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Does Value-at-Risk Really Account for Maximum Market Losses?

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Abstract

Value-at-Risk (VaR) now has become an industry standard that is implemented and used every day by many financial institutions worldwide. The notion of confidence level over a given period as a proxy for market risk is now largely approved by the market and regulators, replacing the notions of worst-case scenarios in risk management. However, VaR only accounts for the possible losses of a portfolio at the end of the holding period. This article demonstrates how VaR severely underestimates the real-world market risk. The true risk of a portfolio during a holding period might well above what we think based upon the VaR methodology. A new risk measure, DVaR, is proposed to take into account the possible assets losses during the holding period.

DVaR is devised to be an alternative risk measure to VaR for market risk exceeding a one-day holding period. Instead of considering only asset return on the last day, DVaR accounts for the possible loss on each day of the holding period. The new risk measure, DVaR, is much closer to our intuition about risk. That is, we might encounter a huge lose that may cause financial distress before the last holding day of the portfolio. DVaR is actually implementing the original idea of VaR that is the maximum possible loss of a portfolio with a level of confidence in a period of time.

This article uses daily data of the index of Dow Jones Industrial Average from 1940 to 1992 and six foreign exchange rates from 1972 to 1998. It is demonstrated that DVaR is up to 100% higher than VaR. This means that capital requirements regulated based upon VaR might be in sufficient to cover possible market losses. Monte Carlo simulation is then conducted to calculate tables of modification factors (always greater than one) that relate VaR to DVaR. With those tables, we can utilize existing VaR systems to calculate DVaR with only minor changes. Practical implications and further research directions in risk management are discussed.

1. Introduction

Market risk is usually measured by how volatile an asset's returns. Major financial institutions could suffer from substantial losses during a period of time. It is necessary to develop a method to estimate the worst expected loss of a portfolio. Value-at-risk (VaR) methodology is developed for regulators and risk managers to deal with the world of uncertainty. VaR has now become one of the most important tools for market risk management. The basic concept of VaR is rather simple and straightforward, that is, the worst expected loss that an institution can suffer over a given time interval under normal market conditions at a given confidence level [1]. However, unrealistic assumptions of VaR methodology also make it vulnerable to criticism from both academics and practitioners. VaR models usually focus on estimating one-day VaR and calculating T-day VaR by multiplying square root of T. There are some problems about this approach. First, the distribution function of expected T-day returns can differ from that of one-day returns, especially at the extreme tails, which form the bases of VaR estimation. Second, financial time series often are serially correlated. It is unrealistic to estimate T-day returns only based upon one-day return. Third, possible losses during the portfolio holding period are underestimated. It is possible that the market losses exceed VaR during a T-day portfolio holding period.

Many experienced risk professionals have challenged VaR on the grounds that it cannot cater for the complex real-world risk factors. Variance-based VaR methods are variably unreliable and that the unreliability is related to sample size, to holding period, and to asset class. There are some problematic assumptions that VaR models are based: 1) Asset returns are normally and independently distributed [2]; 2) The distribution of returns is stationary; and 3) The lower tail of T-day returns PDF represents the possible loss during the holding period [3]. Empirical evidence suggests that none of those assumptions holds true in the financial world. The validity of applying Variance-based VaR for risk measurement is questionable.

Furthermore, to predict the trend of changing volatility is essential to the accuracy of one-day VaR estimation. Volatility is often measured by the standard deviation of portfolio returns. Since volatility cannot be observed directly from the financial time series, it can vary according to its definition such as implied volatility and historical volatility. Volatility of financial time series is often clustered. In certain periods of time, market uncertainty is higher than usual. Volatility of a financial time series is related to the holding period to be considered. Short-term volatility (such as one day) may vary because of local factors. Longer-term volatility (such as one year) is likely determined by the characteristics of asset market. For example, stock markets are more volatile than foreign exchange markets.

