A Study on Optimum Bucket Size

Masamitsu Kiuchi

Tamagawa University, Graduate School of Engineering (kuchm8er@engs.tamagawa.ac.jp)

Abstract

When we make a schedule of jobs, we use the loading technique. In order to load the jobs properly, following problems should be solved; that is, (1)what is the appropriate bucket size , (2) how to estimate the capacity and (3) to what extent the work center is to be loaded. In this paper, the problem of bucket size is taken up. The purpose of this paper is to clarify the relationship between lead time of jobs and bucket size and the relationship between work center utilization and bucket size and to propose a method to determine the optimum bucket size.

In this paper, production system and structure of work are defined as follows. Production system is hierarchically structured, that is the shop is composed of one or multiple work centers, and the work center is composed of work stations. The work has also a hierarchical structure. The operation is the minimum unit of the work. A job is a set of operations, and an order is a set of jobs. The work and production system correspond as follows. An operation is done on a work station. A job is done in a work center. An order is done in the shop.

As the basic research on this problem, a simple model is built. Used this model some experiments are conducted.

From the results of experiment, as the relationship between mean lead time and bucket size followings are found. When bucket size is small, mean lead time is long. As bucket size become large, mean lead time decreases, and it become shortest when bucket size is a certain length. And there exist the bucket size which makes mean lead time shortest. As the relationship between work center utilization and bucket size, followings are found. As bucket size increases, utilization gradually reaches to arrival load ratio. And from a certain bucket size, utilization equals to arrival load ratio. Due to the relationship between optimum bucket size based on lead time and optimum bucket size based on utilization, the optimum bucket size exists in the zone of the optimum bucket size based on the utilization. It is found that optimum bucket size based on lead time is considered to be the optimum bucket size which take two measures of performance into consideration. An estimation equation of optimum bucket size is deducted. It is found that this equation gives fairly correct estimation of the optimum bucket size.

1. Introduction

Generally speaking, the loading method is used in job scheduling system. In order to load the jobs properly, following problems should be solved; that is, (1)what is the appropriate bucket size, (2) how to estimate the capacity and (3) to what extent the work center is to be loaded. In this paper, the problem of bucket size is taken up. The purpose of this paper is to clarify the relationship between lead time of jobs and bucket size and the relationship between work center utilization and bucket size and to propose a method to estimate the optimum bucket size.

First of all, the scheduling model which reflects the real scheduling situation is constructed, and the simulation model is built according to the scheduling model. Secondly, the experiments are conducted by using this simulation model, and the relationship between lead time and bucket size and the relationship between work center utilization and bucket size are made clear. Lastly, an estimation equation to determine the optimum bucket size is to be proposed.

2. Model of the research

2.1 Structure of scheduling

In this paper, the production system and the work are defined as follows.

(1) Production system

Generally, many production systems are hierarchically structured. The number of hierarchies differs company by company according to the size of business, the complexity of the structure of the products, and the complexity of the routing to make the products. In this paper, the structure given below is taken up.

a. Work Station : Work station is the minimum unit of production system.

- b. Work Center : Work center is a set of work stations.
- c. Shop : Shop is a set of work centers.

(2) Work

- The work has also a hierarchical structure.
- a. Operation : Operation is the minimum unit of the work.
- b. Job : Job is a set of operations.
- c. Order : Order is a set of jobs.

(3) Relationship between production system and work

- The work and production system correspond as follows.
- a. Work Station and Operation : One operation is performed without stopping on one work station.
- b. Work Center and Job : The set of operations performed on the work station within the same work center successively is a job.
- c. Shop and Order : The set of jobs to be done in the shop is an order.





2.2 Scheduling model

The scheduling model built in this paper is shown in Fig.2.



(1) Job scheduling system

The function of the job scheduling system is to determine the bucket number in which the processing of the job is to be started and finished, that is, the schedule of the job is to be planned. The scheduling method used in job scheduling system is loading. Based on the balance between the load and the capacity, loading makes a plan which indicates the jobs to be processed in the particular period.

The bucket is the period by which jobs are pooled for loading(see Fig.3).



Loading is done as was shown in Fig.4. Suppose that loading is to be done at time t0, some jobs which are already arrived at

the work center but their processing are not started are pooled. Some or all of jobs are loaded to the work center according to





(2) Dispatching system

The schedule made by the scheduling system is sent to work center for processing. Dispatching system determines the sequence of jobs to be done on each work station using some dispatching rule. Due to the sequence determined in dispatching system, operations of each job are actually done(see Fig.5).





