LP based Product Mix Decision Models

for Capacity Constrained Process Industry

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Abstract

We consider two-stage polymer (resin) production system where the products from the first stage (intermediate products) are used for the production in the second stage at the same plant. The first stage is to make various grades of polymer powders using several kinds of raw materials. The second stage is to mix several types polymer powders with additives and colors to produce final products. The percentage of polymer (recipe) is given for each final product. The amount of polymer powders is determined by the production volume of the final products.

It is observed that the several lines for a certain type of polymer production in the first stage (polymerization process) is the bottleneck of the whole process, and the demand of the final products exceeds this bottleneck capacity. Since final products have unequal profit per unit, an optimal product mix problem naturally arises.

Given the recipe matrix, capacity of polymerization process, and profit per unit of final products, we present three LP models, which help identify the groups of products to be produced more or less. The decision variables in the models include annual production volume of each final product, Xj's. We assume that the demand of each final product is controllable within a certain range, which imposes lower and upper limit on Xj's as constraints. The lower and upper limit implies the minimum amount to be supplied to the market, and maximum possible market share, respectively.

The first model (Model 1) maximizes the total profit by changing the production volume of final products. The second model (Model 2) minimizes the usage of bottleneck polymer process while maintaining the current total profit. The third model (Model 3) assesses the amount of additional capacity of the bottleneck process required to balance the first and second stages.

Applying the Model 1 to a resin manufacturer in Korea, additional 3.3 million dollars of profit (approximately 3.7% of the total annual profit) could have been achieved by slightly changing the product mix of the final products. Model 2 suggests that the company could have saved 3.8% of bottleneck capacity while maintaining current level of total profit. Model 3 shows that 15% increased bottleneck capacity results in 18% increase in the total profit while the first and second stages are well balanced. These results can shed lights on resetting the target market.

1. Introduction

We consider two-stage polymer (resin) production system where the products of the first stage (intermediate products) are used for the production of the final products in the same plant. The first stage is called 'polymerization process' and is to make various grades of polymer powders using several different types of raw materials. Polymer powders are then stored in several silos to be used in the second production stage. The second stage is called 'compounding process' and is to mix one or more polymer powders with additives and color to produce final products.

Currently the company operates four production lines in the polymerization process. Line 1 and line 2 produce 6 grades of type A polymer powder and 5 grades of type B polymer powder, respectively. Line 3 and line 4 produce 8 grades of type C. These 19 grades of polymer powders are then used for final products. The amount of each polymer

powder grade compounded into one unit of a final product is predetermined according to the 'recipe'. Each amount of polymer to be produced thus depends on the production volume of the final products. Production of a final product is triggered by customer order. The annual sales volume of the final products heavily depends upon the marketing force of the company. In 1999, the demand of the final products exceeded production capacity.

It is observed that line 1 in the polymerization process is the bottleneck of the whole process. That is, line 1 fully operates throughout the year, while line 2, 3, and 4 are not fully utilized. Since final products have their own recipe matrix and unequal profit per unit, an optimal product mix problem naturally arises. The term 'profit per unit' means the difference of the market price and the direct material cost of a unit of final product. A preliminary analysis shows that some of the 188 final products consume too much bottleneck polymer powders while their unit profit is relatively low. If the annual sales volume of such products is reduced to a certain extent, then the remaining bottleneck capacity can be utilized for high-profit final products to increase the total profit.

Given the recipe matrix, capacity of polymerization process, and profit per unit of final products, we present three LP models, which help identify the groups of products to be produced more and to be produced less. The decision variables in the models include annual production volume of each final product, Xj's. We assume that the demand on each final product is controllable within a certain range, which imposes lower and upper limit on Xj's as constraints. The lower and upper limit implies the minimum amount to be supplied to the market, and maximum possible market share, respectively. This assumption is practically reasonable since marketing force of the company can, to a certain extent, control the annual sales volume of each final product.