2. Developing a New Risk Measure: DVaR

VaR methodology is a radical innovation in financial risk management [4]. The methodology works fine when dealing with the short-term market risk of a portfolio. Unfortunately, it evaluates only the possible losses at the end of a time period. VaR does not consider the possible losses during the time period. The maximum possible loss of a portfolio during a time period is certainly higher than the VaR measure [5]. Considering a T-day time period, returns up to T-day can be expressed as R_1, R_2, ..., R_T, a one-day VaR of confidence level α, measures the 1-α percentile of R_1. A T-day VaR of confidence level α, measures the 1-α percentile of R_T. If we want to determine the capital required to cover the possible losses during a T-day period, R_D defined as the minimum of $\{R_1, R_2, ..., R_T\}$ is a better measure for possible extreme losses of the portfolio.

This research proposes a new risk measure, Duration Value-at-Risk (DVaR), to take into account the possible losses during a time period. A T-day DVaR of confidence level α, measures the 1-α percentile of R_D. DVaR is a better alternative to VaR for the following reasons. First, it can also provide a single number to measure market risk just like VaR. DVaR is always larger than VaR for a same portfolio because DVaR considers possible losses during the period of time. Second, DVaR is a more conservative measure for market risk to account for the duration effect on market risk of holding an asset. Liquidity risk is related to the efficiency of secondary market of an asset that the asset may not be sold in a short period of time without discount. DVaR therefore has the potential to integrate market risk and liquidity risk under an analytical framework [6]. Third, DVaR can provide the risk profile of a portfolio, which can kelp decision-makers to visualize the distribution function of maximum losses of the portfolio during a period of time. Finally, the DVaR system can be easily implemented with the adaptive bootstrapping scheme and be calculated as a byproduct of VaR.

3. Empirical DVaR versus VaR

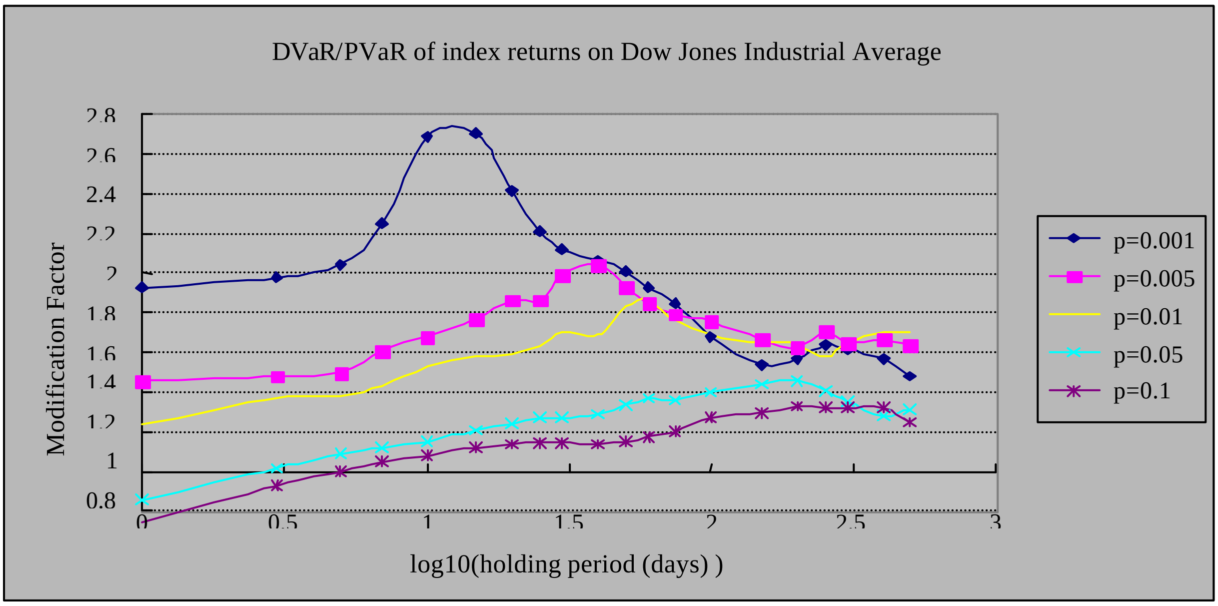
Two typical financial time series, Japanese yen/Canadian Dollar exchange rates (JYCD) and Dow Jones Industrial Average (DJIA), are used to explore empirical relationship between DVaR and VaR. The DJIA daily data are drawn from CRSP database, starting from January 3, 1910 and ending with December 31, 1998. There are 24186 effective observations in the DJIA time series. The JYCD exchange rate data were downloaded from an Internet data source web site, the PACIFIC Exchange Rate Service (URL: http://pacific.commerce.ubc.ca/xr/data.html), hosted by Professor Antweiler from University of British Columbia, Canada. The web site provides extensive foreign exchange data for academic researchers and financial analysts worldwide. The base currency is therefore chosen to be the Canadian Dollar. There are 6693 daily observations in the JYCD time series starting from January 3, 1972 and ending with July 20, 1998.

