Artificial Neural Networks supported by Change-Point Detection for Interest Rates Forecasting

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Abstract

Interest rates are one of the most closely watched variables in the economy. They have been studied by a number of researchers since they strongly affect other economic and financial parameters. Contrary to other chaotic financial data, the movement of interest rates has a series of change points due to the monetary policy of the U.S. government. The basic concept of this proposed model is to obtain intervals divided by change points, to identify them as change-point groups, and to use them in interest rates forecasting. The proposed model consists of three stages. The first stage is to detect successive change points in the interest rates dataset. The second stage is to forecast the change-point group with the backpropagation neural network (BPN). The final stage is to forecast the output with BPN. This study then examines the predictability of the integrated neural network model for interest rates forecasting using change-point detection.

1. Introduction

Interest rates are one of the most closely watched variables in the economy. Their movements are reported almost daily by the news media since they directly affect our everyday lives and have important consequences for the economy. There exist extensive studies in this area using statistical approaches, such as term structure models, vector autoregressive (VAR) models, autoregressive conditionally heteroskedastic (ARCH) - generalized autoregressive conditionally heteroskedastic (GARCH) models and other time series analysis approaches.

Currently, several studies have demonstrated that artificial intelligence (AI) approaches, such as fuzzy theory (Ju et al., 1997) and neural networks (Deboeck and Cader, 1994), can be alternative methodologies for chaotic interest rates data (Larrain, 1991; Peters, 1991; Jaditz and Sayers, 1995). Previous work in interest rates forecasting has tended to use statistical techniques and AI techniques in isolation. However, an integrated approach, which makes full use of statistical approaches and AI techniques, offers the promise of increasing performance over each method alone (Chatfield, 1993). It has been proposed that the integrated neural network models combining two or more models have the potential to achieve a high predictive performance in interest rates forecasting (Kim and Noh, 1997).

In general, interest rates data is controlled by government's monetary policy more than other financial data (Gordon and Leeper, 1994; Strongin, 1995; Christiano et al., 1996; Leeper et al, 1996; Bagliano and Favero, 1999). Especially, banks play a very important role in determining the supply of money. Much regulation of these financial intermediaries is intended to improve their control. One crucial regulation is reserve requirements, which make it obligatory for all depository institutions to keep a certain fraction of their deposits in accounts with the Federal Reserve System, the central bank in the United States (Mishkin, 1995). The government takes intentional action to control the currency flow which has direct influence upon interest rates. Therefore, we can conjecture that the movement of interest rates has a series of change points which occur because of the monetary policy of the government.

Based on these inherent characteristics in interest rates, this study suggests the change-point detection for interest rates forecasting. The proposed model consists of three stages. The first stage is to detect successive change points in the interest rates dataset. The second stage is to forecast the change-point group with BPN. The final stage is to forecast the output with BPN. This study then examines the predictability of the integrated neural network models for interest rates forecasting using change-point detection.

Through the discovery of different patterns in the U.S. Treasury securities, the focus then shifts to the change-point detection-assisted modeling of Treasury bill rates with 1 years' maturity and Treasury bond rates with 30 years' maturity. Input variable selection is based on the causal model of interest rates presented by the econometricians. To explore the predictability, we divided the interest data into the training data over one period and the testing data over the next period. The predictability of interest rates is examined using the metrics of the root mean squared error (RMSE), the mean absolute error (MAE) and the mean absolute percentage error (MAPE).

In section 2, we outline the development of change-point detection and its application to the financial economics. Section 3 describes the proposed integrated neural network model details. Section 4 and 5 report the processes and the results of the case study. Finally, the concluding remarks are presented in Section 6.

2. Change-Point Detection

2.1. Application of Change-Point Detection in the Financial Economics

Financial analysts and econometricians have frequently used piecewise-linear models which also include change-point models. They are known as models with structural breaks in economic literature. In these models, the parameters are assumed to shift — typically once — during a given sample period and the goal is to estimate the two sets of parameters as well as the change point or structural break.

This technique has been applied to macroeconomic time series. The first study in this field is conducted by Rappoport and Reichlin (1989) and Perron (1989, 1990). From then on, several statistics have been developed which work well in a change-point framework, all of which are considered in the context of breaking the trend variables (Banerjee et al., 1992; Christiano, 1992; Zivot and Andrews, 1992; Perron, 1995; Vogelsang and Perron, 1995). In those cases where only a shift in the mean is present, the statistics proposed in the papers of Perron (1990) or Perron and Vogelsang (1992) stand out. However, some variables do not show just one change point. Rather, it is common for them to exhibit the presence of multiple change points. Thus, it may be necessary to introduce multiple change points in the specifications of the models. For example, Lumsdaine and Papell (1997) considered the presence of two or more change points in the trend variables. In this study, it is assumed that the Treasury security rates can have two or more change points as well as just one change point.

There are few artificial intelligence models to consider the change-point detection problems. Most of the previous research has a focus on the finding of unknown change points for the past, not the forecast for the future (Wolkenhauer and Edmunds, 1997; Li and Yu, 1999). Our model obtains intervals divided by change points in the training phase, identifies them as change-point groups in the training phase, and forecasts to which group each sample is assigned in the testing phase. It will be tested whether the introduction of change points to our model may improve the predictability of interest rates.

