APDSI 2000 Full Paper (July, 2000)

A Fuzzy Multiple Attribute Decision Making Approach for Linear Assignment Problems

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Abstract

Decision making is a part of daily life of humans. Almost all decision problems have multiple, usually conflicting, criteria. How to balance these conflicting criteria and achieve the greatest satisfaction is an important issue in the decision-making area. Multiple Attribute Decision Making (MADM) is one of the most effectively and widely used methods to deal with this type of problems. In this paper, we present a new fuzzy MADM method to enhance Bernardo and Blin's linear assignment method [3]. This method can assist the decision maker to determine the ranking order of alternatives under uncertainty.

1. Introduction

The principle of MADM is to select the best alternative from several mutually exclusive alternatives based on their general performance of multiple attributes (or criteria). According to the types and the characteristics of decision problems, different MADM methods have been designed and implemented in various domains such as Linear Assignment method, TOPSIS, ELECTRE, etc.

Among these approaches, linear assignment method [3] uses concordance concept and linear programming (LP) model to determine the rank of alternatives. Since this method only requests the ordinal data instead of cardinal data, it is easy to understand and to be implemented in different domains. However, this approach still has several weaknesses that can be improved. First of all, this approach can only deal with precise information. In reality, that the precise information is available is unrealistic. Secondly, this approach can only deal with two types of attributes: benefit type and cost type. For some problems, the best attribute value is not the highest value or the lowest value, but the target value. The importance of target type's attribute should not be ignored. Thirdly, this approach does not consider actual cardinal difference between alternatives on each attribute. Hence, even two alternatives have the same rank on two attributes the actual cardinal difference between alternatives on each attribute can be quite large.

To overcome the above weaknesses, the objectives of this research is to propose an generalized fuzzy MADM method which can (1) tolerate the uncertain and imprecise attribute values; (2) include the third type of attribute (target type) in addition to benefit and cost types of attribute; and (3) consider the actual cardinal difference between alternatives on each attribute.

2. Literature Review

Generally, fuzzy MADM refers to making selection among some alternatives in the presence of multiple criteria. During last decade, a number of fuzzy MADM methods have been developed and implemented in various fields such as management, decision-making, control engineering. Baas and Kwakernaak [1] was the first to combine α-cut and simple additive weight method to model their fuzzy MADM. Saaty [5] applies the analytic hierarchical process (AHP) approach to formulate the fuzzy MADM model. Takeda and Nishida [7] proposed the use of the degree of concordance and discordance to construct fuzzy outranking relations. A final fuzzy outranking relation is determined by combining the concordance and discordance relation. Siskos et al. [6] also present a fuzzy outranking method by using different concordance and discordance formulas. Bellman and Zadeh [2] apply Maximin concept to model their approach in the fuzzy environment. A detailed survey of fuzzy MADM can be found in Chen and Hwang [4].

3. The Fuzzy MADM Method

The proposed fuzzy MADM method is a fuzzy linear assignment method to determine the proper ranking order of alternatives by applying linear assignment principle. The detailed algorithm of our approach is explained as follows:

Step 1. Identify the alternatives and relevant attributes.

Based on the scope and the limit of the interested decision problem, the decision maker first finds out all possible alternatives (or solutions) regarding to the problem. In addition, the decision maker needs to select the relevant attributes (or criteria) that can help him/her determine the ranking order of alternatives.

Step 2. Determine the type of attributes.

The defined attributes may be either one of quantitative type or qualitative type. If the value of an attribute can be easily determined and assigned by a numerical number (20, 0.8, 500, etc.), the attribute is a quantitative type. In contrast, if the value of attribute is imprecise and vague and difficult to be assigned a numerical number (high, moderate, larger than 4500, etc.), this type of attribute is called qualitative type.

Step 3. Assign proper values to each attribute with regard to all alternatives.

Based on the objective information (e.g. books, manual, report) and the decision maker's own subjective judgment, he/she needs to assign the appropriate numerical numbers (quantitative type of attributes) or linguistic terms (qualitative type of attributes) to each attribute regarding to all alternatives. Thus, the assignment of attribute values generates a decision matrix.

Step 4. Define the membership functions for all linguistic terms.