Since the two time series are long enough, it is reasonable to assume that the average parametric VaR (PVaR) to be the corresponding percentile of an normal distribution with a standard deviation equal to standard deviation estimated from the time series. The Realized VaR (RealVaR) is calculated directly from the empirical distribution functions. DVaR can be also calculated directly as the corresponding percentiles of the distribution functions of maximum asset loss over a T-day period. Financial time series are often characterized as fat-tailed. Parametric VaR often underestimates true VaR. DVaR is always larger than VaR. In order to estimate the difference between DVaR and traditional VaR measures, I use DVaR divided by PVaR and RealVaR respectively as indices. Asset holding time period ranges from 1 to 500 days. Five commonly used confidence levels (α) were selected, that is, 0.999, 0.995, 0.99, 0.95 and 0.9.

Corresponding p-values (1-α) are 0.001, 0.005, 0.01, 0.05, and 0.1.

DVaR/PVaR ratios at different confidence levels of DJIA index are shown in Figure 1. The horizontal axis is the holding period T, which is in a logarithm scale. When p=0.001, DVaR/PVaR ratio is above 1.8 for T < 100 days. It reaches as high as 2.7 when T = 10 days. This means that we can encounter a portfolio loss 2.7 times more than we expected during a certain holding period. When p=0.005, DVaR/PVaR ratio can be above 2.0 as T is around 40 days. When p=0.01, DVaR/PVaR ratio can be above 1.8 as T is around 60 days. When p=0.05, DVaR/PVaR ratio can be above 1.4 as T is between 100 and 250 days. When p=0.1, DVaR/PVaR ratio is not as large but we can see that it increases as T increases. The results suggest that the market risk of DJIA index portfolio have a very complex structure. It varies across holding periods and confidence levels. A simple distribution function such as normal is not likely to capture the whole picture of market risk. The probability of encountering losses exceeding traditional VaR over a specific holding period is far greater than we expected and the magnitude of the extreme losses is far larger.

Figure 1: DVaR/PVaR of index returns on Dow Jones Industrial Average Index



DVaR/RealVaR ratios at different confidence levels of DJIA index are shown in Figure 2. Even though we can develop an ideal VaR model, we can still underestimate possible portfolio losses during the holding period. When p = 0.001, 0.005, and 0.01, the DVaR/RealVaR ratios are mostly below 1.2, which means that the underestimation of market risk seems insignificant. However, the probability of encounter losses can be much higher than expected since p is very small. As p = 0.05 or 0.1, DVaR/RealVaR ratio increases as T increases. It becomes higher than 1.4 when the holding period T is longer than 25 days. The results suggest that VaR can underestimate market risk because extreme losses could occur before the last day of portfolio holding. The underestimation gets worse when p is selected to be larger.

DVaR/PVaR ratios at different confidence levels of JYCD exchange rate are shown in Figure 3. The horizontal axis is the holding period T, which is in a logarithm scale. When p=0.001, DVaR/PVaR ratio is above 1.6 for T < 15 days. It reaches around 2.0 when T = 7 days. When p = 0.005 and 0.01, DVaR/PVaR ratio falls most of the time between 1.4 and 1.6 for all T. When p = 0.05 and 0.1, DVaR/PVaR ratio increases as T increase with roughly a linear trend. Notice that the market risk profile of JYCD is very different from that of DJIA as depicted in Figure 1. The market risk of JYCD also has a very complex structure. It varies across holding periods and confidence levels. However, we observe a similar pattern with the DJIA time series that the probability of encountering losses exceeding traditional VaR over a specific holding period is far greater than we expected and the magnitude of the extreme losses is far larger. Therefore parametric VaR is a very poor market risk estimator. DVaR/RealVaR ratios at different confidence levels of JYCD are shown in Figure 4. Across all holding periods, DVaR/RealVaR ratios are below 1.2. This means that if we can find a good VaR model for JYCD exchange rate, the DVaR measure is not significantly different from VaR.