In this study, a series of change points will be detected by the Pettitt test, a nonparametric change-point detection method, since nonparametric statistical property is a suitable match for a neural network model that is a kind of nonparametric method (White, 1992). In addition, the Pettitt test is a kind of Mann-Whitney type statistic, which has remarkably stable distribution and provides a robust test of the change point resistant to outliers (Pettitt, 1980b). In this point, the introduction of the Pettitt test is fairly appropriate to the analysis of chaotic interest rates data.

2.2. The Pettitt tests

The Pettitt tests assume that the observations form an ordered sequence and that initially the distribution of responses has one median and at some point there is a shift in the median of the distribution. H_0 is the null hypothesis that there is no change in the location parameter (i.e. the median) of the sequence of observations, and H_1 is the alternative hypothesis that there is a change in the location parameter of the sequence.

There are two kinds of change-point detection tests. One is appropriate when the data is binary and consists of observations with some binomial process (Pettitt, 1980a). Another test assumes that the data are continuous (Pettitt, 1979). The logic of the tests is similar although the computational formulas are different. We use the continuous type since we forecast the real value of interest rates. The Pettitt test is explained as follows:

First, each of the observations $X_1, X_2, \ldots, X_N$ must be ranked from 1 to N. Let r_i be the rank associated with the observation X_i. Then at each place j in the series, we calculate

$$W_j = \sum_{i=1}^{j} r_i \,, \quad j = 1, 2, \ldots, N-1 \tag{1}$$

which is the sum of the ranks of the variables at or before point j. Next for each point in the sequence, calculate $2W_j - j(N+1)$. Then set

$$K_{m,n} = \max |\, 2W_j - j(N+1) \,|, \quad j = 1, 2, \ldots, N-1 \tag{2}$$

The value of j where the maximum in Equation (2) occurs is the estimated change point in the sequence and is denoted m. $N - m = n$ is the number of observations after the change point. Thus, $K_{m,n}$ is the statistic which divides the sequence into m and n observations occurring before and after the change respectively.

Whether this value of $K_{m,n}$ is larger than we would expect under H_0 can be tested by referring to a table of the sampling distribution of W_j, the sum of ranks. If W exceeds the tabled value of W at the appropriate significance level, we may reject H_0 that there is no change in distribution.

If N becomes large, W is approximately normally distributed with mean $m(N+1)/2$ and variance $mn(N+1)/12$ under H_0. Thus, when the series is long, the test for change may be done and tested using the standard normal distribution table by transforming W into Z:

$$Z = \frac{W + h - m(N+1)/2}{\sqrt{mn(N+1)/12}} \tag{3}$$

where $h = -0.5$ if $W > m(N+1)/2$ and $h = +0.5$ if $W < m(N+1)/2$.

The Pettitt test detects a possible change point in the time sequence dataset. Once the change point is detected through the test, then the dataset is divided into two intervals. The intervals before and after the change point form homogeneous groups which take heterogeneous characteristics from each other. This process becomes a fundamental part of the binary segmentation method explained in section 3.

3. Model Specification

Statistical techniques and neural network learning methods have been integrated to forecast the Treasury security rates. The advantages of combining multiple techniques to yield synergism for discovery and prediction have been widely recognized (Gottman, 1981; Kaufman et al., 1991). BPN is applied to our model since BPN has been used successfully in many applications such as classification, forecasting and pattern recognition (Patterson, 1996).

In this section, we discuss the architecture and the characteristics of our model to integrate the change-point detection and the BPN. Fig. 1 shows the architecture of our model. Based on the Pettitt test, the proposed model consists of three stages: (1) the change-point detection (CPD) stage, (2) the change-point-assisted group detection (CPGD) stage and (3) the output forecasting neural network (OFNN) stage. The BPN is used as a classification tool in CPGD and as a forecasting tool in OFNN.

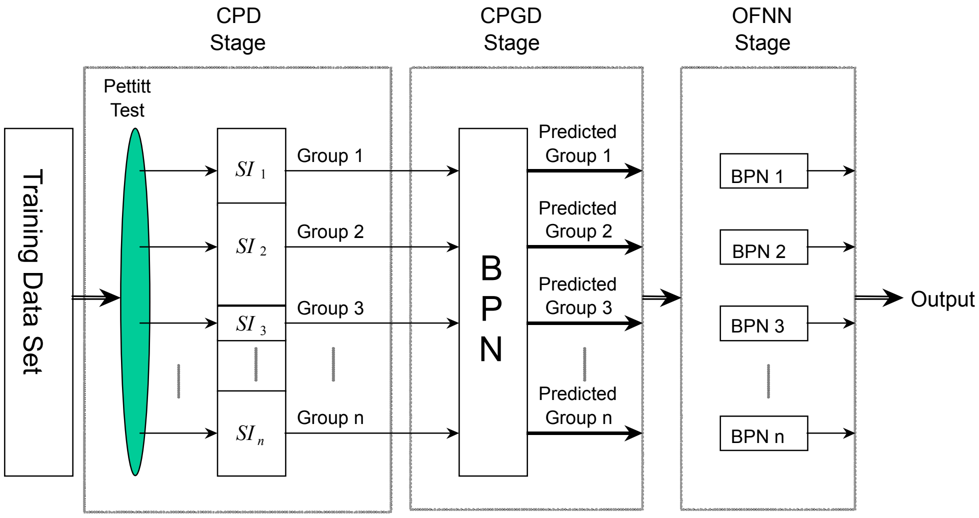


Fig. 1 Architecture of the proposed model using change-point detection

3.1. The CPD stage: Construction and analysis on homogeneous groups

The Pettitt test is a method to find a change-point in time series data (Pettitt, 1979). It is known that interest rates at time t are more important than fundamental economic variables in determining interest rates at time $t+1$ (Larrain, 1991). Thus, we apply the Pettitt test to interest rates at time t in the training phase. The interval made by the test is defined as the significant interval, labeled SI, which is identified with a homogeneous group. Multiple change points are obtained under the binary segmentation method (Vostrikova, 1981) which is explained as follows:

Step 1: Find a change point in $1 \sim N$ intervals by the Pettitt test. If r_1 is a change point, $1 \sim r_1$ intervals are regarded as SI_1 and $(r_1+1) \sim N$ intervals are regarded as SI_2. Otherwise, it is concluded that there does not exist a change point for $1 \sim N$ intervals. $(1 \le r_1 \le N)$

Step 2: Find a change point in $1 \sim r_1$ intervals by the Pettitt test. If r_2 is a change point, $1 \sim r_2$ intervals are regarded as SI_{11} and $(r_2+1) \sim r_1$ intervals are regarded as SI_{12}. Otherwise, $1 \sim r_1$ intervals are regarded as SI_1 like Step 1. $(1 \le r_2 \le r_1)$

Find a change point in $(r_1+1) \sim N$ intervals by the Pettitt test. If r_3 is a change point, $(r_1+1) \sim r_3$ intervals are regarded as SI_{21} and $(r_3+1) \sim N$ intervals are regarded as SI_{22}. Otherwise, $(r_1+1) \sim N$ intervals are regarded as SI_2 like Step 1. $(r_1 \le r_3 \le N)$

Step 3: By applying the same procedure of Step 1 and 2 to subsamples, we can obtain several significant intervals under the dichotomy.

We, first of all, have to decide the number of change points. If just one change point is assumed to occur in a given dataset, only the first step will be performed. Otherwise, all of the three steps will be performed successively. This process plays a role of clustering which constructs groups as well as maintains the time sequence. In this point, the CPD stage is distinguished from other clustering methods such as the k-means nearest neighbor method and the hierarchical clustering method which classify data samples by the Euclidean distance between cases without considering the time sequence. In addition, we analyze the characteristics of groups according to descriptive statistics including the mean and the variance, and also observe the density plot of groups since the classification accuracy is highly sensitive to the density of the samples (Wang, 1995).

3.2. The CPGD stage: Forecast the group with BPN

The significant intervals in the CPD stage are grouped to detect the regularities hidden in interest rates. Such groups represent a set of meaningful trends encompassing interest rates. Since those trends help to find regularity among the related output values more clearly, the neural network model can have a better ability of generalization for the unknown data. This is indeed a very useful point for sample design. In general, the error for forecasting may be reduced by making the subsampling units within groups homogeneous and the variation between groups heterogeneous (Cochran, 1977). After the appropriate groups hidden in interest rates are detected by the CPD stage, BPN is applied to the input data samples at time t with group outputs for $t+1$ given by CPD. In this sense, CPGD is a model that is trained to find an appropriate group for each given sample.

3.3. The OFNN stage: Forecast the output with BPN

OFNN is built by applying the BPN model to each group. OFNN is a mapping function between the input sample and the corresponding desired output (i.e. Treasury security rates). Once OFNN is built, then the sample can be used to forecast the Treasury security rates.

4. Data and Variables

In this study, input variables are selected based on Fisher's theory that nominal interest rates (i.e. monthly U.S. Treasury security rates) consist of expected real interest rates and anticipated inflation:

Nominal Interest Rates = Expected Real Interest Rates + Anticipated Inflation

Many econometricians have conducted the research upon this Fisher-type interest rate equation (Mundell, 1963; Tobin, 1965; Darby, 1975; Feldstein, 1976; Tanzi, 1980; Makin, 1983). They have explained the impact of anticipated inflation on nominal interest rates. Moreover, they have investigated the relationship of money surprise and real GNP growth for the Fisher-type interest rate equation. These relationships are summarized in Fig. 2. In Fig. 2, the straight line is meant to have more causal effects than the dotted line. The causal model like Fig. 2 presents an explanation which would clarify the results (Kim and Park, 1996).

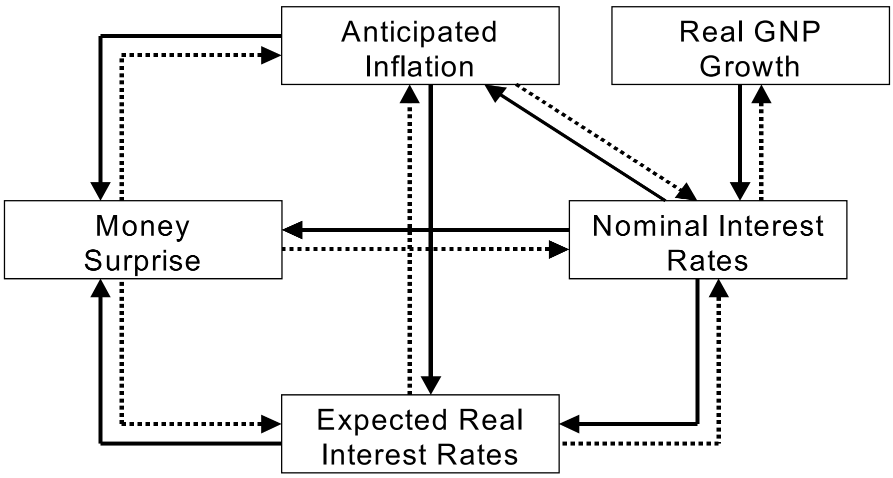


Fig. 2 The economic model under the Fisher-type interest rate equation

The input data sets in this study consist of the figures for the monthly rate of change. Given the data sequence $d_1, d_2, \ldots, d_t$, we form the rate of change at time $t+1$ by dividing the first difference at that time by the datum at time t:

$$\frac{d_{t+1} - d_t}{d_t} \tag{4}$$

The input variables included in this model are anticipated inflation, expected real interest rates, money surprise and real GNP growth which are appeared in Fig. 2. The rate of change of the consumer price index is used as a measure for anticipated inflation while the expected real interest rates is calculated as the difference between the nominal interest rates and the anticipated inflation at time t according to the Fisher-type interest rate equation. M2 and industrial production index are added to input variables as a measure for money surprise and real GNP growth respectively. The list of input variables used in this study is summarized in Table 1.

Table 1 Description of input variables

Variable Name	Description
M2	Money Stock
CPI	Consumer Price Index
ERIR	Expected Real Interest Rates
IPI	Industrial Production Index

The data used in this study is monthly yields on the U.S. Treasury securities from January 1977 to May 1999. As a starting point, we compute descriptive statistics including basic statistics and Pearson correlations among securities. Table 2 shows that the mean and the median change in proportion to maturity. In Table 3, computation on the monthly yields shows that the Pearson correlation between one-year T-bills and thirty-year T-bonds is relatively small except the Federal Funds; The correlation between one-year T-bills and three-year T-notes is 0.97; between one-year T-bills and five-year T-notes, 0.95; between one-year T-bills and ten-year T-notes, 0.92; and between one-year T-bills and thirty-year T-bonds, 0.90. Thus, the forecast of the U.S. Treasury security rates had better not be based on the equivalence alone, but should be performed through individual modeling. In this sense, we build two integrated neural network models for one-year T-bills and thirty-year T-bonds, and establish the experiment interval differently for each model. The motivation for this plan is to see the impact of interval size on the performance and furthermore to demonstrate the generality of the proposed model.

Table 2 Descriptive statistics of the U.S. Treasury monthly yields from January 1977 to May 1999

Statistics	Federal Funds	1-year T-bill	3-year T-note	5-year T-note	10-year T-note	30-year T-bond
Mean	7.68	7.11	8.17	8.39	8.64	8.78
Minimum	2.92	3.06	4.17	4.18	4.53	5.01
Maximum	19.10	14.70	16.22	15.93	15.32	14.68
Range	16.18	11.64	12.05	11.75	10.79	9.67
Median	6.85	6.58	7.73	7.85	8.11	8.27
Lower Quantile	5.40	5.23	6.07	6.40	6.80	7.27
Upper Quantile	9.35	8.58	9.47	9.76	10.28	10.33
Quantile Range	3.96	3.35	3.40	3.36	3.48	3.06
Variance	11.83	6.89	7.54	6.87	6.10	5.07
Standard Deviation	3.44	2.63	2.75	2.62	2.47	2.25
Standard Error	0.21	0.16	0.17	0.16	0.15	0.14
Skewness	1.23	0.82	0.84	0.82	0.74	0.73
Kurtosis	1.55	0.17	0.09	-0.03	-0.21	-0.25

*T-bill means Treasury bill rates; T-note, Treasury note rates; T-bond, Treasury bond rates

Table 3 Pearson correlation matrix of the U.S. Treasury monthly yields form January 1977 to May 1999

	Federal Funds	1-year T-bill	3-year T-note	5-year T-note	10-year T-note	30-year T-bond
Federal Funds	1.0000					
1-year T-bill	0.9735	1.0000				
3-year T-note	0.9314	0.9798	1.0000			
5-year T-note	0.9021	0.9578	0.9951	1.0000		
10-year T-note	0.8674	0.9286	0.9810	0.9949	1.0000	
30-yearT-bond	0.8374	0.9015	0.9644	0.9849	0.9968	1.0000

For one-year T-bills, the training phase involves observations from January 1961 to August 1991 and the testing phase runs from September 1991 to May 1999. For thirty-year T-bonds, the training phase runs from January 1977 to December 1994 and the testing phase runs from January 1995 to May 1999. The interest rates data is presented in Fig. 3. Fig. 3 shows that the movement of interest rates fluctuates highly in both one-year T-bills and thirty-year T-bonds.

