# Lot-streaming with Variable Transfer Batches for a Single Job in a Flow Shop 

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#### Abstract

Lot-streaming is the process of splitting a given lot or job to allow the overlapping of successive operations in a multi-stage production system, thereby reducing the makespan of the corresponding schedule. In the past, the concept of consistent transfer batches is always discussed and applied in the lotstreaming problem. However, the makespan may increase under the constraint of consistent transfer batches. In this paper, we propose a model with the concept of variable transfer batches for a single job in a multi-stage flow shop environment with the objective of minimizing makespan. The results show the proposed model, through flexibly allocating transfer batch sizes on each machine, outperforms others based on the concept of consistent transfer batches in terms of makespan and time complexity.


## 1. Introduction

In the current competitive environment for manufacturing, greater and greater emphasis is being placed on reducing lead times and lowering work-in-process (WIP) inventory levels, both for filling orders for existing products and for bringing new products to market. Therefore, compressing manufacturing lead times and lowering work-in-process inventory levels has given rise to new research problems in planning and scheduling for batch production environment [7].
A technique known as lot streaming or lot splitting has received attention as a scheduling tool to help reduce makespan (i.e., manufacturing lead time) in a batch production environment. Lot streaming is the process of splitting a given lot or job to allow the overlapping of successive operations in multi-stage production systems, thereby reducing the makespan of the corresponding schedule. Scheduling problems for lot streaming have been discussed for a while [1, $2,3,4,5,6,7,8,9]$. Most of the researchers in this area concentrated on consistent transfer batch models. However, the makespan may increase under the constraint of consistent transfer batches. Recently, variable transfer batch models have received a lot of attentions. Trietsch and Baker [7] proposed a variable transfer batch model for lot streaming that can deal with a three-machine problem to minimize the makespan. Since this model cannot solve multi-stage problems when machine (stage) number is more than three, a new lot-streaming model for variable transfer batches is needed.

## 2. The Lot-Streaming Model

In this paper, we address the same single-job, multiple machine flow shop scenario as addressed in [2, 3]. There are $m$ machines and there is a single job containing several identical items, where each item is processed on the machine in the order $1, \ldots, m$. On each machine, the job is to be partitioned into $n$ transfer batches. We assume the transfer batch sizes vary across machines and setup times are not considered. A machine can process a transfer batch only when it has finished processing any previous transfer batch and when the transfer batch has been processed on any previous machines. The detail assumptions are as follows:

1. We are dealing with a flow shop that has a single job, and we want to minimize the makespan.
2. We know the following:
a. number of machines;
b. number of transfer batches;
c. demand, which is deterministic;
d. unit process times, which are constant and deterministic for each machine.
3. The transfer batch sizes may vary between machines.
4. Setup and transfer times are not considered.

After the assumptions are defined, a variable transfer batch model for lot streaming is proposed for a single job in a multi-stage flow shop environment. The procedure of this proposed model is as follows : (1) Determine dominant
machines in an $m$-machine flow shop by using the relaxation algorithm proposed by Glass and Potts [2]. This algorithm performs a systematic search for a machine that is dominated by two adjacent dominant machines in an $m$-machine flow shop. This dominated machine is removed, and repetition of this process continues until no further dominated machines are found. Each pair of adjacent dominant machines structures a critical segment as critical block. Therefore, any production system contains one or more than one critical block. (2) Determine the transfer batch size proportion with a generalization 2-machine problem on each pair of adjacent dominant machines. (3) Reallocate the transfer batch sizes on each dominant machine. Therefore, through reallocating transfer batch sizes, the operations are continuous on each dominant machine and the makespan may reduce. Following are the detail steps:

Step 1 : Initialization
(1). $\quad \mu_{i} \leftarrow i$ for $1 \leq i \leq m ;$
(2). $\quad q_{i} \leftarrow p_{i}$ for $1 \leq i \leq m$;
(3). $\quad l_{i} \leftarrow U$ for $1 \leq i \leq m-1$;
(4). $\quad m^{\prime} \leftarrow m$;
(5). $f \leftarrow 2$ 。

