INFORMATION REVELATION AND VALUE OF INFORMATION IN BILATERAL TRADING

Ching Chyi Lee[1)], Tien Sheng Lee[2)]

[1)]The Chinese University of Hong Kong, Department of Decision Sciences & Managerial Economics, Faculty of Business Administration, Phone: (852) 2609-7763, Fax: (852) 2603-5104, Email: cclee@baf.msmail.cuhk.edu.hk
[2)]The Chinese University of Hong Kong, Department of Decision Sciences & Managerial Economics, Faculty of Business Administration, Phone: (852) 2609-7815, Fax: (852) 2603-5104, Email: tslee@baf.msmail.cuhk.edu.hk

Abstract

One of the well-known facts in single-person decision analysis problem is that value of information cannot be negative. The intuition for this result is quite simple: If an information is judged to be "useless", the decision maker can always choose to "*ignore*" it, and ignoring a useless information should not do any harm to the decision maker. While this intuition is true in the absence of other potential decision-makers, it is not necessary true in decision problems with multiple decision-makers whose interests are in conflict. In such problems, information sometimes can become a burden for the person who possesses it. Even if an information is judged to be useless, a decision-maker often cannot simply ignore it. This is because, knowing that he has the information, the other decision makers may choose their optimal decisions accordingly based on this fact. (Consider as an example, someone who accidentally receives the password to the computer system of national defense can easily become the target of hijackers, even if he personally has no interest at all on the secret of national defense.) To investigate the role of information in two-person decision problems, in this paper, we consider a simple bilateral bargaining and search problem in which the informed player (the buyer) knows his own search cost whereas the uninformed player (the seller) only knows the distribution of the search cost. The result of the analysis indicates that value of private information can indeed sometimes be negative. Based on this basic model, we then consider an information revelation game in which the informed player can choose whether or not to "reveal" his private information to the uninformed player before the game is played. We found that, under some conditions, the informed player will choose to reveal his private information. More interestingly, we found that the buyer's decision to reveal the information about his search cost is not necessarily related to the actual search cost in an intuitive or straightforward way. Depending on the assumption on the distribution of the search cost, we found that sometimes buyers with relatively low search costs would choose to reveal their private information whereas, in other cases, buyers with relatively high search costs would choose to do so.

1. Introduction

One of the well-known facts in single-person decision analysis problem is that value of information cannot be negative. The intuition for this result is quite simple: If an information is judged to be "useless", the decision maker can always choose to "*ignore*" it, and ignoring a useless information should not do any harm to the decision maker. While this intuition is true in the absence of other potential decision-makers, it is not necessary true in decision problems with multiple decision-makers whose interests are in conflict. For example, Ponssard (1981, pp. 85-100) examined some games (involving two players) in which the values of information could be negative. In such problems, information sometimes can become a burden for the person who possesses it. Even if an information is judged to be useless, a decision-maker often cannot simply ignore it. This is because, knowing that he has the information, the other decision makers may choose their optimal decisions accordingly based on this fact. (Consider as an example, someone who accidentally receives the password to the computer system of national defense can easily become the target of hijackers, even if he personally has no interest at all on the secret of national defense.) To investigate the role of information in two-person decision problems, in this paper, we consider a simple bilateral bargaining and search problem in which the informed player (the buyer) knows his own search cost whereas the uninformed player (the seller) only knows the distribution of the search cost. The result of the analysis indicates that value of private information can indeed sometimes be negative. Based on this basic model, we then consider an information revelation game in which the informed player can choose whether or not to "reveal" his private information to the uninformed player before the game is played. We found that, under some conditions, the informed player will choose to reveal his private information. More interestingly, we found that the buyer's decision to reveal the information about his search cost is not necessarily related to the actual search cost in a intuitive or straightforward way. Depending on the assumption on the distribution of the search cost, we found that sometimes buyers with relatively low search costs would choose to reveal their private information whereas, in other cases, buyers with relatively high search costs would choose to do so.

The organization of the paper is the following. In the next section, Section 2, we describe the basic model. The complete characterization and discussion of the equilibrium of the basic model is given in Section 3. Section 4 discusses the "*information revelation game*", and Section 5 concludes.

