Optimal Control Policy for a Supplier Management System with Stochastic Variability

Kenichi NAKASHIMA

Osaka Institute of Technology, Department of Industrial Management
(nakasima@dim.oit.ac.jp)

Abstract

The manufacturing of a product involves using several different kinds parts. Some of these parts are supplied by outside vendors. This adds a layer of complexity to the already challenging problem of managing the logistics of the production operation. It is important to efficiently control the ordering of parts from the outside vendors as their lead times tend to be relatively long (compared to the internal lead times) due to, among other things, the geographical distances of outside vendors from the production facility. Since the production of a product often requires interaction, it is crucial to develop a coordinated supplier management system for the entire operation to work efficiently.

Just-in-time (JIT) production systems have been successfully implemented in many factories. To maintain the JIT production system, two kinds of kanbans, that is, a production-ordering kanban and a withdrawal kanban are used as tools to control the production and withdrawal quantities in each process. In particular, the withdrawal kanban used for withdrawing from a vendor is called a supplier kanban. The number of supplier kanbans attributed to a given part represents the maximum number of units of that part available in stock for the use in production. Once the number of kanbans is agreed upon, the system performs automatically.

The purpose of this paper is to obtain the optimal control policy for a supplier management system under stochastic variability. The supplier management system has a single process with two kinds of kanbans. One is a production-ordering kanban used for production and the other is a supplier kanban used for ordering parts. We assume that there are line stoppages for machine maintenance in the process, which leads to a random production capacity. The problem is formulated into a Markov Decision Process(MDP). It is a class of stochastic sequential processes in which the reward and transition probability depend only on the current state of the system and the current action. The MDP models have gained recognition in such diverse fields as engineering, economics, communications and so on. We can obtain the optimal control policy for the supplier management system that minimizes the expected average cost per period. Comparisons between the optimal control policy and the kanban policy are shown numerically.

1. Introduction

Just-in-time (JIT) production systems have been successfully implemented in many factories. To maintain the JIT production system, two kinds of kanbans, that is, a production-ordering kanban and a withdrawal kanban are used as tools to control the production and withdrawal quantities in each process. In particular, the withdrawal kanban used for withdrawing from a vendor is called a supplier kanban. There are two kinds of withdrawal systems: a constant quantity, nonconstant cycle withdrawal system and a constant cycle, nonconstant quantity withdrawal system. The processes within the Toyota Motor Corporation use the constant quantity withdrawal system, whereas the supplier kanban exclusively uses the constant cycle withdrawal system due to geographical distance[10]. Then supplier kanban system is one of supplier management systems.

Modeling and analysis of JIT under realistic assumptions present a number of challenges, including the ability to:

conduct both qualitative and quantitative analysis of the system, and model control policies[2, 3, 4, 8, 9, 11, 15, 16]. As for the constant cycle withdrawal system, Kimura and Terada[6] have dealt with a serial JIT production system and shown by simulation how fluctuations of demand influence the fluctuations of the production and inventory in the preceding processes. Deleersnyder *et al.*[1] have investigated effects of factors such as the number of kanbans, the machine reliability and the demand variability on the performance of a JIT production system with only the production-ordering kanban using a discrete time Markov chain. Ohno et al.[12] have discussed the JIT production system with the production-ordering and the supplier kanbans with stochastic demand. They have derived its stability condition and determined optimal numbers of two kinds of kanbans that minimize an expected average cost per period. Furthermore, the kanban control with optimal numbers of two kinds of kanbans have been compared with an optimal policy obtained by a Markov decision process(MDP)[5, 14] in Ohno and Nakashima[13]. Kirkavak and Dincer[7] have dealt with a serial JIT production system with the production-ordering and the withdrawal kanbans, and have proposed an approximate decomposition algorithm. They, however, assume that the processing time of each process is exponentially distributed, the lead time of delivery of the parts is negligible and the excess demand is not backlogged.

