_i decreases linearly after ten years till 14 year.

Table 4　The discarded rate of a_i for the life span of a car

Year	0-5	6	7	8	9	10	11	12	13	14
Discarded rate	0.014	0.095	0.134	0.165	0.094	0.097	0.09	0.084	0.08	0.077

The failure rate of a muffler is estimated based on the date of **Table 2**. The MSE method is applied to estimate the Weibull parameter of failure time distribution of the muffler. The obtained values of the shape parameter and scale parameter in Weibull pdf are $\alpha = 2.376, \beta = 0.106$.

This MSE method can obtain unsatisfactory result if the epoch period M is too short to fully reflect the failure of the part. In this case, the procedure presented in this paper can be improved in two aspects. At first, the parameter estimation is updated year by year. Secondly, the interval estimation of the mean failure time of the part can be more easily estimated than the variance.

On the other hand, the MSE method is applied to the data of shipped cars while the mean of failure time is fixed at some point in the estimated interval. This approach enables us to have the flexibility of selecting the weight of the data for the shipped car and the estimated interval of mean based on the failure data.

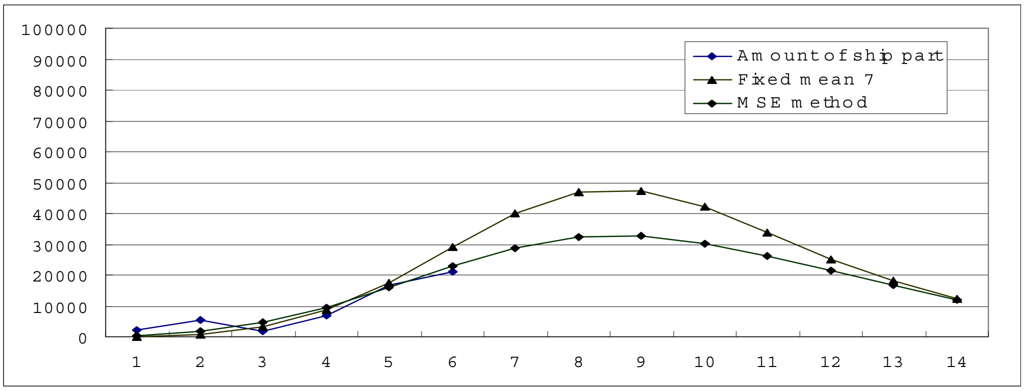

Fig. 1　The two alternative forecasts

Forecast (1) is obtained from MSE and two-dimensional search. Forecast (2) is obtained from MSE and one-dimensional search where the mean is fixed at 7. **Table 5** shows the total amount of demand obtained by the two methods

Table 5　Total amount of demand obtained by 2 methods

	MSE method	MSE and fixed mean of 7
Amount of total demand	255296	325427

5. Conclusion

A model which represents the relationship between the amount of shipped cars and the observed amount of shipped parts is presented by introducing the intermediate variable of age j running car whose part's age is k , discarded rate of a card and the failure rate of a part. The parameter estimation procedure based on the model is presented. The field data of H motor company is analyzed using the model and the estimation procedures are given in this paper. It is shown that the parameter estimation procedure may be improved by using the estimated mean in case that the mean of failure time is known. More extensive research needs to be carried out for forecasting the dependent demand, especially, investigating the relation between the independent item and the dependent item, and comparing the various parameter estimation methods.

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