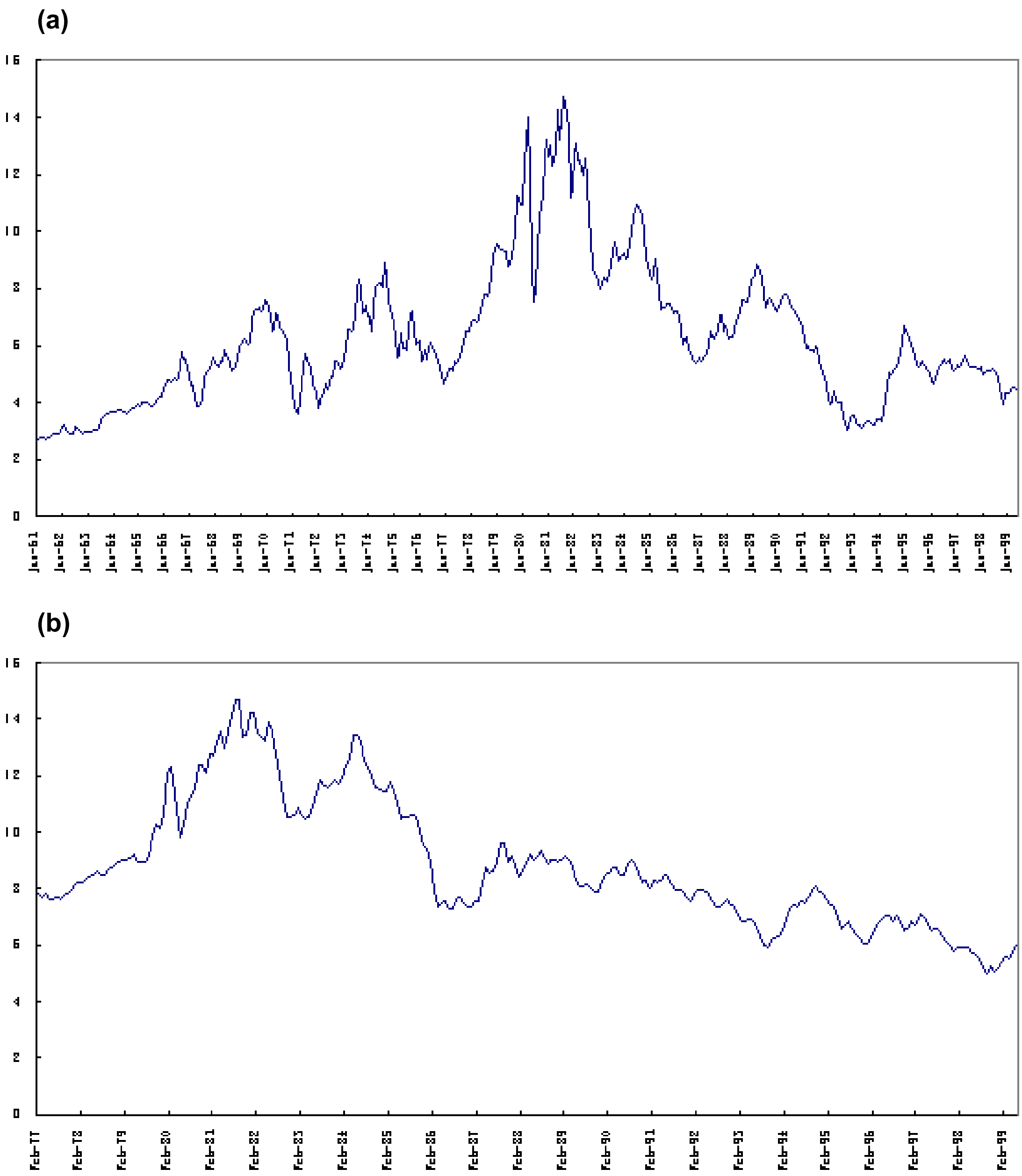


Fig. 3 (a) U.S. Treasury bills with a maturity of 1 year from Jan. 1960 to May 1999 (b) U.S. Treasury bonds with a maturity of 30 years from Jan. 1977 to May 1999

The study employs two neural network models. One model, labeled Pure_NN, involves four input variables at time t to generate a forecast for $t+1$. The input variables are M2, CPI, ERIR and IPI. The second type, labeled BPN_NN, is the two-step BPN model that consists of three stages mentioned in section 3. The first step is the CPGD stage that forecasts the change-point group while the next step is the OFNN stage that forecasts the output. For validation, two learning models are also compared.

5. Empirical Results

The Pettitt test is applied to the interest rates dataset. Since the interest dataset is about forty years long for one-year T-bills, it is considered that there exist three or more change points. It is further assumed that there exist two change points because of the small size of data for thirty-year T-bonds. Table 4 shows these results for one-year T-bills and thirty-year T-bonds.

Table 4 (a) Period and descriptive statistics of groups for the training phase, Jan. 1961 - Aug. 1991, in one-year T-bills

	Group 1	Group 2	Group 3	Group 4
Periods	Jan. 61 – Nov. 65	Dec. 65 – Feb. 73	Mar. 73 – May 78	Jun. 78 – Aug. 91
Minimum	2.720	3.600	4.640	5.260
Maximum	4.230	7.610	8.880	14.700
Range	1.510	4.010	4.240	9.440
Mean	3.378	5.419	6.507	8.654
Variance	0.219	0.938	1.008	5.240
Standard Deviation	0.468	0.969	1.004	2.289
Skewness	0.147	0.496	0.363	0.781
Kurtosis	-1.544	-0.361	-0.575	-0.135

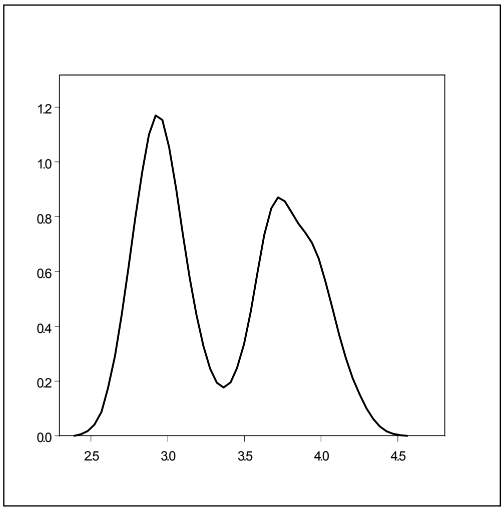
(b) Period and descriptive statistics of groups for the training phase, Jan. 1961 – Dec. 1994, in thirty-year T-bonds