2. The Basic Model

The model we consider here is a simplified version of the bargaining and search model of Lee (1994) and Chatterjee and Lee (1998). The major differences between the model discussed here and the models of Lee and Chatterjee and Lee are the following. First of all, in this paper, we only consider a single stage bargaining and search problem whereas both Lee and Chatterjee and Lee considered two-stage problems. Second, while Lee considers a complete information model and Chatterjee and Lee consider a model with asymmetric information in search outcome, this paper considers a model with asymmetric information in search cost. We now describe the basic model below.

Consider a bilateral trading problem in which one seller and one buyer are bargaining over the price of an indivisible good. The seller announces a price, p, initially. The buyer can then choose (1) to accept p, in which case the game ends immediately, or (2) to reject p initially and then to "search" for an outside offer, in which case he draws a sample of outside offers, x, at cost c. The realized value of x is observable to both parties. Once x is observed, the buyer can then choose either to accept p, x, or to reject both.

The seller's and the buyer's valuations of the good are 0 and 1, respectively. Both players are assumed to be risk neutral. Their objectives are to maximize their expected gains, where "*gains*" are defined as the differences between their reservation prices and the price that they agree upon. Outside offer, x, is governed by the absolutely continuous and differentiable distribution function, $F(\cdot)$, with support on [0, 1]. As for the information about the buyer's search cost c, we consider two cases: the first case is when c is known to both parties (**the public information case**), and the second case is when c is only known to the buyer himself (**the private information case**). In the second case, although not knowing the exact value of c, the seller knows the distribution of c, which is described by a continuous distribution function $G(\cdot)$ with support on [0, b], where b is strictly greater than zero. The density functions of $G(\cdot)$ and $F(\cdot)$ are denoted as $g(\cdot)$ and $f(\cdot)$, respectively. The players' valuations of the good, their risk attitudes, the distribution of the outside offers x, and the distribution of c are all common knowledge.

3. The Analysis

In this section, we shall try to characterize the equilibrium of the game. The public information case has been examined by Lee (1994). To facilitate discussion, we shall provide a brief review of Lee's analysis. After the discussion of the public information case, we will extend the analysis to the private information case. The buyer's and the seller's expected payoffs from each of these two different cases will then be compared.

3.1 The Public Information Case

Following backward induction, let us consider the buyer's strategy first. Obviously, in this case, the buyer will accept the seller's offer p if and only if $1 - p \geq \Phi(p)$, where $\Phi(p)$ is the buyer's expected payoff from search given p and is defined as

$$\Phi(p) \equiv -c + \int_0^p (1-x)f(x)dx + \int_p^1 (1-p)f(x)dx \tag{1}$$

Let p^* be the price which makes the buyer indifference between "*search*" and "*not search*", i.e., $1 - p^* = \Phi(p^*)$. Then, it is clear that the buyer will accept the seller's offer p if and only if $p \le p^*$. Notice that $1 - p \ge \Phi(p)$ is equivalent to

$$c \ge \int_0^p F(x)dx \equiv \Psi(p) \tag{2}$$

Now, let us consider the seller's strategy. If the seller's offer p is accepted immediately, she gets p and the buyer gets $1 - p$. However, if the buyer rejects p and searches, the seller's expected payoff is $p[1 - F(p)]$ because p may be accepted later, with probability $p[1 - F(p)]$, if the buyer is unable to get better offer from search. Hence, the seller chooses p to maximize $S(p)$, where

$$S(p) = \begin{cases} p[1-F(p)] & \text{if } p > p^* \\ p & \text{if } p \le p^* \end{cases} \tag{3}$$

Let $\bar{p}$ be the price maximizing $p[1 - F(p)]$, then the seller's optimal strategy is to offer p^* if $p^* \ge \bar{p}[1 - F(\bar{p})]$ and to offer $\bar{p}$ otherwise.