The first model (Model 1) maximizes the total profit by slightly changing the production volume of final products. The second model (Model 2) minimizes the usage of bottleneck polymer process while maintaining the current total profit. The third model (Model 3) assesses the amount of additional capacity of the bottleneck process required to balance the first and second stages.

2. Models

2.1 Model 1: maximizing total profit

In this model we find an optimal product mix of the final products, which maximizes the total annual profit under the assumption that the marketing department could have controlled the sales volume by \pm 10% last year. The model is given below.

max	$\Sigma b_j X_j$	(1)
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st. $\Sigma_j a_{ij} X_j = c_i$ (2)

 $\Sigma_{i} c_{i}/k_{i} \leq T, i = 1 \text{ to } 6$ (3)

- $\Sigma_{i} c_{i}/k_{i} \leq T, i = 7 \text{ to } 11$ (4)
- $\sum_{i} c_{i}/k_{i} \le 2T, i = 12 \text{ to } 19$ (5) $0.9 X^{0} \le X_{i} \le 1.1 X^{0}$ (6)

$$\begin{array}{l} 0.5 \ X_j \leq 1.1 \ X_j \\ X_i \geq 0, \end{array} \tag{6}$$

where

 X_i : production volume of final product j, j = 1, 2, ..., 188,

X_i⁰: actual production(sales) volume of final product j in 1999,

 c_i : production quantity of polymer powder i, i = 1, 2, ..., 19,

 k_i : hourly production rate of polymer powder i, i = 1, 2, ..., 19,

a_{ii} : amount of polymer powder grade i which is used to produce one unit of final product j,

b_i : unit profit of final product j, and

T: total annual production time (in hours) per polymer production line

Constraints (3) and (4) are related to the capacity limit of line 1 and line 2 for the production of type A and type B polymer grades in the polymerization process, respectively. Constraint (5) is related to the capacity limit of line 3 and 4 for the production of type C polymer grades. Constraint (6) assumes that the marketing force could have changed the sales volume of each final product by \pm 10% last year.

It is found that it would have been beneficial if 54 of 188 final products were sold 10% less in last year while the rest of the final products were sold 10% more. The company could have earned additional 3.3 million dollars (approximately 3.7% of the total profit) last year without additional investments for capacity expansion. In this scenario, the bottleneck polymer production line, line 1, is also fully utilized while other lines are not. Table 1 summarizes the results of model 1.

	As is in 1999	Model 1: Max total profit
total amount of final products	199,155.9 ton	202,077,4 ton
utilization of line 1	100 %	100 %
type A grades used	55,229 ton	55,325 ton
type B grades used	8,051 ton	8,388 ton
type C grades used	99,891 ton	100,099 ton
total annual profit	88.8 million dollars	92.1 million dollars

Table 1 Results of Model 1

2.2 Model 2: minimizing the usage of bottleneck polymer powders while maintaining total profit

In this model we find an optimal product mix of the final products, which minimizes the bottleneck polymer powder grades while maintaining the level of the total profit as in 1999. We assume that the marketing department could have controlled the sales volume of the last year by \pm 10%. The model is given below.

min	$\Sigma_{i} \Sigma_{j} a_{ij} X_{j}$	(7))
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- st. $\Sigma_j a_{ij} X_j = c_i, i = 7 \text{ to } 19$ (8)
 - $\Sigma_{i} c_{i}/k_{i} \leq T, i = 7 \text{ to } 11$ $\Sigma_{i} c_{i}/k_{i} \leq 2T, i = 12 \text{ to } 19$ (9)
 (10)
 - $$\begin{split} & \Sigma_i \, c_i / k_i \leq 2 \mathrm{T}, \, i = 12 \text{ to } 19 \end{split} \tag{1} \\ & \Sigma_i \, b_i \, X_i \geq \mathrm{P}, \end{split} \tag{1}$$
 - $\sum_{j} b_{j} X_{j} \ge P,$ (11) 0.9 $X_{i}^{0} \le X_{i} \le 1.1 X_{i}^{0}$ (12)
 - $X_i \ge 0,$

where P is the total profit in 1999.