From the empirical results from the two typical time series, DJIA and JYCD, we learn that financial time series are quite different in nature. To adopt a simple approach in evaluating market risk for a variety of financial assets is not likely to be very effective. Serial correlation exists and plays an important role in determining the market risk over a holding period longer than one day. Parametric models may be suitable in certain circumstances but can fail in the others. The first thing to estimate market risk of a portfolio is to explore the characteristics of its historical returns.

Figure 2: DVaR/RealVaR of index returns on Dow Jones Industrial Average Index

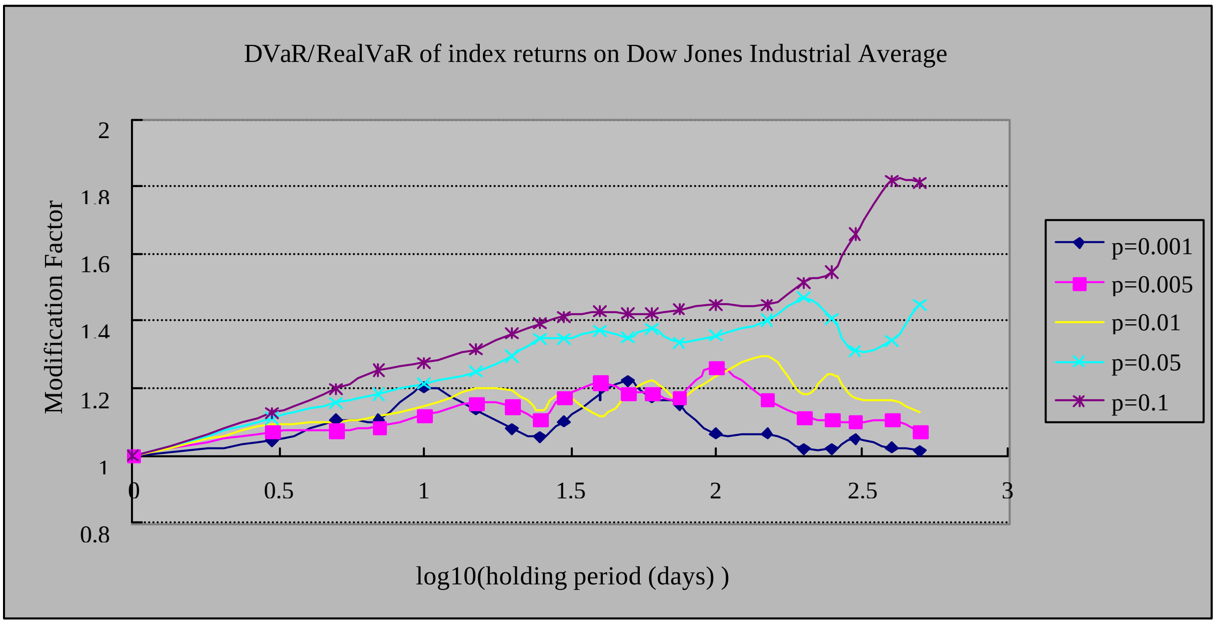


Figure 3: DVaR/PVaR of index returns on Japanese Yen/Canadian Dollar Exchange Rates