Since the linguistic terms in the created decision matrix is relatively difficult to perform mathematic operations, they need to be properly transformed into some numerical numbers without losing their imprecision. The induction of fuzzy set seems natural and reasonable. The trapezoidal fuzzy number can well represent the vagueness of the linguistic terms. For example, the membership function of attribute value 'high' may be defined as trapezoidal fuzzy number (0.6, 0.7, 0.8, 0.9) in terms of 0-1 scale.

Step 5. Determine the weight and category of evaluation attribute.

The decision maker should assign appropriate numerical values (fuzzy singleton) to describe the weight of individual attribute. In addition, the decision analyst needs to determine the type of evaluation attributes. There are three types of attributes: benefit attribute, cost attribute, and target attribute. For the benefit attribute, the ranking order of an alternative will increase when the attribute value become larger. By contrast, for the cost attribute, the ranking order of an alternative will decrease when the attribute value becomes larger. For the target attribute, the decision maker first specifies a target level (or the preferred value) for that attribute. If the attribute value of an alternative is further from the target level, the ranking order of an alternative will decrease.

Step 6. Normalize the decision matrix.

For later comparison purpose, the decision matrix obtained should be properly normalized. The normalization formulas are listed as follows:

(a) For benefit type

$$r_{ij} = \frac{x_{ij}}{\max_i \; x_{ij}}$$

(b) For cost type

$$r_{ij} = \frac{\min_i \; x_{ij}}{x_{ij}}$$

(c) For target type

$$d_{ij} = \left| x_{ij} - x_j^{target} \right|$$

x_j^{target} = the target value of x_j

$$r_{ij} = 1 - \frac{d_{ij}}{\max_i \; x_{ij}}$$

The above three formulas works when x_{ij} is crisp. However, when x_{ij} is fuzzy, its corresponding r_{ij} is also fuzzy. The above formulas are replaced by the following fuzzy operations.

Let $X_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$

$$\max_i X_{ij} = (a_j^*, b_j^*, c_j^*, d_j^*)$$

$$\min_i X_{ij} = (a_j^-, b_j^-, c_j^-, d_j^-)$$

(a') X_j is a benefit attribute

$$r_{ij} = \left(\frac{a_{ij}}{d_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{b_j^*}, \frac{d_{ij}}{a_j^*}\right)$$

(b') X_j is a cost attribute

$$r_{ij} = \left(\frac{a_j^-}{d_{ij}}, \frac{b_j^-}{c_{ij}}, \frac{c_j^-}{b_{ij}}, \frac{d_j^-}{a_{ij}}\right)$$

(c') X_j is a target attribute

We can employ Zadeh's similarity concept to measure the difference between two fuzzy numbers.

$$D_{ij} = 1 - \left\{ \sup_X \left[\mu(X_{ij}) \wedge \mu(X_j^{target}) \right] \right\}$$

$$= 1 - L_{ij}$$

Where L_{ij} is the highest degree of similarity of x_{ij} and X_j^{target}.

$$r_{ij} = 1 - \frac{1 - L_{ij}}{(a_j^*, b_j^*, c_j^*, d_j^*)}$$

$$= \left(\frac{a_j^* - 1 + L_{ij}}{a_j^*}, \frac{b_j^* - 1 + L_{ij}}{b_j^*}, \frac{c_j^* - 1 + L_{ij}}{c_j^*}, \frac{d_j^* - 1 + L_{ij}}{d_j^*}\right)$$

Step 7. Convert the fuzzy decision matrix into the crisp decision matrix.

Since the decision matrix consists of fuzzy and crisp data, it should be appropriately transformed into crisp matrix for comparing the attribute values. Until now, more than two dozens of ranking methods have been designed and applied in the fuzzy environment. However, there is no mathematic proof showing that some ranking method is better than the others. Here, we arbitrary select Chen and Hwang's ranking method for comparing all alternatives. The conversion formula is listed as follows:

If a trapezoidal fuzzy number M

M = (a, b, c, d)

The corresponding crisp value C of the fuzzy number M is

$$C = \frac{1}{2}\left(\frac{d}{1 - c + d} + \frac{b}{b - a + 1}\right)$$

Thus, we can determine the ranking orders of alternatives for each attribute.

Step 8. Determine the ranking order of alternatives for each attribute.