When $F(\cdot)$ is uniform, the equilibrium strategies and payoffs of both players can be calculated explicitly. In this case, $p^* = (2c)^{1/2}$, $\bar{p} = ½$, and $\bar{p}[1 - F(\bar{p})] = ¼$. Hence, if $p^* = (2c)^{1/2} \ge \bar{p}[1 - F(\bar{p})] = ¼$, which implies $c \ge 1/32$, the seller will offer p^* and the buyer will accept; otherwise, the seller will offer $\bar{p}$ and the buyer will search. In the first case, the equilibrium payoffs are $(2c)^{1/2}$ for the seller and $1 - (2c)^{1/2}$ for the buyer. In the second case, the actual equilibrium payoffs depend on the value of x realized. However, the *ex ante* equilibrium expected payoffs are $\bar{p}[1 - F(\bar{p})] = ¼$ for the seller and $\Phi(½) = 5/8 - c$ for the buyer. The players' equilibrium expected payoffs for this case are depicted in Figure 1.

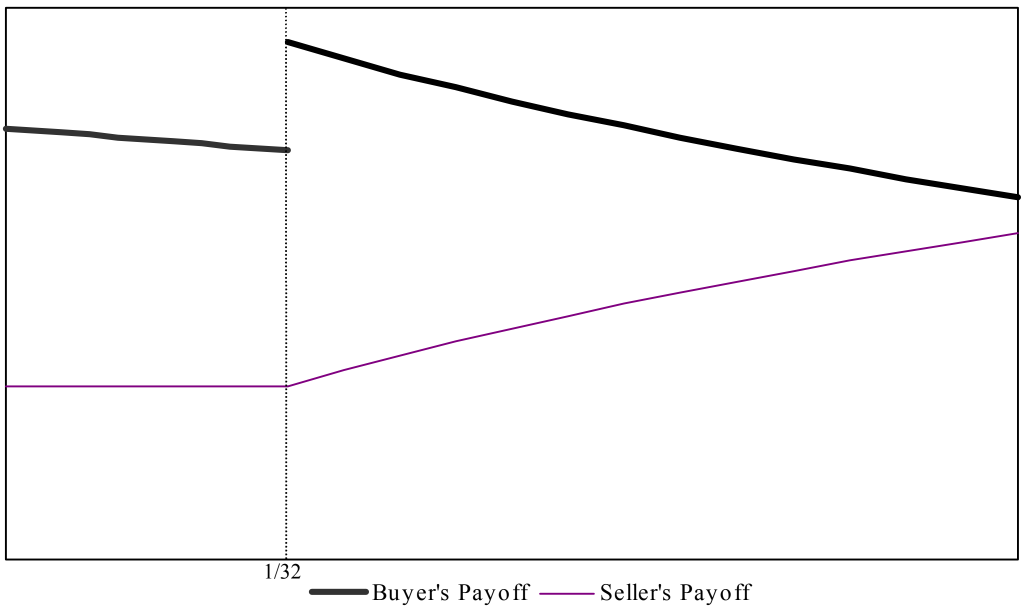

Figure 1: Players' Equilibrium Expected Payoffs in the Public Information Case

One of the most intriguing results of the public information case is that the buyer's equilibrium expected payoff, as depicted in Figure 1, is not monotonically decreasing in c. In other words, the buyer may not necessarily get worse payoff as c increases. The buyer's equilibrium expected payoff is higher for c in some intermediate range than for small c. In fact, the buyer's expected payoff is the highest when $c = 1/32$ (or, in general, when $c = c^*$ for other distributions of outside options). This naturally raises the interesting question regarding how the buyer will deal with the situation when (1) c is his private information and (2) c is equal to or slightly greater than c^*. In this case, will the then buyer have the incentive to reveal his private information about c to the seller? Obviously, if the seller's knows that c is equal to c^*, she will then offer p^*, and, in which case, the buyer will get better payoff. However, if the seller does not know the value of c, she will probably offer something different from p^* which makes the buyer's expected payoff lower. To answer this question, we shall fist analyze the case when c is the buyer's private information in the next Section.

3.2 The Private Information Case

The buyer's optimal strategy in the private information case is the same as that described in the public information case. We now consider the seller's strategy. Let $S'(p)$ be the seller's expected payoff in this private information case. The seller's problem is then to choose p to maximize $S'(p)$, where

$$S'(p) = \begin{cases} p[1 - F(p)] & \text{if } \Psi(p) > b \\ \int_0^{\Psi(p)} p[1 - F(p)]g(c)dc + \int_{\Psi(p)}^{b} pg(c)dc & \text{if } \Psi(p) \leq b \end{cases} \tag{4}$$

and $\Psi(p)$ is as defined in (2).