	Group 1	Group 2
Periods	Jan. 77 – Feb. 86	Mar. 86 – Dec. 94
Minimum	7.640	5.940
Maximum	14.680	9.610
Range	7.040	3.670
Mean	10.819	7.995
Variance	3.862	0.676
Standard Deviation	1.965	0.822
Skewness	0.011	-0.365
Kurtosis	-1.062	-0.262

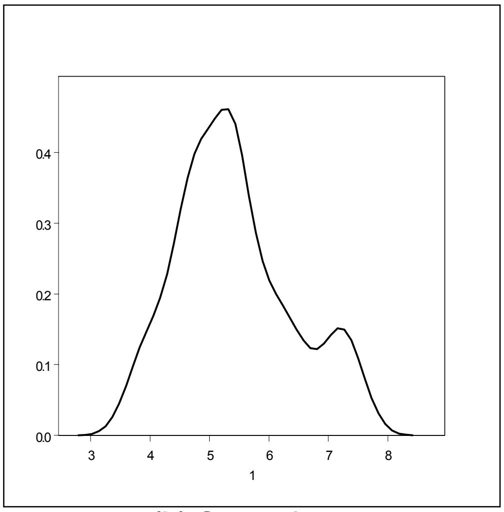
For the case of one-year T-bills, Table 4(a) also presents descriptive statistics including the mean and the variance. Group 1 is the stable interval that has small variance. Group 2 and 3 have more fluctuated intervals than Group 1 in terms of the variance. Group 4 fluctuates highly. The values of skewness and kurtosis indicate that the four groups have similar attributes in distribution. Fig. 4 depicts the density plot for each group. By Fig. 4, Group 2 and 4 are considered to have similar distribution in terms of the shape.

In the case of thirty-year T-bonds, Table 4(b) shows that Group 2 is the stable interval with small variance while Group 1 fluctuates heavily with a big range. Fig. 5 presents the density plot for each group. Through Fig. 5, Group 1 and 2 are recognized to have the distinctive distribution.

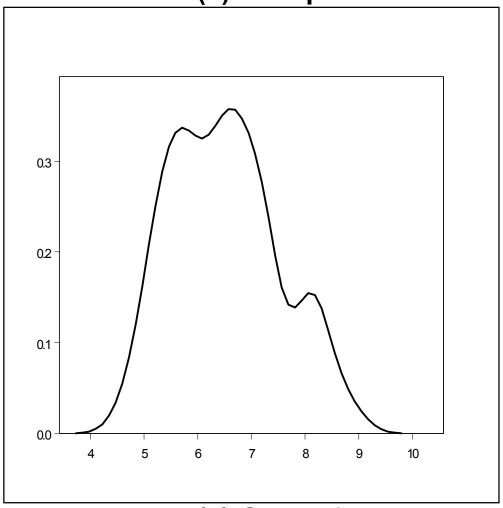
To highlight the performance of the models, the actual values of interest rates and their predicted values are shown in Fig. 6. For one-year T-bills, the predicted values of the pure BPN model (i.e. Pure_NN) moves apart from the actual values in some intervals. In the case of thirty-year T-bonds, the predictability of the two models falls down even though the predictive values of the proposed model (i.e. BPN_NN) comes closer to the actual values than that of the pure BPN model. It is inferred that this phenomenon is caused by the long-term maturity of T-bonds.