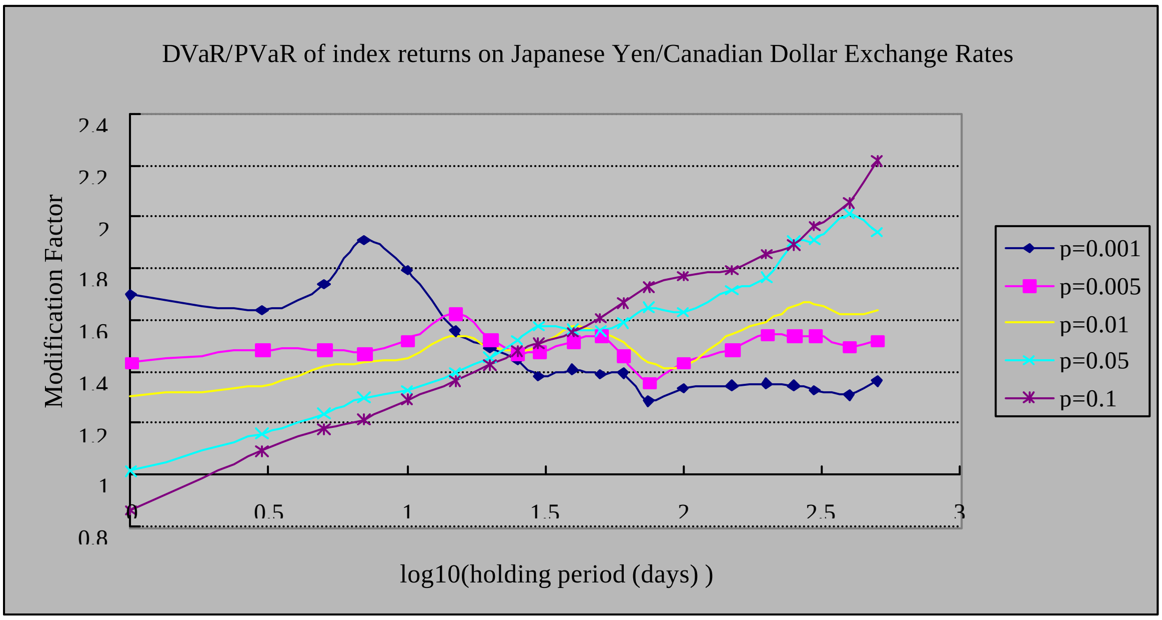
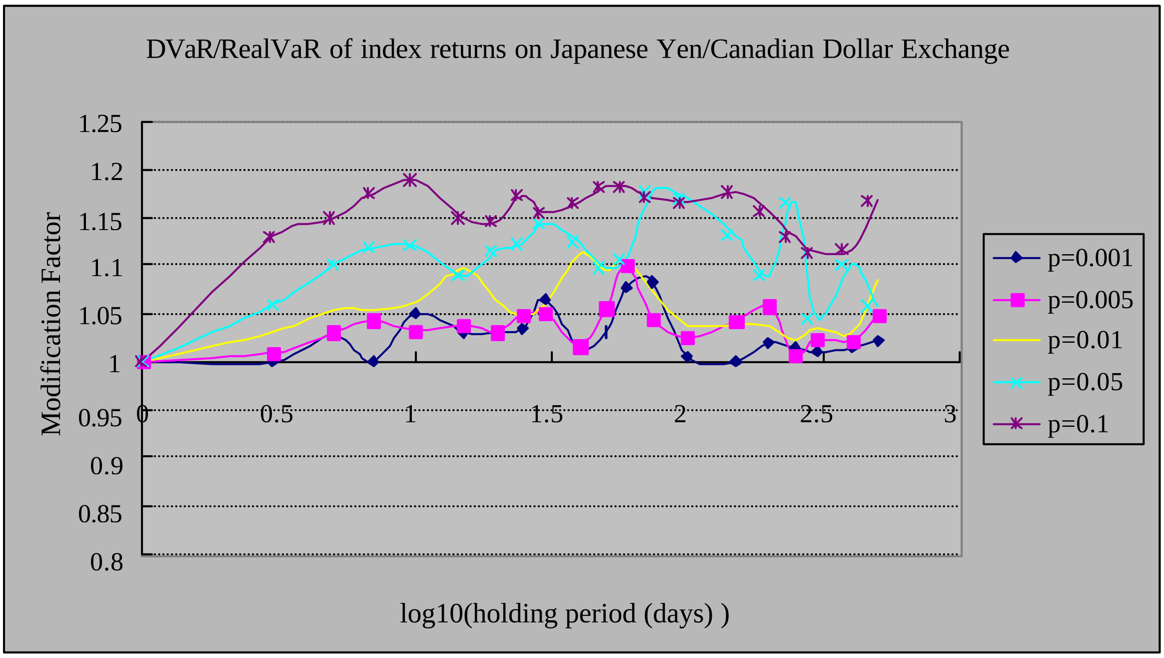