Let $\tilde{p}$ be the value of p maximizing (4). Then, in equilibrium, the seller will offer $\tilde{p}$. The buyer will accept $\tilde{p}$ if $c \geq \Psi(\tilde{p})$; otherwise he will will search if $c < \Psi(\tilde{p})$.

When $F(\cdot)$ and $G(\cdot)$ are both uniform distributions (i.e., outside option x is uniformly distributed between 0 and 1, and search cost c is uniformly distributed between 0 and b), the equilibrium strategies and payoffs can be calculated explicitly. In this case, expression (4) becomes the following:

$$S'(p) = \begin{cases} p[1-p] & \text{if } p > \sqrt{2b} \\ p - \dfrac{p^4}{2b} & \text{if } p \leq \sqrt{2b} \end{cases} \tag{5}$$

Notice that, in (5), both $p[1 - p]$ and $p - p^4/(2b)$ are concave in p. Then, it is easy to verify that

$$\tilde{p} = \begin{cases} 1/2 & \text{if } b < 2/27 \\ (b/2)^{1/3} & \text{if } b \geq 2/27 \end{cases} \tag{6}$$

In equilibrium, the seller's *ex ante* expected payoff is

$$S'(\tilde{p}) = \begin{cases} 1/4 & \text{if } b < 2/27 \\ (3/4)(b/2)^{1/3} & \text{if } b \geq 2/27 \end{cases} \tag{7}$$

The buyer's expected payoff, denoted as $B'(p)$, is slightly more complicated.

When $b < 2/27$, since $\tilde{p} = 1/2$, the buyer will always search no matter what the value of c is. The buyer's expected payoff is thus:

$$B(\tilde{p}) = \Phi(\tfrac{1}{2}) = 5/8 - c \tag{8}$$

When $b \geq 2/27$, since $\tilde{p} = (b/2)^{1/3}$, the buyer will accept the offer immediately if and only if $c \geq ½(b/2)^{1/3}$, otherwise he will search. Hence, the buyer's expected payoff is

$$B(\widetilde{p}) = \begin{cases} 1-(b/2)^{1/3} & \text{if } c > (1/2)(b/2)^{2/3} \\ 1-c-(b/2)^{1/3}+(1/2)(b/2)^{2/3} & \text{if } c \le (1/2)(b/2)^{2/3} \end{cases} \tag{9}$$

Having worked out the private information case, we are now ready to compare the players' payoffs under these two different cases. This is discussed in the next section.

3.3. Comparison Between Public and Private Information Cases

It is interesting to note that the buyer's expected payoff under this private information case may be lower than his expected payoff under the public information one. To see how this can happen, we now compare the players' payoffs in each of the two cases for different values of b and c. In the following, p is the equilibrium offer and $B(p)$ and $S(p)$ are the buyer's and the seller's equilibrium expected payoff respectively.

Case 1: $b \in [0, 1/32]$

Public Information: $p = 1/2$, $B(p) = 5/8 - c$, $S(p) = 1/4$.
Private Information: $p = 1/2$, $B(p) = 5/8 - c$, $S(p) = 1/4$.

Case 2: $b \in [1/32, 2/27]$

a) $c < 1/32$

Public Information: $p = 1/2$, $B(p) = 5/8 - c$, $S(p) = 1/4$.
Private Information: $p = 1/2$, $B(p) = 5/8 - c$, $S(p) = 1/4$.

b) $c \in (1/32, b]$

Public Information: $p = (2c)^{1/2}$, $B(p) = 1 - (2c)^{1/2}$, $S(p) = (2c)^{1/2}$.
Private Information: $p = 1/2$, $B(p) = 5/8 - c$, $S(p) = 1/4$.

Since $1/2 > (2c)^{1/2}$, for c in this range, the buyer can get better price (and hence better payoff) by revealing c. This can also be seen by compare the buyer's payoff in each of the two cases. Since $1 - (2c)^{1/2}$ is strictly greater than $5/8 - c$ for $c \in (1/32, b]$, the buyer is better off to be in public information case.