Based on the crisp decision matrix, we can generate an attributewise ranking matrix. For example, a crisp decision matrix is given below:

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{pmatrix} X_1 & X_2 \\ 0.7 & 0.8 \\ 0.5 & 0.8 \\ 1 & 0.6 \end{pmatrix}$$

Its attributewise ranking matrix will become

$$\begin{array}{c} \\ 1st \\ 2nd \\ 3rd \end{array} \begin{pmatrix} X_1 & X_2 \\ A_3 & A_1 \text{or} A_3 \\ A_1 & \\ A_2 & A_3 \end{pmatrix}$$

Step 9. Assign the proper weight for each rank of individual alternative.

The traditional linear assignment model neglects the importance of actual cardinal difference between alternatives on each attribute. The proposed approach will consider actual cardinal difference and compute the weight for each rank of individual alternative. For the above example, assume the weight for X1 and X2 is 0.6 and 0.4, respectively. The above ranking matrix will become

$$\begin{array}{c} \\ 1st \\ 2nd \\ 3rd \end{array} \begin{pmatrix} X_1 & X_2 \\ W(A_3) = 0.6 & W(A_1) = W(A_2) = 0.4 \\ W(A_1) = (0.7/1) \times 0.6 = 0.42 & - \\ W(A_2) = (0.5/1) \times 0.6 = 0.3 & W(A_3) = (0.6/0.8) \times 0.4 = 0.3 \end{pmatrix}$$

Step 10. Convert the weight matrix into concordance matrix.

Based on the weight matrix, we can construct a concordance (square) matrix F, whose element f_{ij} is the summation of the weights for all attributes where alternative i is ranked j. For instance, the concordance matrix of above example will be computed as below:

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{pmatrix} 1st & 2nd & 3rd \\ 0.4 & 0.42 & 0 \\ 0.4 & 0 & 0.3 \\ 0.6 & 0 & 0.3 \end{pmatrix}$$

Step 11. Form the linear assignment (LP) model.

$$\max \sum_{i=1}^{m} \sum_{j=1}^{m} f_{ij} p_{ij}$$

$$S.T.$$

$$\sum_{i=1}^{m} p_{ij} = 1, j = 1,2, \mathrm{K}\ m.$$

$$\sum_{j=1}^{m} p_{ij} = 1, i = 1,2, \mathrm{K}\ m.$$

Where $\boldsymbol{p}_{ij} = 1$ if $\boldsymbol{A}_i$ is assigned to overall rank j. Otherwise, $\boldsymbol{p}_{ij} = 0$.

Using the above example, the formed LP model will become

$$\begin{aligned} \mathrm{Max}\quad & 0.4P_{11} + 0.42P_{12} + 0P_{13} \\ & + 0.4P_{21} + 0P_{22} + 0.3P_{23} \\ & + 0.6P_{31} + 0P_{32} + 0.3P_{33} \end{aligned}$$

$$S.T.$$

$$P_{11} + P_{12} + P_{13} = 1$$

$$P_{21} + P_{22} + P_{23} = 1$$

$$P_{31} + P_{32} + P_{33} = 1$$

$$P_{11} + P_{21} + P_{31} = 1$$

$$P_{12} + P_{22} + P_{32} = 1$$

$$P_{13} + P_{23} + P_{33} = 1$$

$$\forall P_{ij} = 0 \text{ or } 1$$

Step 12. Solve the LP model.

The overall ranking order of alternatives is obtained by solving the LP formulation.

4. Example

An example has been designed and developed to illustrate the proposed fuzzy MADM approach step by step. A toy company wants to by a CNC lathe machine for dealing with numerous orders.

Step 1. Identify the alternatives and relevant attributes.

After detail survey, they found three companies (A1, A2, A3) are available. After discussion, the company considers four attributes; the price (X1), compatibility (X2), feature (X3), and quality (X4) should be used to evaluate these six machines.

Step 2. Determine the type of attributes.

Since only the price (X1) of CNC machine can be clearly determined, and the value of the rest three attributes (X2, X3, and X4) are hard to judge. Thus, X1 is a quantitative type of attribute and the rest attributes are qualitative type.

Step 3. Assign proper values to each attribute with regard to all alternatives.

The assignment result creates the following decision matrix D.

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{pmatrix} X_1 & X_2 & X_3 & X_4 \\ 150 & \text{High} & \text{Average} & \text{Good} \\ 145 & \text{Average} & \text{Average} & \text{Good} \\ 175 & \text{Average} & \text{Good} & \text{Average} \end{pmatrix}$$

Step 4. Define the membership functions for all linguistic terms.