Step 2 : Identifying dominated machines
If $\frac{l_{f-1}+q_{f}}{q_{f-1}+l_{f-1}} \leq \frac{l_{f}+q_{f+1}}{q_{f}+l_{f}}$, the following variables are reset:
(1). $\quad \iota_{f-1} \leftarrow l_{f-1}+q_{f}+l_{f}$;
(2). $\quad \mu_{i^{\prime}} \leftarrow \mu_{i^{\prime}+1}$ for $f \leq i^{\prime} \leq m^{\prime}-1$;
(3). $\quad q_{i^{\prime}} \leftarrow q_{i^{\prime}+1}$ for $f \leq i^{\prime} \leq m^{\prime}-1$;
(4). $\quad l_{i^{\prime}}, \leftarrow l_{i^{\prime}+1}$, for $f \leq i^{\prime} \leq m^{\prime}-2$
(5). $\quad m^{\prime} \leftarrow m^{\prime}+1 ; f \leftarrow \max \{1, f-2\}$ 。

Step 3 : Loop and termination
If $f=m^{\prime}-1$, go to Step 4. Otherwise, execution follows:
(1). Let $f \leftarrow f+1$;
(2). Go to step 2

Step 4 : Calculating allocation ratio in each critical block

$$
R_{i^{\prime}} \leftarrow \frac{l_{i^{\prime}}+q_{i^{\prime}+1}}{q_{i^{\prime}}+l_{i^{\prime}}} \text { for } 1 \leq i^{\prime} \leq m^{\prime}-1
$$

Step 5 : Calculating transfer batch sizes in each critical block
If $R_{i^{\prime}}=\frac{l_{i^{\prime}}+q_{i^{\prime}+1}}{q_{i^{\prime}}+l_{i^{\prime}}} \neq 1$, then execution follows:
(1). Calculating the first transfer batch size

$$
L_{1}^{i^{\prime}} \leftarrow\left(\frac{1-R_{i^{\prime}}}{1-R_{i^{\prime}}^{n}}\right) D
$$

(2). Calculating the other transfer batch sizes

$$
L_{j}^{i^{\prime}} \leftarrow L_{1}^{i^{\prime}} R_{i^{\prime}}^{j-1} \text { for } 2 \leq j \leq n
$$

Else $R_{i^{\prime}}=\frac{l_{i^{\prime}}+q_{i^{\prime}+1}}{q_{i^{\prime}}+l_{i^{\prime}}}=1$, then
(1). Calculating each transfer batch size

$$
L_{j}^{i^{\prime}} \leftarrow \frac{D}{n} \text { for } 1 \leq j \leq n
$$

Step 6: Calculating the makespan

$$
\text { Makespan }=\sum_{i=1}^{m^{\prime}-1} L_{1}^{i^{\prime}}\left(q_{i^{\prime}}+l_{i^{\prime}}\right)+q_{m^{\prime}} D
$$

## 3. Time Complexity Analysis

In this section, the time requirement of this model is analyzed as following:

1. The initialization step: Clearly, it requires $O(m)$ time.
2. The steps of identifying dominated machines and loop and termination: each iteration of the identifying dominated machines step can be implemented in $O(1)$ time. If the identifying dominated machines step fails to detect any dominated machines, then it is executed $m-2$ times. Moreover, for each dominated machine that is found, at most an extra iteration of the identifying dominated machines step is necessary. Since at most $m-2$ machines can be dominated, we deduce that no more $2 m-4$ iterations of identifying dominated machines step are necessary. Therefore, the overall time requirement of the identifying dominated machines step to the loop and termination step is $O(m)$.
3. Calculating allocation ratio and transfer batch sizes in each critical block step: Clearly, they can be implemented in $O\left(m^{\prime} n\right)$ time.
4. Calculating the makespan step: Clearly, it can be implemented in $O(1)$ time.

Concluding the time complexity analysis, the overall time requirement of this proposed model is $O\left(m+m^{\prime} n\right)$.