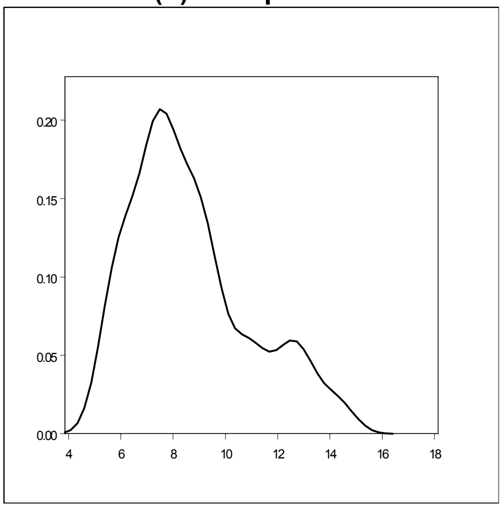
(a) Group 1



(b) Group 2

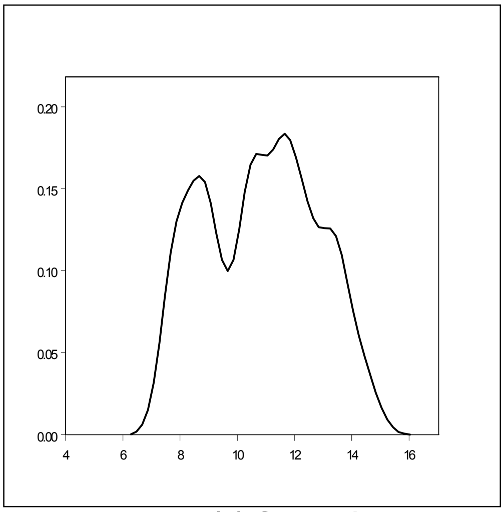


(c) Group 3

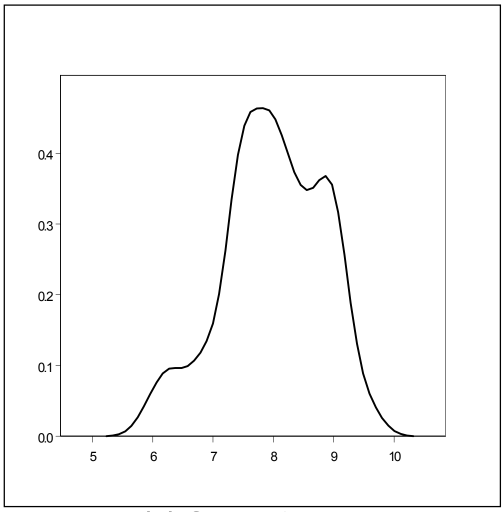


(d) Group 4

Fig. 4 Density plot of four homogeneous groups for one-year T-bills



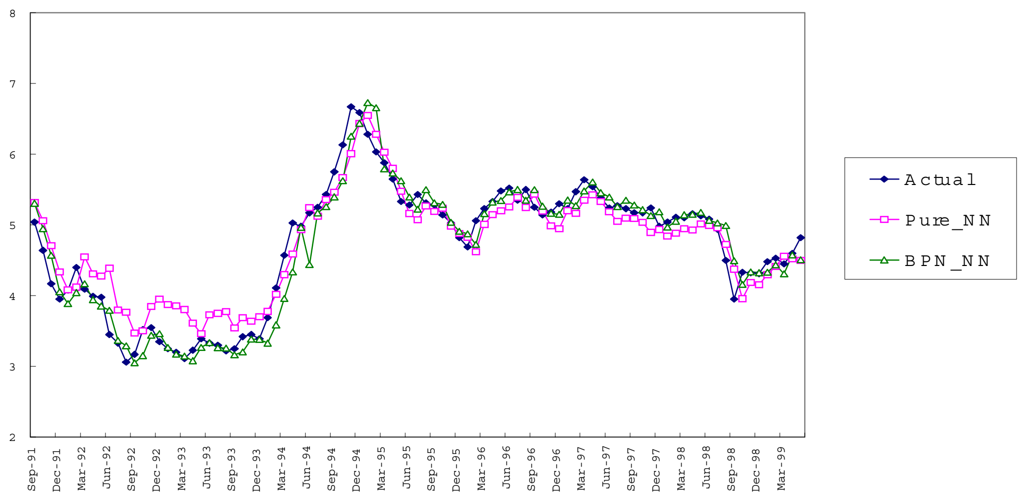
(a) Group 1



(b) Group 2

Fig. 5 Density plot of two homogeneous groups for thirty-year T-bonds

(a)



(b)

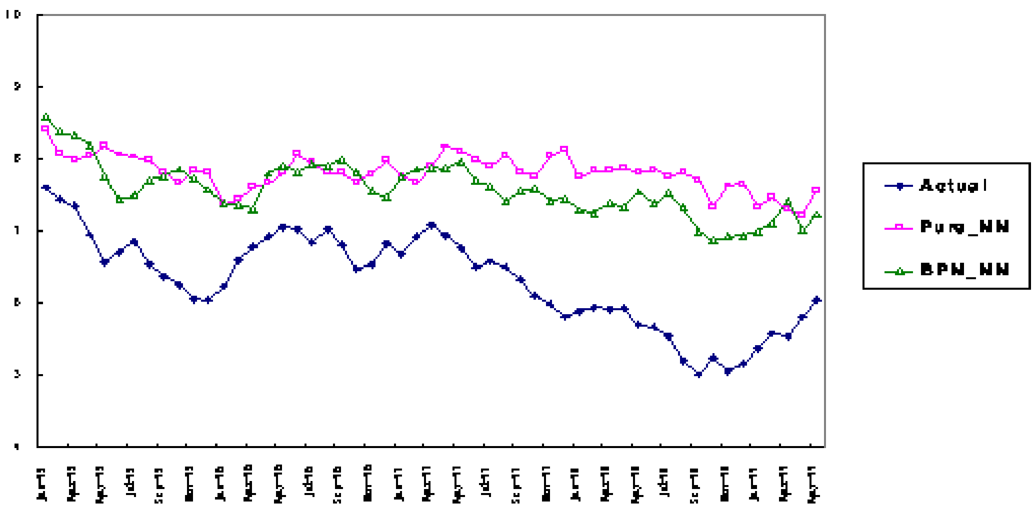


Fig. 6 (a) Actual vs predicted values due to the models for one-year T-bills (b) Actual vs predicted values due to the models for thirty-year T-bonds

Numerical values for the performance metrics by the predictive model are given in Table 5. Fig. 7 presents histograms of RMSE, MAE and MAPE for the forecast of each learning model in the cases of one-year T-bills and thirty-year T-bonds. According to RMSE, MAE and MAPE, the outcomes indicate that the proposed neural network model is superior to the pure BPN model for both of the interest rates.