Case 3: $b \in [2/27, 1/4]$

a) $c < 1/32$

Public Information: $p = 1/2$, $B(p) = 5/8 - c$, $S(p) = 1/4$.
Private Information: $p = (b/2)^{1/3}$, $B(p) = 1 - c - (b/2)^{1/3} + ½(b/2)^{2/3}$, $S(p) = ¾\,(b/2)^{1/3}$.

Notice that, under private information, the offer $p = (b/2)^{1/3}$ will be rejected by the buyer if $c < 1/32$. This is because the buyer will only accept all $p \le (2c)^{1/2}$. Since $(b/2)^{1/3} > (2c)^{1/2}$ for $c < 1/32$ and $b \in [2/27, 1/4]$, the seller offer's $p = (b/2)^{1/3}$ will be rejected. Since $(b/2)^{1/3} \le 1/2$, the buyer is better off to be in the private information case, and hence, he will have no incentive to reveal his information.

b) $c \in (1/32, 2/27]$

Public Information: $p = (2c)^{1/2}$, $B(p) = 1 - (2c)^{1/2}$, $S(p) = (2c)^{1/2}$.
Private Information: $p = (b/2)^{1/3}$, $S(p) = ¾\,(b/2)^{1/3}$.

In the private information case, whether or not the offer $p = (b/2)^{1/3}$ will be accepted depends on c. If $(b/2)^{1/3} \le (2c)^{1/2}$, or equivalently, $c \ge ½(b/2)^{2/3}$, the buyer will accept, otherwise he will reject. Hence, the buyer's expected payoff is

$$B(\tilde{p}) = \begin{cases} 1-(b/2)^{1/3} & if\, c > (1/2)(b/2)^{2/3} \\ 1-c-(b/2)^{1/3}+(1/2)(b/2)^{2/3} & if\, c \le (1/2)(b/2)^{2/3} \end{cases}$$

The condition, $c \geq ½(b/2)^{2/3}$, also turns out to be the cutoff value determining whether the buyer will want to reveal his information about c. If $c \geq ½(b/2)^{2/3}$, he will not reveal; otherwise, if $c < ½(b/2)^{2/3}$, he will.

c) $c \in (2/27, b]$

This case is basically the same as the previous case. Hence,

Public Information: $p = (2c)^{1/2}$, $B(p) = 1 - (2c)^{1/2}$, $S(p) = (2c)^{1/2}$.
Private Information: $p = (b/2)^{1/3}$, $S(p) = ¾\,(b/2)^{1/3}$, and the buyer's payoff is

$$B(\tilde{p}) = \begin{cases} 1-(b/2)^{1/3} & if\, c > (1/2)(b/2)^{2/3} \\ 1-c-(b/2)^{1/3}+(1/2)(b/2)^{2/3} & if\, c \le (1/2)(b/2)^{2/3} \end{cases}$$

Case 4: $b > 1/4$.

This case is still the same as in the previous case, i.e.,

Public Information: $p = (2c)^{1/2}$, $B(p) = 1 - (2c)^{1/2}$, $S(p) = (2c)^{1/2}$.
Private Information: $p = (b/2)^{1/3}$, $S(p) = ¾\,(b/2)^{1/3}$, and the buyer's payoff is

$$B(\tilde{p}) = \begin{cases} 1-(b/2)^{1/3} & if\, c > (1/2)(b/2)^{2/3} \\ 1-c-(b/2)^{1/3}+(1/2)(b/2)^{2/3} & if\, c \le (1/2)(b/2)^{2/3} \end{cases}$$

However, the offer $p = (b/2)^{1/3} > 1/2$ due to large b. Hence, this gives the buyer incentive to reveal c. Since, in the public information, all buyers with $c \leq 1/8$ can expect to get an offer that is as good as, or better than, 1/2, they will certainly want to reveal their information. For buyers with $c > 1/8$, they will reveal their information as long as $c < ½(b/2)^{2/3}$.

Summarizing the above cases, we have identified the following cases in which the buyer is strictly better off in the public information case (i.e., he is strictly better off to announce the value of c publicly):

1) when $1/32 < b < 2/27$ and $1/32 < c \leq b$,
2) when $2/27 \leq b \leq ¼$ and $1/32 < c < ½(b/2)^{2/3}$, and
3) when $b > ¼$ and $c < ½(b/2)^{2/3}$.