Figure 4: DVaR/RealVaR of index returns on Japanese Yen/Canadian Dollar Exchange Rates



4. A Simulation Study

According to the concepts of DVaR and VaR, DVaR should be always larger than VaR. But how much larger is not fully understood and depends on the characteristics of the financial time series. To evaluate the possible underestimation of VaR measures across different time duration, this study conducts two simulation models [7]. In the first simulation model, we first assume that log-returns of financial time series have a normal distribution and follow an AR(1) process with an auto correlation coefficient, which ranges from 0 to 0.3. In the second simulation model, we assume that log-returns of financial time series have a student-t distribution with degree of freedom 5, and follow an AR(1) process with an auto correlation coefficient , which also ranges from 0 to 0.3. This study simulates 100,000 sample paths of holding time periods ranging from 5 to 500 days. The simulated time series have a standard deviation (std.) equal to 0.005 or 0.01. Five commonly used confidence levels () were selected, that is, 0.999, 0.995, 0.99, 0.95 and 0.9 and the corresponding p-values (1-) are 0.001, 0.005, 0.01, 0.05, and 0.1. The simulation results are shown in Table 1.

Several interesting observations can be derived the simulation results. First, when p-value is small, ρ is small, and the distribution function is normal, parametric VaR does not much deviate from DVaR. This means that parametric VaR is a good risk measure when asset returns are normal and independently and identically distributed. Second, as the autocorrelation coefficient ρ increases, parametric VaR tends to underestimate the true market risk of a portfolio. The underestimation gets worse when we select a more lenient level of confidence such as 0.9 or 0.95. Third, as the holding period T increases, the DVaR/PVaR values increase in general. This means we have to consider the possible losses during a longer holding period when a long-term investment strategy is adopted. Forth, when the distribution function is student-T, when the distribution function is student-T, the pattern of DVaR/PVaR is different from that of a normal distribution. The VaR measure is not a good risk measure when ρ is large and T is small.

From the exploratory simulation study, we observe that the conventional VaR measure tends to underestimate the true market risk to various degrees depending on the nature of financial time series [8]. It is widely recognized that asset returns have more mass in the tail areas than would be predicted by a normal distribution. The student-T distribution seems to be a better alternative to the normal distribution in describing asset returns. The empirical DVaR/PVaR of DJIA and JYCD time series exhibit a similar pattern to that of simulated student-T distribution. Empirical time series are far more complex than an AR(1) process. Serial correlation structure can vary over time. However, once we know the true underlying stochastic model of a financial time series, the corresponding relationship between DVaR and parametric VaR can be constructed.

Table 1: DVaR/PVaR Simulation Results

DVaR/PVaR (Normal)