## 4. Example

Consider a seven-machine problem with $n=2, D=210,\left(p_{1}, p_{2}, p_{3}, p_{4}, p_{5}, p_{6}, p_{7}\right)=(1,2,3,2,4,3,7)$. The solution procedures are described as follows:
Step 1 : Initialization

|  | $i^{\prime}=1$ | $i^{\prime}=2$ | $i^{\prime}=3$ | $i^{\prime}=4$ | $i^{\prime}=5$ | $i^{\prime}=6$ | $i^{\prime}=7$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $q_{i^{\prime}}$ | 1 | 2 | 3 | 2 | 4 | 3 | 7 |
| $l_{i^{\prime}}$ | 0 | 0 | 0 | 0 | 0 | 0 |  |
| $R_{i^{\prime}}$ | 2 | $3 / 2$ | $2 / 3$ | 2 | $3 / 4$ | $7 / 3$ |  |
| P | $\{1,2,3,4,5,6,7\}$ | $m^{\prime}=7 ; f=2$ |  |  |  |  |  |

Step 2 : Identifying dominated machines
$f=2$, because $R_{f-1}>R_{f} \Rightarrow \frac{2}{1}>\frac{3}{2}$, no dominated machine is detected, then go to Step 3.
Step 3: Loop and termination
$f=2$, because $f=2<m^{\prime}-1=6$, execution follows:
(1). Let $f=2+1=3$
(2). Go to Step 2

Step 2: Identifying dominated machines
$f=3$, because $R_{f-1}>R_{f} \Rightarrow \frac{3}{2}>\frac{2}{3}$, no dominated machine is detected, then go to Step 3.
Step 3 : Loop and termination $f=3$, because $f=3<m^{\prime}-1=6$, execution follows:
(1). Let $f=3+1=4$
(2). Go to Step 2

Step 2 : Identifying dominated machines
$f=4$, because $R_{f-1}<R_{f} \Rightarrow \frac{2}{3}<\frac{4}{2}$, we update the following information:

|  | $i^{\prime}=1$ | $i^{\prime}=2$ | $i^{\prime}=3$ | $i^{\prime}=4$ | $i^{\prime}=5$ | $i^{\prime}=6$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $q_{i^{\prime}}$ | 1 | 2 | 3 | 4 | 3 | 7 |
| $l_{i^{\prime}}$ | 0 | 0 | 2 | 0 | 0 |  |
| $K_{i^{\prime}}$ | 2 | $3 / 2$ | $6 / 5$ | $3 / 4$ | $7 / 3$ |  |
| P | $1,2,3,5,6,7$ |  |  |  |  |  |
|  |  |  | $m^{\prime}=6 ; f=2$ |  |  |  |

Step 3 : Loop and termination $f=2$, because $f=2<m^{\prime}-1=5$, execution follows:
(1)
Let $f=2+1=3$
(2). Go to Step 2

Step 2 : Identifying dominated machines

$$
f=3, \text { because } R_{f-1}>R_{f} \Rightarrow \frac{3}{2}>\frac{2+4}{3+2} \text { then go to Step } 3 .
$$

Step 3: Loop and termination
$f=3$, because $f=3<m^{\prime}-1=5$, execution follows:
(1). Let $f=3+1=4$
(2). Go to Step 2

Step 2 : Identifying dominated machines

$$
f=4, \text { because } R_{f-1}>R_{f} \Rightarrow \frac{2+4}{3+2}>\frac{3}{4} \text { then go to Step } 3 .
$$

Step 3 : Loop and termination
$f=4$, because $f=4<m^{\prime}-1=5$, execution follows:
(1). Let $f=4+1=5$
(2). Go to Step 2

Step 2 : Identifying dominated machines
$f=5$, because $R_{f-1}<R_{f} \Rightarrow \frac{3}{4}<\frac{7}{3}$, we update the following information:

|  | $i^{\prime}=1$ | $i^{\prime}=2$ | $i^{\prime}=3$ | $i^{\prime}=4$ | $i^{\prime}=5$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $q_{i^{\prime}}$ | 1 | 2 | 3 | 4 | 7 |  |
| $l_{i^{\prime}}$ | 0 | 0 | 2 | 3 |  |  |
| $R_{i^{\prime}}$ | 2 | $3 / 2$ | $6 / 5$ | $10 / 7$ |  |  |
| P | $1,2,3,5,7$ |  | $m^{\prime}=5 ; f=3$ |  |  |  |

Step 3: Loop and termination $f=3$, because $f=3<m^{\prime}-1=4$, execution follows:
(1). Let $f=3+1=4$
(2). Go to Step 2