We can interpret the above result as the following: since the buyer is strictly better off in the public information case, if he were given the chance to announce his information publicly (i.e., to let the seller know his search cost), he will be happy to do so.

We also identified the following cases in which the buyer is strictly better off in the private information case:

1) when $b \in [2/27, 1/4]$ and $c < 1/32$,
2) when $2/27 \leq b \leq ¼$ and $½(b/2)^{2/3} < c < b$, and
3) when $b > ¼$ and $c > ½(b/2)^{2/3}$.

For these cases, if the buyer could choose whether or not to reveal his private information, he will choose not to do so. Of course, here by the way the comparisons are done, we are assuming that it is common knowledge that the uninformed player (the seller) does not know that the informed player (the buyer) is informed. It will be interesting to see how the result will be affected if we replace the above assumption with another assumption, namely, although the uninformed player does not know the value of c, she does know the distribution of c and also she does know that the buyer is informed. This leads us to the discussions in the next session – the information revelation game.

4. Information Revelation Game

Let us first describe the game. The game involves the following moves in sequence:

1. The nature first draws a random number c from a uniformly distribution with support on $[0, b]$.
2. The buyer then observes c. Although the seller cannot observe c, she knows the distribution from which c is drawn and she also knows that the buyer knows the value of c.
3. After observing c, the buyer then decides whether or not to reveal the value of c to the seller. If the buyer decides to reveal c, he must do so truthfully and the seller knows this also.
4. If the buyer reveals c, the seller and the buyer will then play the public information game described in the previous section; otherwise, if the buyer does not reveal c, they will play the private information game.

To simplify the analysis, we introduce two assumptions: the first one is a tie-breaking rule and the second one is used to avoid the complicated situations in which the set of all buyers who will reveal (or will not reveal) information in equilibrium may consists of disjoint subsets.

A1. If the buyer is indifferent between "revealing" and "not revealing" the information, he will not reveal the information.

A2. If an equilibrium requires both buyers with c_1 and c_2 to reveal (or not to reveal) the information, then all buyers with $c = \alpha c_1 + (1-\alpha)c_2$ for all $\alpha \in [0, 1]$ must also reveal (or not reveal) the information.

Proposition 0 through Proposition 3 describe the equilibrium of the information revelation game under different parameter values.

Proposition 0: If $b \leq 1/32$, the buyer is indifferent between "*revealing*" and "*not revealing*". Revealing or not, the seller always offers $p = ½$. Every thing is in equilibrium. (Proof is omitted)

Proposition 1: If $1/32 < b \leq 1/8$, then the buyer will not reveal if $c \leq 1/32$, and he will reveal if $c > 1/32$. If the buyer reveals his c, the seller's optimal offer will follow that described in the public information model (i.e., offering $p = (2c)^{1/2}$ if $c > 1/32$ or offering $p = ½$ if $c \leq 1/32$). If the buyer does not reveal, believing $c \in [0, 1/32]$, the seller will offer $p = ½$.[1]

Proof:

Consider the seller's strategy first. If the buyer reveals c, the seller's optimal offer follows that described in the public information model (i.e., offering $p = (2c)^{1/2}$ if $c > 1/32$ or offering $p = ½$ if $c \leq 1/32$). If the buyer does not reveal, believing $c \in [0, 1/32]$, the seller will then offer $p = ½$ according to the analysis given in the private information model (the case when $b < 2/27$).

Now, consider the buyer's strategy. Let first consider the case when $c > 1/32$. In this case, by revealing, the buyer will receive $p = (2c)^{1/2}$ from the seller, whereas, by not revealing, the buyer will receive $p = ½$. For $b \leq 1/8$, $(2c)^{1/2} \leq ½$. Since the buyer's expected payoff $\Phi(p)$ is strictly decreasing in p, the buyer is better off revealing c if $c > 1/32$. Consider the other case when $c \leq 1/32$. In this case, from our previous analysis, we know that, revealing or not, the buyer will always receive $p = ½$ from the seller. Since the buyer is indifferent between revealing and not revealing, by A1, the buyer will not reveal if $c \leq 1/32$. ***Q.E.D.***

---

[1] Notice that this proposition requires assumptions A1 and A2. Without A1 (A2 is still required), this proposition can be rewritten as the following and the proof is straightforward: ***Proposition 1A:*** If $1/32 < b \leq 1/8$, then the buyer will not reveal if $c \leq c^*$ ($c^* \leq 1/32$), and he will reveal if $c > c^*$. If the buyer reveals his c, the seller's optimal offer will follow that described in the public information model. If the buyer does not reveal, believing $c \in [0, c^*]$, the seller will offer $p = ½$.