For example, the linguistic terms of attribute 'compatibility' (Figure 2) can be defined as follows:

For X2:

'Very High' = (0.8, 0.9, 1.0, 1.0)
'High' = (0.6, 0.7, 0.8, 0.9)
'Average' = (0.4, 0.5, 0.5, 0.6)
'low' = (0.1, 0.2, 0.3, 0.4)

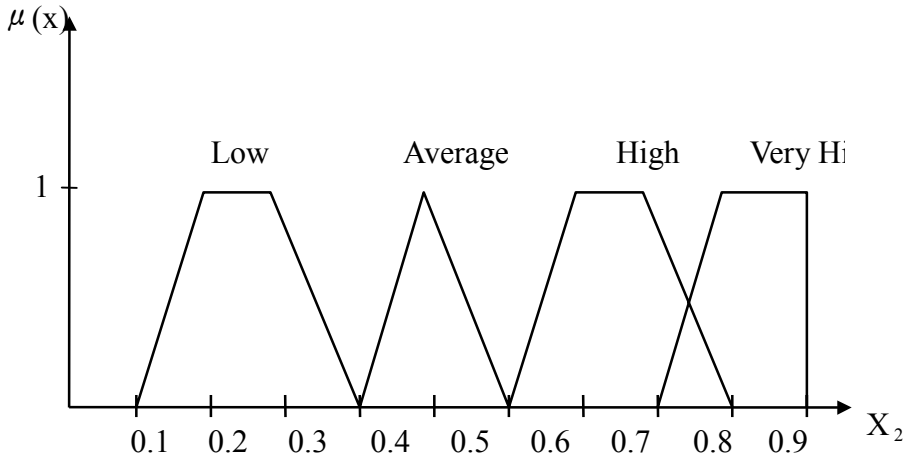

Figure 2. The membership functions of linguistic terms for attribute 'compatibility'

Similarly, the membership function of the linguistic terms for attribute 'feature' and 'quality' can be defined as

For X3:

'Good' = (0.7, 0.8, 0.8, 0.9)
'Average' = (0.4, 0.5, 0.5, 0.6)
'Worse' = (0.2, 0.3, 0.3, 0.4)
'Very Worse' = (0, 0, 0.1, 0.2)

For X4:

'Good' = (0.7, 0.8, 0.8, 0.9)
'Average' = (0.3, 0.4, 0.6, 0.7)
'Worse' = (0.1, 0.2, 0.3, 0.4)

The decision matrix then becomes

	X_1	X_2	X_3	X_4
A_1	150	(0.6,0.7,0.8,0.9)	(0.4,0.5,0.5,0.6)	(0.7,0.8,0.8,0.9)
A_2	145	(0.4,0.5,0.5,0.6)	(0.4,0.5,0.5,0.6)	(0.7,0.8,0.8,0.9)
A_3	175	(0.4,0.5,0.5,0.6)	(0.7,0.8,0.8,0.9)	(0.3,0.4,0.6,0.7)

Step 5. Determine the weight and category of evaluation attribute.

Assume attributes X_1 and X_2 are benefit type, X_3 is cost type, and X_4 is target type with target level (0.65, 0.75, 0.75, 0.85). Further, decision maker assigns his subjective estimate of the weight of these four attributes. Suppose the weight of these four attributes, X1, X2, X3, and X4, is 0.35, 0.25, 0.3, and 0.1, respectively.

Step 6. Normalize the decision matrix.

For later computation purpose, the decision matrix should be normalized. For example, the attribute values of X_1 are crisp. We can apply the formula listed in the previous section. Since X_1 is a benefit attribute, the normalization formula will become

$$r_{ij} = \frac{x_{ij}}{\max_i \; x_{ij}}$$

Thus

$$r_{12} = \frac{150}{175} = 0.857$$

$$r_{21} = \frac{145}{175} = 0.829$$

For benefit attribute X_2, the attribute values in the decision matrix are fuzzy. The normalization formula will be replaced by the following operation.

$$r_{ij} = \left(\frac{a_{ij}}{d_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{b_j^*}, \frac{d_{ij}}{a_j^*}\right)$$