Step 2: Identifying dominated machines
$f=4$, because $R_{f-1}<R_{f} \Rightarrow \frac{2+4}{3+2}<\frac{3+7}{4+3}$, we update the following information:

|  | $i^{\prime}=1$ | $i^{\prime}=2$ | $i^{\prime}=3$ | $i^{\prime}=4$ |
| :---: | :---: | :---: | :---: | :---: |
| $q_{i^{\prime}}$ | 1 | 2 | 3 | 7 |
| $l_{i^{\prime}}$ | 0 | 0 | 9 |  |
| $K_{i^{\prime}}$ | 2 | $3 / 2$ | $4 / 3$ |  |
| P | $1,2,3,7$ |  | $m^{\prime}=4 ; f=2$ |  |

Step 3: Loop and termination
$f=2$, because $f=2<m^{\prime}-1=3$, execution follows:
(1). Let $f=2+1=3$
(2). Go to Step 2

Step 2 : Identifying dominated machine
$f=3$, because $R_{f-1}>R_{f} \Rightarrow \frac{3}{2}>\frac{2+4+3+7}{3+2+4+3}$ then go to Step 3.
Step 3 : Loop and termination
$f=3$, because $f=3=m^{\prime}-1$ then go to Step 4.

Step 4 : Calculating allocation ratio in each critical block

|  | $i^{\prime}=1$ | $i^{\prime}=2$ | $i^{\prime}=3$ |
| :---: | :---: | :---: | :---: |
| $K_{i^{\prime}}$ | 2 | $3 / 2$ | $4 / 3$ |
| P | $1,2,3,7$ |  | $m^{\prime}=4 ; f=2$ |

Step 5: Calculating transfer batch sizes in each critical block
(1) $L_{1}^{1}=\left[\frac{1-2}{1-2^{2}}\right] \times 210=70$
(2) $L_{2}^{1}=2^{1} \times 70=140$

| $L_{j}$ | $i^{\prime}=1$ | $i^{\prime}=2$ | $i^{\prime}=3$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{~J}=1$ | 70 | 84 | 90 |
| $\mathrm{~J}=2$ | 140 | 126 | 120 |

Step 6 : Calculating the makespan
(1). Scheduling information

| $\mathrm{S}_{\mathrm{i}, \mathrm{j}}$ | $\mathrm{I}=1$ | $\mathrm{i}=2$ | $\mathrm{i}=3$ | $\mathrm{i}=4$ | $\mathrm{i}=5$ | $\mathrm{i}=6$ | $\mathrm{i}=7$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{j}=1$ | 0 | 70 | 238 | 508 | 688 | 1048 | 1318 |
| $\mathrm{j}=2$ | 70 | 210 | 490 | 868 | 1108 | 1588 | 1948 |


| $\mathrm{C}_{\mathrm{i}, \mathrm{j}}$ | $\mathrm{I}=1$ | $\mathrm{i}=2$ | $\mathrm{i}=3$ | $\mathrm{i}=4$ | $\mathrm{i}=5$ | $\mathrm{i}=6$ | $\mathrm{i}=7$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{j}=1$ | 70 | 210 | 490 | 688 | 1084 | 1318 | 1948 |
| $\mathrm{j}=2$ | 210 | 490 | 868 | 1108 | 1588 | 1948 | 2788 |

(2). Makespan $=\mathrm{C}_{7,2}=2788$

Note:

1. The optimal solution for the consistent transfer batch model [3] is 2820.
2. The heuristic solution for the equal transfer batch model is 3045 .

## 5. Conclusions

In this paper, a variable transfer batch model and solution algorithm is proposed to solve an $m$-machines lot-streaming problem, where the objective is to minimize the makespan. We found that the solution of the lot-streaming problem depends on whether there are "dominated" machines in a production system as indicated in step 2 and step 3. Any production system dealt by step 2 and step 3 must structure more than one critical block. The solution of each critical block is found by a generalization of the 2-machine procedure. Therefore, the makespan of our model is smaller than model with consistent transfer batches in the flow shop production system. The time complexity of our model for solving transfer batch sizes is $O\left(m+m^{\prime} n\right)$, where $m^{\prime}$ is the number of dominant machines. The proposed model is more efficient than Glass and Potts [2] approach whose time complexity is $O\left(m+m^{\prime} n^{m-\alpha}\right)$.

Three related research directions are as follows: (1) Develop an optimal variable transfer batch model for a single job in a flow shop, (2) Develop a multiple-job model with variable transfer batches, (3) Evaluate models' performance based on different objective functions, such as minimizing mean flow time or WIP inventory levels.

## Reference

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