The objective function in (7) is to minimize the total amount of type A polymer powder grades. Constraints (9) and (10) are related to the capacity limit of line 1 and line 2 for the production of type B and type C polymer grades in the polymerization process, respectively. Constraint (11) is related to the capacity limit of line 3 and 4 for the production of type C polymer grades. Constraint (12) assumes that the marketing force could have changed the sales volume of each final product by \pm 10% last year.

Results suggest thatthe company could have saved 3.8% of the bottleneck capacity while maintaining current level of total profit with slight change in product mix last year. The remaining capacity could have been devoted to more profitable products. This unused capacity is worth approximately 3 million dollars. To achieve this, 58 of 188 final products should have been sold 10% less while the rest of the final products were sold 10% more last year. It is noted that the total amount of final products decreases in model 1. That indicates the possibility of additional capacity savings in the stage of final production. Also the company could have saved some capacity in the production of type C grades. Table 2 summarizes the results of model 2.

	As is in 1999	Model 2: Min bottleneck polymer
total amount of final products	199,155.9 ton	194,160.6 ton
utilization of line 1	100 %	96.2 %
type A grades used	55,229 ton	53,119 ton
type B grades used	8,051 ton	8,297 ton
type C grades used	99,891 ton	96,347 ton
total annual profit	88.8 million dollars	88.8 million dollars

Table 2 Results of Model 2

2.3 Model 3: capacity expansion in bottleneck process

Model 3 assesses the amount of additional capacity of the bottleneck process required to balance the first and second production stages. A preliminary analysis shows that the total capacity of the final production lines far exceeds the polymerization process, and that at least 10% increase in the capacity of polymerization line 1 is needed to balance the two production stages. With an assumption that the marketing department could have controlled the sales volume of the final products by \pm 20% last year, we try to maximize the total profit. The model is given below.

max $\Sigma b_i X_i$	(13)
st. $\Sigma_i a_{ii} X_i = c_i$	(14)
$\Sigma_i c_i / k_i \le 2T, i = 1$ to 6	(15)
$\Sigma_i c_i / k_i \le 2T, i = 7$ to 11	(16)
$\Sigma_{i} c_{i}/k_{i} \leq 4T, i = 12 \text{ to } 19$	(17)
$0.8 X^0 \le X \le 1.2 X^0$	(18)

$$0.8 X_j^{\circ} \le X_j \le 1.2 X_j^{\circ}$$
(18)
$$\Sigma X \le F$$
(19)

 $X_j \ge 0,$

where F is the total production capacity of the final products.

Note that constraints (15) to (17) allow enough capacity in the polymerization process so that the amount of additional capacity needed can be measured. Constraint (19) limits the total mount of final products to F.

It is found that additional 15% of the capacity in the bottleneck polymerization process is required to balance the first and the second production stages. Production lines 2, 3, and 4 for type B and C grades remain unsaturated even in this case. Furthermore the company could have increased the total annual profit by 18%. Table 2 summarizes the results of model 3.

	As is in 1000	Model 3: max profit while
	AS 15 III 1999	balancing two production stages
total amount of final products	199,155.9 ton	230,170.5 ton
utilization of line 1	100 %	115 %
type A grades used	55,229 ton	63,531.3 ton
type B grades used	8,051 ton	9,638.9 ton
type C grades used	99,891 ton	114,452.2 ton
total annual profit	88.8 million dollars	104.8 million dollars

Table 3 Results of Model 3

3. Conclusion

Three LP models are developed to suggest better product mix policies for a process industry in Korea. The first two models show similar results in identifying groups of final products to be produced more and to be produced less. The results can shed lights on resetting the target market. These two models can be further combined into a goal programming model. The third model assesses additional capacity of the bottleneck process to balance the first and second stages of production.

Motivated by the results of the models, the company is currently in the process of shifting its target market towards more profitable buyers with an effort to increase the bottleneck capacity in the polymer powder production process. And the technology division of the company is reformulating the recipe of the polymer powders to increase the market value of the final products. With these effort and changes in recipe and capacity, new product mix decision problem is being developed.

References

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