Thus

$$r_{12} = \left(\frac{0.6}{0.9}, \frac{0.7}{0.8}, \frac{0.8}{0.7}, \frac{0.9}{0.6}\right)$$

$$= (0.667, 0.875, 1.143, 1.5)$$

$$r_{22} = \left(\frac{0.4}{0.9}, \frac{0.5}{0.8}, \frac{0.5}{0.7}, \frac{0.6}{0.6}\right)$$

$$= (0.444, 0.625, 0.714, 1)$$

$$r_{32} = (0.444, 0.625, 0.714, 1)$$

Where $\max_i X_{ij} = (0.6, 0.7, 0.8, 0.9)$

For the cost attribute X_3, the attribute values in the decision matrix are still fuzzy. The normalization formula will become

$$r_{ij} = \left(\frac{a_j^-}{d_{ij}}, \frac{b_j^-}{c_{ij}}, \frac{c_j^-}{b_{ij}}, \frac{d_j^-}{a_{ij}}\right)$$

Thus

$$r_{13} = \left(\frac{0.4}{0.6}, \frac{0.5}{0.5}, \frac{0.5}{0.5}, \frac{0.6}{0.4}\right)$$

$$= (0.667, 1, 1, 1.5)$$

$$r_{23} = \left(\frac{0.4}{0.6}, \frac{0.5}{0.5}, \frac{0.5}{0.5}, \frac{0.6}{0.4}\right)$$

$$= (0.667, 1, 1, 1.5)$$

$$r_{33} = (0.444, 0.625, 0.625, 0.857)$$

Where $\min_i X_{ij} = (0.4, 0.5, 0.5, 0.6)$

For attribute X_4, the attribute values are fuzzy and this attribute is a target attribute. The following normalization formula will be used.

$$r_{ij} = 1 - \frac{1 - L_{ij}}{(a_j^*, b_j^*, c_j^*, d_j^*)}$$

$$= \left(\frac{a_j^* - 1 + L_{ij}}{a_j^*}, \frac{b_j^* - 1 + L_{ij}}{b_j^*}, \frac{c_j^* - 1 + L_{ij}}{c_j^*}, \frac{d_j^* - 1 + L_{ij}}{d_j^*}\right)$$

Thus,

$$r_{14} = \left(\frac{0.7 - 1 + 0.75}{0.7}, \frac{0.8 - 1 + 0.75}{0.8}, \frac{0.8 - 1 + 0.75}{0.8}, \frac{0.9 - 1 + 0.75}{0.9}\right)$$

$$= (0.643, 0.688, 0.688, 0.722)$$

$$r_{24} = (0.643, 0.688, 0.688, 0.722)$$

$$r_{34} = \left(\frac{0.7 - 1 + 0.25}{0.7}, \frac{0.8 - 1 + 0.25}{0.8}, \frac{0.8 - 1 + 0.25}{0.8}, \frac{0.9 - 1 + 0.25}{0.9}\right)$$

$$= (-0.071, 0.063, 0.063, 0.167)$$

$$r_{14} = 0.75,\ r_{24} = 0.75,\ r_{34} = 0.25$$

$$\max_i X_{ij} = (0.7, 0.8, 0.8, 0.9)$$

Therefore, the normalized decision matrix will become

$$
\begin{array}{c}
 \\ A_1 \\ A_2 \\ A_3
\end{array}
\begin{pmatrix}
X_1 & X_2 & X_3 & X_4 \\
0.857 & (0.667,0.875,1.143,1.5) & (0.667,1,1,1.5) & (0.643,0.688,0.688,0.722) \\
0.829 & (0.444,0.625,0.714,1) & (0.667,1,1,1.5) & (0.643,0.688,0.688,0.722) \\
1 & (0.444,0.625,0.714,1) & (0.444,0.625,0.625,0.827) & (-0.071,0.063,0.063,0.167)
\end{pmatrix}
$$

Step 7. Convert the fuzzy decision matrix into the crisp decision matrix.

$$C = \frac{1}{2}\left(\frac{d}{1-c+d} + \frac{b}{b-a+1}\right)$$

Since the attribute values of X_2, X_3, and X_4 are fuzzy, these three attributes are required to converted. For example,

r_{12} = (0.667, 0.875, 1.143, 1.5)

The converted crisp values of r_{12} is computed as follows:

$$r_{12} = \frac{1}{2}\left(\frac{1.5}{1-1.43+1.5} + \frac{0.875}{0.875-0.667+1}\right)$$

$$= 0.857$$

Similarly, the rest r_{ij} values can be converted in the same way. The converted decision matrix is listed below:

$$
\begin{array}{c}
 \\ A_1 \\ A_2 \\ A_3
\end{array}
\begin{pmatrix}
X_1 & X_2 & X_3 & X_4 \\
0.857 & 0.915 & 0.875 & 0.675 \\
0.829 & 0.653 & 0.875 & 0.675 \\
1 & 0.653 & 0.612 & 0.105
\end{pmatrix}
$$

Step 8. Determine the ranking order of alternatives for each attribute.