2.3 Conditions of the model

As the basic study on optimum bucket size, the conditions of the model taken up are as follows.

(1) Criteria

As the criteria of the optimum bucket size, mean lead time and work center utilization are adopted. Lead time of the job is time period between its arrival time to completion time. So mean lead time is expressed as equation(1).

$$L = \Sigma l i / n \tag{1}$$

Where L : mean lead time l i : lead time of job i

n : number of sample

Work center utilization is the rate of total work load done at the work center to the capacity of the work center. Then it is written as equation(2).

$$U = W / C \tag{2}$$

where U: work center utilization

W: total work load done

C : capacity of the work center

(2) Factor concerned with production system

a. structure of shop : The shop is composed of only one work center.

b. number of work station : from one to eight.

c. capacity of the work center : The capacity of the work center is given by equation(3).

$$C = b \times m$$
 (3)

where b : bucket size

m : number of work stations in the work center

(3) Factors concerned with job

a. due date : In this paper, due date is not considered.

b. processing time : The distribution of the processing time is Erlang distribution of phase four.

c. number of operation : The number of operation of job equals to the number of work stations.

d. routing of job : The flow type is taken up in this paper.

e. job arrival pattern to the work center : The jobs arrive at work center at random.

f. arrival load ratio : The ratio of the total work to be loaded on the work center to the capacity of the work center is called arrival load ratio in this paper. Arrival load ratio is from 0.7 to 0.95.

(4) Loading method

a. Finite loading is adopted.

b. As priority rule, FCFS rule is adopted.

(5) Dispatching method

dispatching rule : Due time rule is adopted. Due time is given by job scheduling system.

(6) Other assumptions

a. The break down of the work station is not considered.

b. The estimated processing time has no error.

2.4 Approach

As this paper takes up a dynamic situation, it seems that analytical approach can not be taken. Therefore, simulative approach is adopted.

3. Experimental results

The experiments are conducted by simulation.

3.1 Relationship between mean lead time and bucket size

The relationship between mean lead time and bucket size in case of a certain number of work stations is shown in Fig. 7. And the relationship between mean lead time and bucket size in case of a certain arrival load ratio is shown by Fig. 8. As was shown by Fig.7 and Fig.8, when bucket size is small, mean lead time is long. As bucket size become large, mean lead time decreases, and it become shortest when bucket size is a certain length. From that point, it increases gradually. It is found that there exist the bucket size which makes mean lead time shortest. This is true regardless to arrival load ratio and to number of work stations.

From Fig.7, it is found that as arrival load ratio rises, the bucket size which makes mean lead time shortest becomes large.

From Fig.8, it is found that as number of work stations increases, the bucket size which makes mean lead time shortest becomes large.



Fig. 6 Effect of arrival load ratio on relationship between lead time and bucket size



Fig. 7 Effect of number of work stations on relationship between lead time and bucket size

3.2 Relationship between utilization and bucket size

The relationship between work center utilization and bucket size in case of a certain number of work stations is shown in Fig.8. And the relationship between work center utilization and bucket size in case of a certain arrived load ratio is shown in Fig. 9. From Fig.8 and Fig.9, it is found that as bucket size increases, the work center utilization becomes high and it reaches gradually to the arrival load ratio. This is true regardless of arrival load ratio and of number of work stations.

From Fig. 8, it is found that the higher arrival load ratio is, the large bucket size from where utilization equals to arrival load ratio is.

From Fig. 9, it is found that as number of work stations increases, the bucket size at which utilization equal to arrival load ratio become large.



Fig. 8 Effect of arrival load ratio on relationship between utilization and bucket size



Fig. 9 Effect of number of work stations on relationship between utilization and bucket size

3.3 Optimum bucket size

Optimum bucket size can be defined in two ways. One is based on mean lead time and the other is based on work center utilization. Optimum bucket size based on mean lead time is defined as the bucket size when the mean lead time is shortest. Table 1 shows the optimum bucket size based on mean lead time obtained from the results of experiments.

Optimum bucket size based on the work center utilization is defined as the smallest bucket size where the work center utilization equals to arrival load ratio. In table 2, the smallest bucket size at which work center utilization equals to arrival load ratio is shown.