Table 5 (a) Performance results of one-year Treasury bill rate forecasting based on the root mean squared error (RMSE), the mean absolute error (MAE) and the mean absolute percentage error (MAPE)

Model	RMSE	MAE	MAPE
Pure_NN	0.0973	0.2506	5.969%
BPN_NN	0.0584	0.1745	3.746%

(b) Performance results of thirty-year Treasury bond rate forecasting based on the RMSE, the MAE and the MAPE

Model	RMSE	MAE	MAPE
Pure_NN	2.5462	1.4976	24.828%
BPN_NN	1.7553	1.2668	20.836%

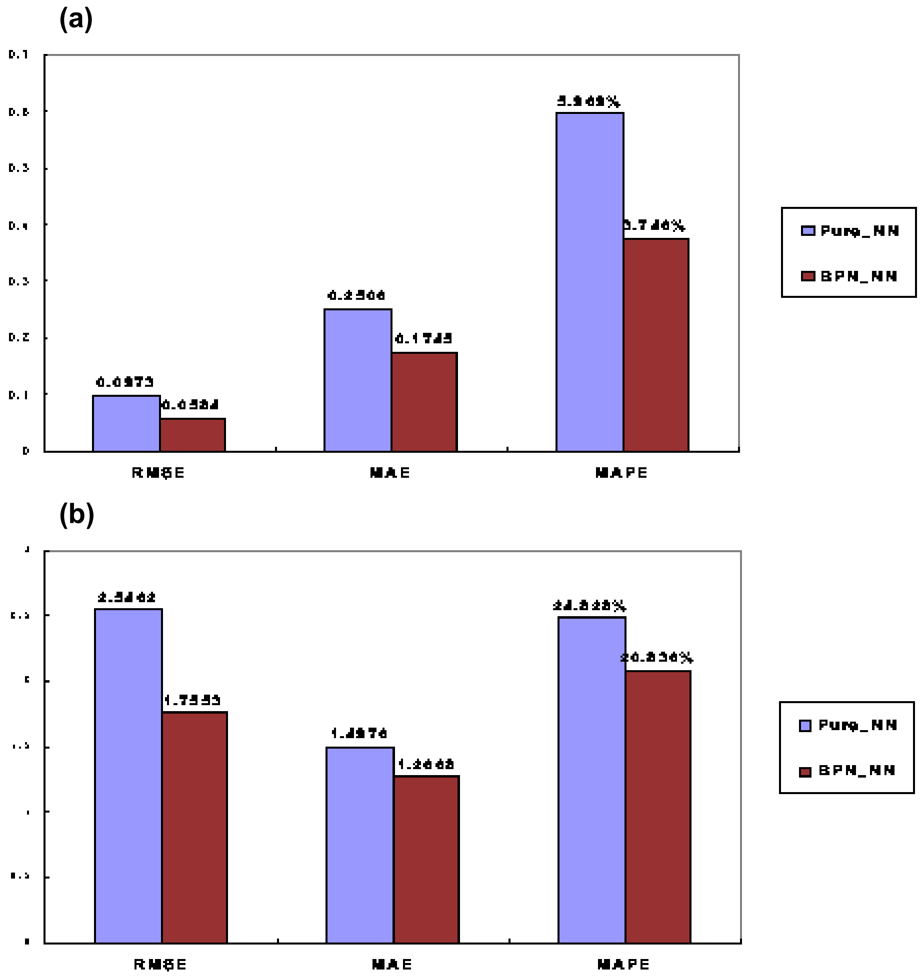


Fig. 7 (a) Histogram of RMSE, MAE and MAPE resulting from forecasts of one-year T-bills (b) Histogram of RMSE, MAE and MAPE resulting from forecasts of thirty-year T-bonds

We use the pairwise t-test to examine whether the differences exist in the predicted values of models according to the absolute percentage error (APE). This metric is chosen since it is commonly used (Carbone and Armstrong, 1982) and is highly robust (Armstrong and Collopy, 1992; Makridakis, 1993). Since the forecasts are not statistically independent and not always normally distributed, we compare the APEs of forecast using the pairwise t-test. Where sample sizes are reasonably large, this test is robust to the distribution of the data, to nonhomogeneity of variances, and to statistical dependence (Iman and Conover, 1983). Table 6 shows t-values and p-values. The neural network models using change-point detection perform significantly better than the pure BPN model at a 1% significant level. Therefore, the proposed model is demonstrated to obtain improved performance using the change-point detection approach.

Table 6 Pairwise t-tests for the difference in residuals between the pure BPN model and the proposed neural network model for one-year T-bills and thirty-year T-bonds based on the absolute percentage error (APE) with the significance level in parentheses.

Interest Rates	Test Value
One-year T-bills	3.43 (0.000)***
Thirty-year T-bonds	5.85 (0.000)***

*** Significant at 1%

In summary, the neural network models using the change-point detection turns out to have a high potential in interest rates forecasting. This is attributable to the fact that it categorizes the interest rates data into homogeneous groups and extracts regularities from each homogeneous group. Therefore, the neural network models using change-point detection can cope with the noise or irregularities more efficiently than the pure BPN model.

6. Concluding Remarks

This study has suggested change-point detection to support neural network models in interest rates forecasting. The basic concept of this proposed model is to obtain significant intervals divided by the change points, to identify them as change-point groups, and to use them in interest rates forecasting. We propose the integrated neural network model which consists of three stages. In the first stage, we conduct the nonparametric statistical test to construct the homogeneous groups. In the second stage, we apply BPN to forecast the change-point group. In the final stage, we also apply BPN to forecast the output.

The neural network models using change-point detection perform significantly better than the pure BPN model at a 1% significant level. These experimental results imply the change-point detection has a high potential to improve the performance. Our integrated neural network model is demonstrated to be a useful intelligent data analysis method with the concept of change-point detection. In conclusion, we have shown that the proposed model improves the predictability of interest rates significantly.

The proposed model has the promising possibility of improving the performance if further studies are to focus on the optimal decision of the number of change point and the various approaches in the construction of change-point groups. In the OFNN stage, other intelligent techniques besides BPN can be used to forecast the output. In addition, the proposed model may be applied to other chaotic time series data such as stock market prediction and exchange rate prediction.

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