For instance, the ranking order of alternatives for X_1 is, from high to low, A_3, A_1, and A_2 (1 > 0.857 > 0.829). Similarly, the ranking order of alternatives for the other attributes can be determined in the same way. The complete attributewise ranking matrix is listed as follows:

$$
\begin{array}{c}
 \\ 1st \\ 2nd \\ 3rd
\end{array}
\begin{pmatrix}
X_1 & X_2 & X_3 & X_4 \\
A_3 & A_1 & A_1 \text{or} A_2 & A_1 \text{or} A_2 \\
A_1 & A_2 \text{or} A_3 & - & - \\
A_2 & - & A_3 & A_3
\end{pmatrix}
$$

Step 9. Assign the proper weight for each rank of individual alternative.

For example, the weight of each rank for X1 is computed as follows:

The weight of the first rank = 0.35

The weight of the second rank = (0.857 / 1) × 0.35 = 0.3

The weight of the first rank = (0.829 / 1) × 0.35 = 0.29

We can compute the weight of the rest attributes in a similar way. The complete weight matrix will become

$$
\begin{array}{c}
 \\ 1st \\ 2nd \\ 3rd
\end{array}
\begin{pmatrix}
X_1 & X_2 & X_3 & X_4 \\
W(A_6)=0.35 & W(A_1)=0.25 & W(A_1)= W(A_2)=0.3 & W(A_1)= W(A_2)=0.1 \\
W(A_1)=0.3 & W(A_2)= W(A_6)=0.18 & - & - \\
W(A_2)=0.29 & - & W(A_6)=0.21 & W(A_6)=0.015
\end{pmatrix}
$$

Step 10. Convert the weight matrix into concordance matrix.

The concordance matrix is computed as follows:

$$
\begin{array}{c}
 \\ A_1 \\ A_2 \\ A_6
\end{array}
\begin{pmatrix}
1st & 2nd & 3rd \\
0.25+0.3+0.1=0.65 & 0.3 & 0 \\
0.3+0.1=0.4 & 0.18 & 0.29 \\
0.35 & 0.18 & 0.21+0.015=0.225
\end{pmatrix}
$$

Step 11. Form the linear assignment (LP) model.

$$
\begin{aligned}
\text{Max} \quad & 0.62P_{11} + 0.3P_{12} + 0P_{13} \\
& + 0.4P_{21} + 0.18P_{22} + 0.29P_{23} \\
& + 0.35P_{31} + 0.18P_{32} + 0.225P_{33}
\end{aligned}
$$

$$
\begin{aligned}
&S.T.\\
&P_{11}+P_{12}+P_{13}=1\\
&P_{21}+P_{22}+P_{23}=1\\
&P_{31}+P_{32}+P_{33}=1\\
&P_{11}+P_{21}+P_{31}=1\\
&P_{12}+P_{22}+P_{32}=1\\
&P_{13}+P_{23}+P_{33}=1\\
&\forall P_{ij}=0 \text{ or } 1
\end{aligned}
$$

Step 12. Solve the LP model.

The result is computed as below:

p11 = 1 p12 = 0 p13 = 0
p21 = 0 p22 = 0 p23 = 1
p31 = 0 p32 = 1 p33 = 0

or in the matrix form

$$
\begin{array}{c}
 \\ A_1 \\ A_2 \\ A_3
\end{array}
\begin{array}{c}
\begin{array}{ccc} \text{1st} & \text{2nd} & \text{3rd} \end{array} \\
\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}
\end{array}
$$

From the priority matrix, it indicates that the priority of alternatives, from high to low, is A_1, A_3, and A_2, respectively. Therefore, A_1 is the best alternative we should employ.

5. Conclusions

In this paper, we present a new fuzzy MADM method to enhance Bernardo and Blin's linear assignment method. This method can assist the decision maker to determine the ranking order of alternatives under uncertainty.

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