The purpose of this paper is to obtain the optimal control policy for a supplier management system with temporary line stoppage under stochastic variability. We also compare the optimal policy with kanban policy by numerical experiments. In §2, we describe the single-process single-item supplier management system, and explain the operation of the kanban policy. In §3, we formulate it into an MDP without using suppiler kanbans that determines an optimal ordering policy minimizing the expected average cost per period. The problem is solved by the policy iteration method[5]. In §4, we numerically compare the kanban control with the optimal ordering policy obtained by the PIM.

2. Supplier Management System with Kanbans

The manufacturing of a product involves using several different kinds parts. Some of these parts are supplied by outside vendors. This adds a layer of complexity to the already challenging problem of managing the logistics of the production operation. It is important to efficiently control the ordering of parts from the outside vendors as their lead times tend to be relatively long (compared to the internal lead times) due to, among other things, the geographical distances of outside vendors from the production facility. Since the production of a product often requires interaction, it is crucial to develop a coordinated supplier management system for the entire operation to work efficiently. In order to control the quantities and timing of the parts from outside vendors, supplier kanbans can be used [10]. The number of kanbans attributed to a given part represents the maximum number of units of that part available in stock for the use in production. Once the number of kanbans is agreed upon, the system performs automatically.

We consider a single-process single-item supplier management system with the production-ordering and supplier kanbans. That is, the preceding process is a vendor and the constant cycle withdrawal system is used. For simplicity, the constant cycle is taken as one period. Denote by M and N the number of production-ordering kanbans and that of supplier kanbans, respectively. A product is put in a container with one production-ordering kanban. When a subsequent process carrier or the demand withdraws products, the production-ordering kanbans are detached from containers. The process produces the products according to the ordinal sequence of the detached production-ordering kanbans. A part used for production is also placed in a container with one supplier kanban. When parts are consumed for production, the supplier kanbans are detached from containers. Then, the quantity of the parts used in period $n(=1, 2, \cdots)$ is ordered to the vendor by the detached supplier kanbans at the beginning of period $n+1$, and the order is delivered at the beginning of period $n+L+1$. That is, the lead time of the delivery is L. It is assumed that the demand of the product in each period is independent and identically distributed(*i.i.d.*) with mean D and that the excess demand is backlogged.

Let C be the maximum production capacity of the process per period. We assume that there exist temporary line stoppages for adjusting a machine in the system. We consider the effect of them as $\alpha_n C$ in period n, where $0 \leqq \alpha_n \leqq 1$. The following notations are used: ,

D_n = the demand in period n,
I_n = the inventory level of the parts at the beginning of period n,
J_n = the number of detached production-ordering kanbans at the beginning of period n,
B_n = the backlogged demand at the beginning of period n,
O_n = the quantity of the parts ordered at the beginning of period n,

and

P_n = the production quantity in period *n*.

Since the quantity of the parts used for production in period *n-1* is ordered to the vendor at the beginning of period *n*, the following relation holds:

$$O_n = P_{n-1}. \tag{1}$$

Since the lead time of the delivery is *L* and the number of detached supplier kanbans in period *j* is P_j , it holds that for *n=1,2,* · · ·

$$N = I_n + \sum_{j=n-L}^{n-1} P_j. \tag{2}$$

where P_0, P_{-1}, P_{-L+1} are given. Moreover, the process should produce the products to the number of detached production-ordering kanbans, if it is under the production capacity and the parts are available. Therefore,

$$P_n = \min(I_n, J_n, \alpha_n C). \tag{3}$$

Notice that $0 \leqq J_n \leqq M$ under the determined number of production-ordering kanban, *M*. Then, production and ordering quantities at the beginning of period *n* are self-regulatively determined by the kanbans. We define equation(3) the kanban production policy. On the other hand, it follows from equation (1) and (2) that

$$O_n = N - I_n - \sum_{j=n-L+1}^{n-1} O_j. \tag{4}$$

This control policy is called the kanban ordering policy.

3. Optimal Control Policy

Let us consider an optimal control problem without using supplier kanbans of the production system shown in Figure1. Let I_{max} be the maximum possible stock of the parts. Denote by U_n the total inventory level of products which takes a negative value if the backlogged demand occurs. Let U_{max} be the maximum possible stock of products. The production quantity in period *n*, P_n is determined by equation(3). Since the supplier kanbans are not used, both the ordering quantity at the beginning of period *n*, O_n should be determined by a state of the system observed at the beginning of period *n*. That is, the problem is formulated into the MDP to determine the optimal ordering policy that minimizes the expected average cost per period.