 Table 1
 Optimum bucket size based on lead time

ρ	_m	1	2	3	4	5	6	7	8
0.70)	11.0	11.0	11.0	11.0	11.0	11.0	11.7	13.4
0.75	5	9.0	9.3	9.0	9.9	11.4	14.2	15.7	18.8
0.80)	9.0	9.3	11.0	13.4	16.4	20.0	22.0	27.7
0.85	5	10.0	13.0	17.8	21.1	26.2	31.9	38.5	45.7
0.90)	10.0	20.8	37.7	39.1	51.3	61.9	72.3	88.4
0.95	5	17.0	56.5	73.4	100.9	136.3	141.9	155.9	178.7

 Table 2
 Optimum bucket size based on the utilization

ρ	m	1	2	3	4	5	6	7	8
0.	.70	9.0	9.0	9.0	11.0	13.0	15.0	15.8	18.8
0.	.75	9.0	9.8	11.4	13.4	16.8	19.3	20.8	25.2
0.	.80	9.0	12.6	15.2	19.0	22.6	25.9	29.4	36.4
0.	.85	10.0	17.5	22.3	29.0	34.9	39.4	46.6	57.1
0.	.90	11.8	26.8	37.9	51.7	65.3	72.6	85.3	97.8
0	.95	22.5	59.5	79.8	105.3	135.1	149.5	158.7	187.5

3.4 Relationship optimum bucket size based on lead time and optimum bucket size based on utilization

The relationship between optimum bucket size based on lead time and optimum bucket size based on utilization is shown in Fig.10. As was shown by Fig.10, the optimum bucket size based on lead time exists in the zone of the optimum bucket size based on the utilization.

In concluding, it is found that optimum bucket size based on lead time is considered to be optimum bucket size which take two measures of performance into consideration.



Fig. 10 Relationship between bucket size and each certification

4. Estimation equation of optimum bucket size

4.1 Estimation equation of optimum bucket size based on lead time

Fig.11 shows the relationship between the optimum bucket size and number of work stations in the work center.



Fig. 11 Relationship between the optimum bucket size and number of work stations

As was shown in this figure, the optimum bucket size increases linearly as the number of work stations become large. Therefore, optimum bucket size can be written as equation.(4)

$$B = a \times m \tag{4}$$

Where B : optimum bucket size

m : number of work stations in work center

a : gradient





Applied the linear estimation method, the gradient is found and shown in Fig.12. As the arrival load ratio approaches to 1.0, the gradient increases exponentially. If the arrival load ratio equals to one, the system never reaches to steady state, therefore, the gradient is infinite. Then, the gradient can be expressed as equation(5).

$$a = K \times \frac{1}{(1 - \rho)} \tag{5}$$

Where K: constant

 ρ : arrival load ratio

Figure 13 shows the relationship between arrival load ratio and the constant K in equation(5).



Fig. 13 Relationship between K and arrival load ratio

From the results of curve fitting, equation(6) is obtained.

$$K = 2.143 \rho - 0.7716 \tag{6}$$

Then

$$a = \frac{(2.143\rho - 0.7716)}{(1 - \rho)} \tag{7}$$

Due to the results above mentioned, estimation equation of optimum bucket size is expressed as follows.

$$B = \frac{(2.143\rho - 0.7716)}{(1 - \rho)} m$$
(8)

Table 3 shows the difference between the estimated value by equation(8) and that of the experiment. As is shown by Table 3, there is a small error. However, from Fig.10, near the optimum bucket size, there is little difference in mean lead time. Accordingly, it is said that this error has little influence on lead time. So equation(8) gives fairy correct estimation.

ρ m	1	2	3	4	5	6	7	8
0.70	0.0	0.0	0.0	-1.3	-0.9	-0.4	1.2	0.6
0.75	0.0	-3.1	-1.4	0.0	-0.1	0.8	2.6	1.5
0.80	0.0	-3.2	-1.1	-0.1	1.0	2.4	3.6	1.3
0.85	-3.0	-3.5	-1.3	-1.0	0.1	2.6	2.4	-1.1
0.90	-0.2	-3.7	-3.2	-5.4	-7.4	-3.1	-4.3	-5.2
0.95	2.8	-8.9	-3.9	-4.2	-8.7	2.2	18.3	14.8

Table 3 Difference between value of estimation equation and value of experiment

5. Conclusion

In this paper, the problem of bucket size in loading is taken up. The purpose of this paper is proposed an estimation equation of optimum bucket size. As the first step, structure of scheduling is defined, and simulation model is build. As next step, the experiments are conducted by simulation model. Due time to the results of experiments, an estimation equation of optimum bucket size is proposed. And it is found that this equation is correct.