Proposition 2: If $1/8 < b < ¼$, then the buyer will not reveal if $c \geq \underline{c}$ $(1/8 < \underline{c} \leq b)$, and he will reveal if $c < \underline{c}$. If the buyer reveals his c, the seller's optimal offer will follow that described in the public information model (i.e., offering $p = (2c)^{1/2}$ if $c > 1/32$ or offering $p = ½$ if $c \leq 1/32$). If the buyer does not reveal, believing $c \in [\underline{c}, b]$, the seller will offer $p = (2\underline{c})^{1/2}$.

Proof:

We first consider the seller's strategy. When the buyer reveals c, his strategy follows that described in the public information game. If the buyer does not reveal c, then, believing $c \in [\underline{c}, b]$, where $1/8 \leq \underline{c} \leq b \leq ¼$, the seller's problem is to maximize $S'(p)$, where

$$S'(p) = \begin{cases} p[1-F(p)] & \text{if } \Psi(p) \geq b \\ \int_{\underline{c}}^{\Psi(p)} p[1-F(p)]g(c)dc + \int_{\Psi(p)}^{b} pg(c)dc & \text{if } \underline{c} \leq \Psi(p) \leq b \\ p & \text{if } \Psi(p) \leq \underline{c} \end{cases} \tag{10}$$

When both $F(\cdot)$ and $G(\cdot)$ are uniform distribution functions, $S'(p)$ becomes the following:

$$S'(p) = \begin{cases} p(1-p) & \text{if } \tfrac{1}{2}p^2 \geq b \quad (\text{or if } p \geq \sqrt{2b}) \\ \dfrac{1}{b-\underline{c}}\left[\int_{\underline{c}}^{\frac{1}{2}p^2} p(1-p)dc + \int_{\frac{1}{2}p^2}^{b} pdc\right] & \text{if } \underline{c} \leq \tfrac{1}{2}p^2 \leq b \quad (\text{or if } \sqrt{2\underline{c}} \leq p \leq \sqrt{2b}) \\ p & \text{if } \tfrac{1}{2}p^2 \leq \underline{c} \quad (\text{or if } p \leq \sqrt{2\underline{c}}) \end{cases} \tag{11}$$

Notice that the first part of $S'(p)$, $p(1 - p)$, is decreasing in p if $p \geq (2b)^{1/2}$. This is easy to check since if $p \geq (2b)^{1/2}$ and $b \geq 1/8$, the first derivative of $p(1 - p)$, $1 - 2p$, is less than or equal to 0. Hence, this part of $S'(p)$ is maximized at $p = (2b)^{1/2}$.

The last part of $S'(p)$ is apparently maximized at $p = (2\underline{c})^{1/2}$.

We now consider the second part of $S'(p)$. Notice that, after integration, it is equivalent to

$$p + \frac{\underline{c}}{b-\underline{c}}p^2 - \frac{1}{2(b-\underline{c})}p^4 \qquad \text{if } \sqrt{2\underline{c}} \leq p \leq \sqrt{2b}. \tag{12}$$

We now show that this function is decreasing in p if $(2\underline{c})^{1/2} \leq p \leq (2b)^{1/2}$ and $1/8 \leq \underline{c} \leq b \leq ¼$ and, hence, is maximized at $p = (2\underline{c})^{1/2}$. The first derivative is

$$\begin{aligned} &1 + \frac{2p}{b-\underline{c}}\left(\underline{c} - p^2\right) \\ &\leq 1 + \frac{2p}{b-\underline{c}}\left(\underline{c} - 2\underline{c}\right) \qquad (\Theta\ p \geq \sqrt{2\underline{c}}) \\ &= 1 - \frac{2p\underline{c}}{b-\underline{c}} \\ &\leq 0 \end{aligned}$$

The last inequality follows because $1/8 \leq \underline{c} \leq b \leq ¼$ and $(2\underline{c})^{1/2} \leq p \leq (2b)^{1/2}$ imply that $b - \underline{c} \leq 1/8$ and $2p\underline{c} \geq 1/8$.