Denote by s_n the state of the system at the beginning of period *n*. The state s_n consists of the vector of quantities of the parts ordered from period *n-L+1* to period *n-1*, the inventory level of parts and the total inventory level of products at the beginning of period *n*. Consequently,

$$s_n = \min(O_{n-L+1}, \mathbf{L}, O_{n-1}, I_n, U_n). \tag{5}$$

In particular, if *L=1*, then s_n=(I_n, U_n). Denote by *S* and |*S*| a set of all possible states and the total number of states, respectively.

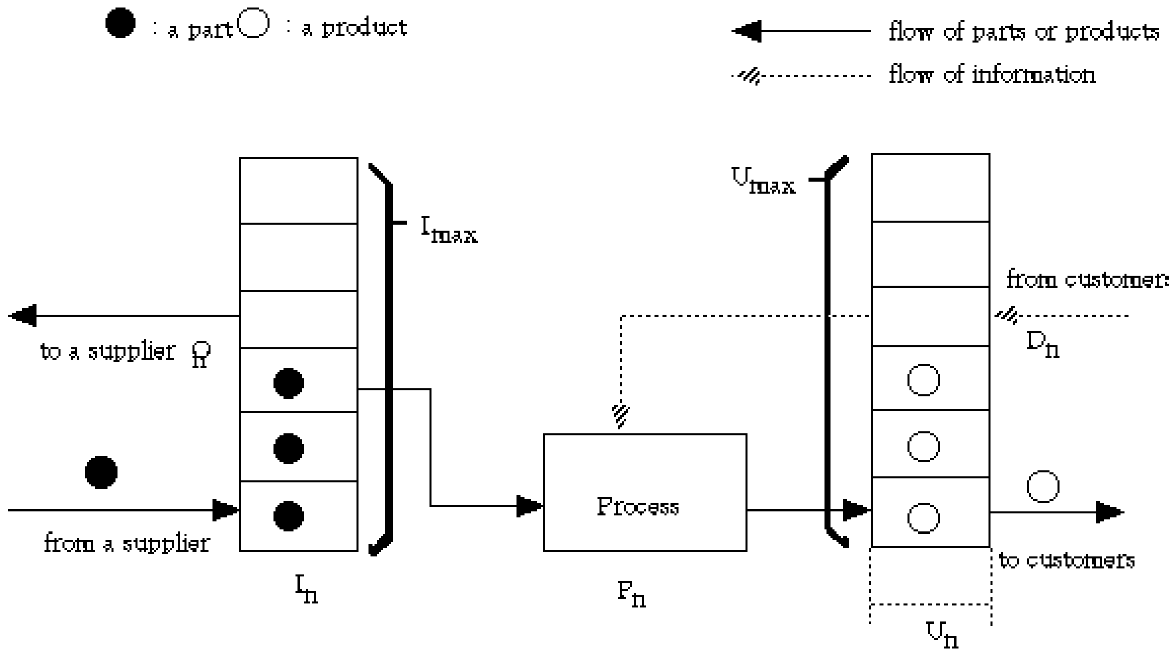

Figure1: A Production system

Let $K(s_n)$ be a set of possible ordering quantities in state s_n . That is,

$$K\ (s_n) = \left\{0, 1, \cdots, I_{\max} - I_n - \sum_{j=n-L+1}^{n-1} O_j\ \right\}.$$

If L=1, then $K(s_n) = \{\ 0,\ 1,\ \cdots, I_{max}\text{-}I_n\ \}$.