		Std. = 0.005							Std. = 0.01						
		T=5	T=10	T=20	T=50	T=100	T=200	T=500	T=5	T=10	T=20	T=50	T=100	T=200	T=500
P=0.001	=0	1.012	1.028	1.049	1.032	1.035	1.048	1.038	1.023	1.035	1.022	1.039	1.046	1.031	1.039
	=0.1	1.083	1.106	1.120	1.125	1.146	1.134	1.139	1.087	1.098	1.129	1.116	1.134	1.127	1.105
	=0.2	1.161	1.196	1.225	1.229	1.245	1.241	1.227	1.173	1.201	1.196	1.232	1.238	1.202	1.190
	=0.3	1.244	1.291	1.332	1.353	1.354	1.363	1.315	1.245	1.281	1.323	1.313	1.311	1.295	1.257
P=0.005	=0	1.011	1.030	1.046	1.056	1.066	1.073	1.066	1.029	1.035	1.048	1.059	1.060	1.052	1.058
	=0.1	1.091	1.127	1.146	1.158	1.153	1.170	1.164	1.098	1.120	1.153	1.148	1.155	1.141	1.141
	=0.2	1.171	1.214	1.242	1.266	1.272	1.279	1.254	1.173	1.202	1.224	1.246	1.259	1.233	1.218
	=0.3	1.253	1.315	1.364	1.363	1.386	1.380	1.358	1.267	1.313	1.349	1.347	1.357	1.342	1.297
p=0.01	=0	1.019	1.042	1.060	1.071	1.083	1.088	1.081	1.028	1.042	1.062	1.076	1.072	1.068	1.072
	=0.1	1.102	1.136	1.157	1.177	1.173	1.184	1.180	1.108	1.127	1.151	1.161	1.173	1.160	1.157
	=0.2	1.185	1.220	1.253	1.269	1.293	1.298	1.283	1.182	1.219	1.242	1.260	1.275	1.253	1.239
	=0.3	1.254	1.331	1.374	1.378	1.401	1.402	1.379	1.267	1.319	1.358	1.367	1.367	1.362	1.318
p=0.05	=0	1.062	1.091	1.117	1.140	1.143	1.155	1.161	1.062	1.091	1.115	1.136	1.146	1.140	1.140
	=0.1	1.143	1.190	1.220	1.251	1.256	1.254	1.264	1.137	1.179	1.209	1.240	1.250	1.243	1.243
	=0.2	1.219	1.278	1.329	1.354	1.372	1.383	1.377	1.216	1.274	1.308	1.345	1.356	1.350	1.327
	=0.3	1.294	1.380	1.436	1.483	1.499	1.506	1.484	1.300	1.371	1.426	1.458	1.467	1.469	1.424
p=0.1	=0	1.110	1.153	1.187	1.211	1.226	1.244	1.240	1.111	1.155	1.184	1.209	1.224	1.225	1.221
	=0.1	1.189	1.249	1.293	1.333	1.345	1.350	1.361	1.184	1.237	1.280	1.322	1.326	1.334	1.328
	=0.2	1.269	1.340	1.399	1.437	1.461	1.477	1.478	1.261	1.337	1.383	1.429	1.448	1.444	1.421
	=0.3	1.336	1.439	1.512	1.577	1.595	1.615	1.598	1.335	1.436	1.500	1.555	1.568	1.576	1.536

DVaR/PVaR (Student T, d.o.f.=5)

		Std. = 0.005							Std. = 0.01						
		T=5	T=10	T=20	T=50	T=100	T=200	T=500	T=5	T=10	T=20	T=50	T=100	T=200	T=500
P=0.001	=0	1.175	1.156	1.097	1.062	1.043	1.047	1.054	1.237	1.125	1.089	1.076	1.048	1.037	1.029
	=0.1	1.256	1.211	1.212	1.162	1.161	1.153	1.135	1.268	1.220	1.178	1.145	1.132	1.130	1.104
	=0.2	1.389	1.326	1.290	1.279	1.263	1.256	1.238	1.389	1.305	1.264	1.270	1.231	1.209	1.178
	=0.3	1.489	1.468	1.394	1.395	1.396	1.346	1.318	1.469	1.447	1.375	1.340	1.328	1.288	1.251
p=0.005	=0	1.111	1.098	1.073	1.075	1.060	1.072	1.068	1.113	1.097	1.076	1.070	1.065	1.054	1.056
	=0.1	1.179	1.174	1.192	1.168	1.172	1.177	1.146	1.180	1.180	1.174	1.160	1.154	1.147	1.134
	=0.2	1.277	1.263	1.272	1.275	1.276	1.270	1.259	1.281	1.262	1.260	1.272	1.244	1.240	1.211
	=0.3	1.374	1.394	1.389	1.392	1.396	1.388	1.353	1.358	1.368	1.366	1.359	1.349	1.331	1.286
p=0.01	=0	1.088	1.086	1.070	1.079	1.083	1.080	1.083	1.088	1.080	1.071	1.074	1.079	1.068	1.074
	=0.1	1.148	1.156	1.188	1.180	1.179	1.187	1.171	1.158	1.178	1.170	1.166	1.166	1.168	1.154
	=0.2	1.242	1.252	1.276	1.286	1.292	1.284	1.284	1.243	1.261	1.263	1.278	1.267	1.253	1.228
	=0.3	1.331	1.372	1.388	1.397	1.421	1.404	1.385	1.321	1.358	1.360	1.381	1.367	1.358	1.309
p=0.05	=0	1.054	1.089	1.106	1.135	1.146	1.159	1.156	1.048	1.086	1.112	1.127	1.140	1.143	1.142
	=0.1	1.121	1.166	1.216	1.242	1.252	1.260	1.265	1.120	1.175	1.204	1.234	1.238	1.243	1.232
	=0.2	1.199	1.261	1.312	1.352	1.367	1.370	1.378	1.192	1.263	1.303	1.341	1.352	1.345	1.327
	=0.3	1.282	1.366	1.421	1.467	1.495	1.497	1.487	1.266	1.355	1.404	1.464	1.465	1.460	1.426
p=0.1	=0	1.073	1.130	1.166	1.203	1.223	1.230	1.236	1.069	1.127	1.166	1.194	1.217	1.217	1.218
	=0.1	1.138	1.208	1.271	1.317	1.331	1.348	1.352	1.134	1.218	1.263	1.306	1.320	1.328	1.319
	=0.2	1.214	1.301	1.378	1.437	1.461	1.464	1.476	1.201	1.303	1.364	1.419	1.441	1.443	1.423
	=0.3	1.285	1.408	1.483	1.554	1.595	1.607	1.599	1.276	1.397	1.475	1.549	1.565	1.567	1.536