Now, based on the above discussions, $S'(p)$ can be simplified as follows:

$$S'(p) = \begin{cases} \sqrt{2b}(1-\sqrt{2b}) & \text{if } p = \sqrt{2b} \\ \sqrt{2\underline{c}} & \text{if } p = \sqrt{2\underline{c}} \end{cases}$$

Clearly, since $1/8 \leq \underline{c} \leq b \leq 1/4$, $S'(p)$ is maximized at $p = (2\underline{c})^{1/2}$.

Now, consider the buyer's strategy. If the buyer's search cost is less than $\underline{c}$, he will get either $p = 1/2$ (if $c \leq 1/32$) or $p = (2c)^{1/2}$ if he reveals c, whereas he will get $p = (2\underline{c})^{1/2}$ if he does not reveal. Clearly, since $\underline{c} \geq 1/8$, $(2\underline{c})^{1/2} > (2c)^{1/2}$ (since $c < \underline{c}$) and $(2\underline{c})^{1/2} \geq 1/2$. Hence, if the buyer's search cost is less than $\underline{c}$, he is better off revealing c.

If the buyer's search cost is greater than $\underline{c}$, he will get $p = (2c)^{1/2}$ if he reveals c whereas he can get $p = (2\underline{c})^{1/2}$ if he does not reveal. Clearly, since $c > \underline{c}$, the buyer will not reveal c. ***Q.E.D.***

Proposition 3: If $1/4 < b < 1/2$, then the buyer will not reveal if $c \geq \underline{c}$ ($b/2 < \underline{c} \leq b$) will not reveal, and he will reveal if $c < \underline{c}$. If the buyer reveals his c, the seller's optimal offer will follow that described in the public information model (i.e., offering $p = (2c)^{1/2}$ if $c > 1/32$ or offering $p = 1/2$ if $c \leq 1/32$). If the buyer does not reveal, believing $c \in [\underline{c}, b]$, the seller will offer $p = (2\underline{c})^{1/2}$.

Proof: Since the proof of this proposition is similar to that of Proposition 2, it is omitted.

5. Discussion

From the preceding analysis, it is interesting to note that there is no general rule on how the buyer's optimal decision to reveal or not to reveal the information depends on the values of c and b. In some cases (e.g., Proposition 0), the seller's equilbrium strategy and the equilibrium outcomes are independent of the buyer's information revelation decision. In other cases, such as those described in Proposition 1, only buyers with relatively large c values will reveal their private information. However, there are also cases (e.g., Propositions 2 and 3), only buyers with relatively small c values will reveal their private information.

Zwick and Lee (1999) has found some empirical support for the discontinuous payoff functions under public information case shown in Figure 1. Hence, to some extent, even though the game involves non-trivial analysis, players are able to recognize their different strategic positions under different parameter values. Now, given the unintuitive and intriguing results of the information revelation game discussed in Section 4, the next question we are interesting in asking is: will the players still be able to recognize their different strategic positions under different parameter values. Specifically, in the next phase of our research, we are interested in finding out how, in the real life situations, the players will actually handle their private information, given different parameter values.

References

[1] Chatterjee K. and Ching C. Lee, "Bargaining and Search with Incomplete Information about Outside Options", *Games and Economic Behavior* **22**: 203-237, 1998.

[2] Lee, Ching C., "Bargaining and Search with Recall -- A Two-Period Model with Complete Information", *Operations Research* **42**: 1100-1109, 1994.

[3] Ponssard, Jean Pierre, *Competitive strategies : an advanced textbook in game theory for business students*, North-Holland, 1981.

[4] Zwick, R., and Ching C. Lee (1999), "Bargaining and Search: An Experimental Study", *Group Decision and Negotiation* **8**: 463-387, 1999.