Selected controls, $k \in K(s_n)$ in state s_n at the beginning of period n become O_n . Then, the following relations hold:

$$O_n = k,\ P_n = \min(I_n, J_n, \alpha_n C\ ). \tag{6}$$

$$I_{n+1} = I_n + O_{n-L+1} - P_n, \tag{7}$$

and

$$U_{n+1} = U_n + P_n - D_n. \tag{8}$$

Hence, the transition probability from s_n to s_n+1 is given by

$$P_{s_n s_{n+1}}(k_1, k_2) = \begin{cases} P_r\{D_n = d,\ \alpha_n = \alpha\} & s_{n+1} = [O_{n-L+2}, \cdots, k, I_n + O_{n-L+1} - P_n, U_n + P_n - d\ ], \\ 0, & otherwise. \end{cases} \tag{9}$$

Moreover, when $k = O_n$ in state s_n, the expected average cost per period, $r_{s_n}(k)$ is given using the following cost parameters as:

$$r_{s_n}(k\ ) = C_I I_n + C_U \max\{0, U_n\} + C_B \max\{0, -U_n\} + C_O H\ (U_n < 0), \tag{10}$$

where C_I= inventory cost of one part per period, C_U= inventory cost of one product per period, C_B= backlogged cost of one product per period, and C_O= backlogged cost per once. *H(E)* is the indicator function of event *E*, that is, *H(E)*=1 ,if event *E* occurs; =0, otherwise.

Let number the state s_n by s (= 1, $\cdots$, $|S|$). An undiscounted MDP that minimizes the expected average cost per period, g is formulated as the following optimality equation:

$$g + v_s = \min_{k \in K(s)} \left\{ r_s(k) + \sum_{s' \in S} P_{ss'}(k) v_{s'} \right\} (s \in S) \tag{11}$$

where v_s denotes the relative value when the production system starts from state s [5]. An optimal ordering policy is determined as k that minimizes the right-hand side of equation (11) for each state s. The problem can be solved by the policy iteration method(PIM).

4. Numerical Results

Consider the production system with I_{max} =14 and D =3. Suppose that the maximum backlogged demand is denoted by *Umin* , where we set it as -4. The cost parameters are set as C_I=1.0, C_U=2.0, C_B=10.0, C_O=100.0. The distribution of the demand, D_n(n =1, 2, $\cdots$) is given by :

$$P_r \left\{ D_n = D - \frac{1}{2}Q + i \right\} = \binom{Q}{i} \left(\frac{1}{2}\right)^Q, 0 \le i \le Q$$

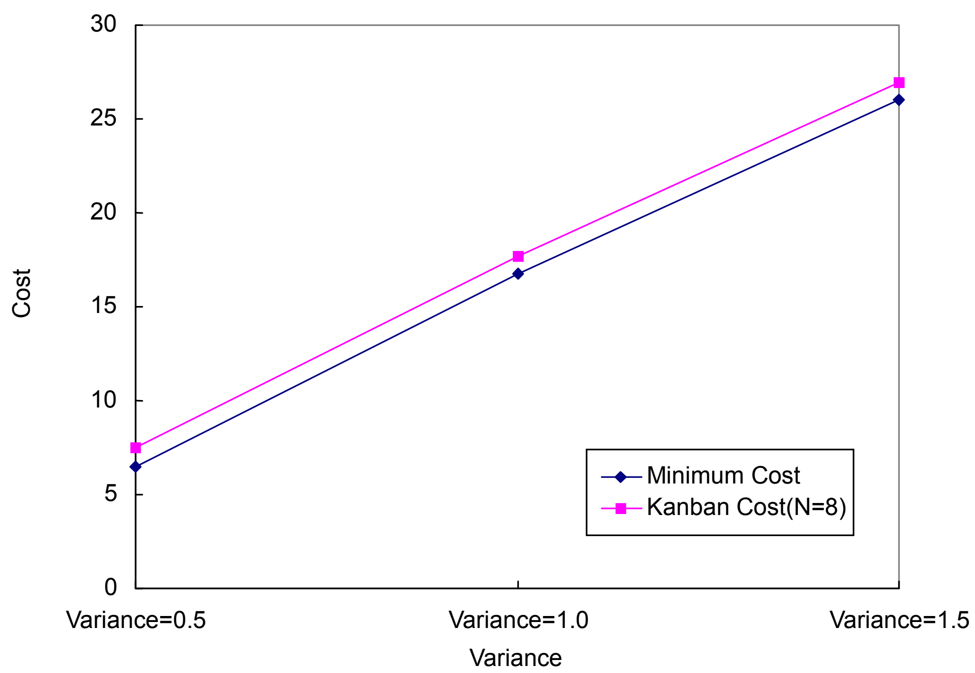

Figure2: Comparison between optimal policy and kanban policy with M=4

where Q is an even number less than or equal to 2D. The computation was carried on a VP 2100/10RE computer at the Computation Center of Kyoto University.