5. Discussion and Conclusion

VaR is widely accepted as a standard tool for financial institutions to evaluate the risk exposure of their portfolios. This study argues that VaR underestimates possible market risk during the asset holding period [9]. Therefore we develop a new risk measure, duration VaR, to account for possible losses within the portfolio holding period. There is a one-to-one correspondence between VaR and DVaR. By its definition, DVaR is always larger than VaR. If we know the true model of portfolio returns, a simple modification factor can be used to calculate the DVaR from a known VaR. A simulation study is conducted to demonstrate the relationship when asset returns are normal or student-T distributed with a small autocorrelation. This suggests that we can utilize the existing models to estimate VaR and multiply a modification factor to get DVaR. The modification factor depends on the nature of portfolio returns. One way to estimate the modification factor is historical simulation by assuming that historical returns can best describe the future behavior of portfolio returns. Therefore, we can calculate a reference chart like Fig. 1 or Fig. 2 for specific financial portfolios. Modification factors can then be selected directly from the chart. In some sense, the chart represents the characteristics of time series structure of portfolio returns. Certainly, we can also calculate DVaR directly from historical simulation and bypass the need to calculate VaR. Another approach to estimate modification factors is to select an appropriate time series model and calculate a table like Table 1. The corresponding relationship between VaR and DVaR can be constructed.

In this study, we have shown that conventional VaR methodology severely underestimates possible losses of typical portfolios, especially when the holding period is longer than 10 days and the level of confidence is high. We also find that different financial time series have different characteristics. The corresponding relationships between VaR and DVaR are quite different for Dow Jones and foreign exchanges. Therefore, we might need different parametric models to describe time series of distinct natures. The result of simple simulation study suggests that a time series with log-returns of a student-T distribution and weakly auto-correlated can better describe the empirical linkage between VaR and DVaR than one with a normal distribution and without autocorrelation. For extremely volatile financial assets, one-day VaR is a very good tool to monitor daily market risk. The portfolio size of any median-size financial institution can easily exceed billions of dollars. It is hardly to imagine the institution can turn the portfolio into cash within one trading day even all assets are traded in the most liquid markets. Measuring portfolio risk from a perspective of a longer time horizon, such as 25 days (on month), is probably more practical. Duration VaR can serve as a complementary tool to the conventional VaR when assessing a portfolio containing assets which are not highly liquid or when a longer-term investment position is adopted.

One possible further research direction is to find the relationship between VaR and DVaR for different time series and how those relationships change as the portfolio composed of different assets. Volatility of financial time series is not constant over time but is often clustered. Christoffersen and Diebold [10] report that when the forecast horizon exceeds 10 days, models adopting conditional prediction perform no better than those adopting the unconditional distribution do. One-day VaR based on volatility dynamics may not be appropriate for estimating market risk when the holding period is longer than 10 days. DVaR based upon historical simulation can provide a more robust market risk estimate.

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