Figure 2 shows the expected average costs of the optimal policy and the kanban policy with L=1 and $Pr\{\alpha_n$=0.95$\}$=1, respectively. Hence, we can see that the kanban control is sub-optimal, or near optimal. In addition, when the variance of the demand increases, the each cost increases in proportion. This shows the importance of the smoothing of production bases in the kanban system.

5. Conclusion

In this paper, we consider the supplier management system with stochastic variability. The optimal control problem without using kanbans is formulated into the MDP to determine the optimal policy that minimizes the expected average cost per period. Comparisons between optimal policies obtained by MDP and the kanban policy show the properties of the supplier kanban system.

References

[1] Deleersnyder, J. L., Hodgson, T. J., Muller(-Malek), H. and O'Grady, P. J.: Kanbancontrolled pull systems: an analytic approach, Management Science, Vol.35, pp.1079-1091, 1989

[2] Gupta, S.M. and Al-Turki, Y. A. Y.: An algorithm to dynamically adjust the number of kanbans in stochastic processing times and variable demand environment, Production Planning and Control, Vol. 8(2), pp.133-141, 1997

[3] Gupta, S.M. and Al-Turki, Y. A. Y.: Adapting Just-In-Time Manufacturing Systems to Preventive Maintenance Interruption, Production Planning and Control, Vol. 8(4), pp.349-359, 1998

[4] Gupta, S.M., Al-Turki, Y. A. Y. and Perry, R. F.: Flexible Kanban System, International Journal of Operations and Production Management, Vol. 19(7), pp.1065-1093, 1998

[5] Howard, R. A.: Dynamic Programming and Markov Processes (Cambridge: The M. I. T. Press), 1960

[6] Kimura, O., and Terada H.: Design and analysis of pull system, a method of multi-stage production control,International Journal of Production Research, Vol.19, pp.241-253, 1981

[7] Kirkava, N., and Dinçer, C.: Performance evaluation models for single-item periodic pull production sys-tems, Journal of the Operational Research Society, Vol.47, pp.239-250, 1996

[8] Mitra, D., and Mitrani, I.: Analysis of a kanban discipline for cell coordination in production lines. I,Management Science, Vol.36, pp.1548-1566, 1990

[9] Mitra, D., and Mitrani, I.: Analysis of a kanban discipline for cell coordination in production lines. II,Operations Research,Vol.39, pp.807-823, 1991

[10] Monden, Y.: Toyota Production System, second ed.(Norcross, Georgia: Institute of Industrial Engineers,1993

[11] Moore, K. E. and Gupta, S.M.: Stochastic Colored Petri Net (SCPN) Models of Traditional and Flexible Kanban Systems, International Journal of Production Research, Vol.37(9), pp.2135-2158, 1999

[12] Ohno, K., Nakashima, K., and Kojima, M.: Optimal numbers of two kinds of kanbans in a JITproduction system, International Journal of Production Research, Vol.33, pp.1387-1401, 1995

[13] Ohno, K. and Nakashima, K.: Optimality of a Just-in-Time production system, Proceedings of APORS'94 (Singapore:World Scientific), pp.390-398, 1995

[14] PutermanUTERMA, M. L.: Malkov Decision Process, Discrete Stochastic Dynamic Programming, New York; John Wiley & Sons, 1994

[15] Tayur, S. R.: Properties of serial kanban systems, Queueing Systems, Vol.12, pp.297-318, 1992

[16] Tayur, S. R.: Structural properties and a heuristic for kanban-controlled serial lines, Management Science,Vol.39, pp.1347-1368, 1993

[17] Veatch, M. H., and Wein, L. M.: Optimal control of a two-station tandem production/inventory system,Operations Research, Vol. 42, pp.337-350, 1994