The choice of a personnel problem:
An application of an interactive group fuzzy AHP method

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Abstract

Given that personnel assessment and selection problems are ambiguous, vague and full of conflicting judgments among evaluators, the disagreement or contradictory frequently occurred in the group judgment. Hence, a good decision of personnel selection problem must be evaluated by multiple criteria in a subjective/objective and quantitative/qualitative assessment framework. In contrast to conventional PSP methods, which are usually treated with simple additive weighting or statistical techniques, this research presents a fuzzy group decision model for conducting PSP assessment. This study takes Shih Chien University as a case study and the selected faculty reflects the preference of the majority of a panel of reviewers from pair-by-pair comparison of a set of criteria assuming group decision-making and an uncertain assessment environment. The application of a group fuzzy AHP methodology studied in this personnel selection problem can also be used in the business, governmental, and other organizational practices.

1. Introduction

Personnel selection problems (PSP) are very crucial and not easy to treat, such as choosing an adequate Asia-Pacific president from the senior management, appointing a promising government spokesperson from a couple of candidates, recruiting a talented new faculty for a university, or even dis missing some incompetent employees due to sales shrinkage or downsizing in a company. No matter in the business, educational, governmental or any other organizational activities, PSP may not be complicated, but require a objective evaluation process in order to convince also-runners and prevent conflicts among the recruiting committee or top decision makers. To neglect the group consensus, the selected person may stick in a questionable or hostile working environment that hampers the efficiency of the organizational operation.

Owing to that the best PSP decision in real world heavily depends on available personnel data, a critical role of a recruiting committee is how to make an optimization choice under an uncertain and noisy environment. To reduce the subjective prejudice, emotional affection, and limitation of the human habitual domain, a systematic and objective methodology always is better utilized in the personnel selection process in order to supplement group consensus and human information processing capabilities. Consequently, in reality, a good decision-making model needs to consider the group consensus so as to assist the policy implementation successfully, e.g. in the event of Peru hostage crisis in 1997, the negotiation strategy of President Bush in the incident US-China plane crush etc.

Pioneered by Satty in 1971, the AHP is a convenient and widespread decision making analysis method that models unstructured economic, social, political and management science problems based on multiple criteria. AHP elicits a corresponding priority vector representing the preferred information of the evaluator(s), based on the pair-by-pair comparison values for a set of objects. By considering qualitative and quantitative aspects, the AHP methodology can easily adjust evaluation constructs and increase or decrease criteria/sub-criteria.

Since pairwise comparison values are the judgments obtained from the assessor(s) using an adequate semantic scale, in practice a assessor very often give some or all pairwise comparison values with an uncertain degree rather than precise ratings. Hence, in 1980, Professor Graan presented a fuzzy AHP method and received lots of attention immediately. Since then, numerous methods in the literatures have been developed to generate the priority vector from comparing all pairs of criteria and decision alternatives under a fuzzy environment. Four major steps in the fuzzy AHP method are listed below:

(1) To build a hierarchy structure;

(2) To do pair-by-pair comparison for a set of objects;

(3) To treat fuzzy values; and

(4) To calculate priority weights among a number of alternatives.

Despite these efforts, current fuzzy AHP methods and applications ignore the interactions between group consensus and individual evaluator's opinion as well as the interactions among fuzzy comparisons and evaluators. By taking the interaction among every panel reviewer's opinion and group consensus into account, this study applies a fuzzy AHP method to rank the most suitable faculty with different group consensus level. The derived priority vector reflects the majority of the involved individual's preference and is progressively less vulnerable to conflicting group judgments. The University of Shih Chien (USC) is used as case study herein.

The rest of this paper is organized as follows. Section 1 introduces the motivations for the research. Section 2 then presents an overview of the PSP problem occurred in the USC. Section 3 presents several propositions for treating a fuzzy rating, and also proposes a fuzzy decision-making model to measure the performance of every marketing strategy. By considering the consensus of a reviewing committee, Section 4 develops an interactive fuzzy group decision-making algorithm for assessing, choosing, and ranking five prospective candidates in this case study. Concluding remarks are finally made in Section 5.

2. Overview of the Case Study

The university number in Taiwan has markedly grown up from 25 to 175 during the last two decades. Meanwhile, the graduate student number of high schools has decreased from 0.3 million down to 0.25 million. Hence, in the recent years how to differentiate teaching characteristics, enhance the research competitiveness, and attract excellent students to apply USC has become the vital task for the USC board. Due to that human resource is not only the one of the most valuable assets in an organization, but also is the key factor to surpass your competitors especial in the era of knowledge economics. The board of USC plans to build several objective criteria and a public procedure to assist each department or institute to recruit talented and adequate professors.

As is widely known, a faculty selection may not be complicated, but require an objective evaluation process in order to convince also-runners and prevent conflicts among the teacher recruiting committee. To neglect the group consensus, the selected person may stick in a questionable or hostile working environment that hampers the department operation and competitiveness. Thus, this study employs the merit of a fuzzy AHP methodology to treat the problem of choosing an appropriate faculty for a professorship position in the graduate school of Internet Trade of Shih Chien University.

Following extended discussion among senior scholars, alumni and industry managers, four decision criteria are suggested. They are research potentiality, creativity in Internet business models, creativity in international trade models, and maturity and personal integrity. After several competitive screening interviews, only five serious candidates remain, referred to herein as A, B, C, D, and E. To identify which applicant is the best qualified for the job, the committee has been installed to provide advice. The committee has six members and they assess the five candidates by the mentioned four decision criteria. Since the disagreement or contradictory frequently occurred in the group judgment, how to treat the group consensus is also included in this investigation.

3. Treating a fuzzy decision making problem

Among numerous current AHP methods for deriving the priority vector by comparing all pairs of criteria and decision alternatives, the eigenvector method (EM), least squares method (LSM), and logarithmic least squares method (LLSM) are the three most common and popular approaches for calculating the priority vector [5, 18, 36]. In evaluating various methods of deriving the priority, Fichtner [18] proposed four criteria for deciding the "best", as follows:

Criterion 1: Correctness in the consistent cases. The method should produce the correct priority vector if the provided matrix A is consistent.

Criterion 2: Comparison order variance. The priority vector should not vary with the indexing order.

Criterion 3: Smoothness. Small changes in input variables should not cause big changes in output variables.

Criterion 4: Power invariance. The resulting priority vector of the latter case should be the inverse of that of the former case.

Fichtner [18] demonstrated that both EM and LSM satisfy the first three criteria and only LLSM satisfies all four criteria. LLSM formulates an AHP problem as follows [36]:

<u>Model 1</u>

Minimize $\sum_{1 \le i<j \le n} (\log a_{ij} - \log(\frac{w_i}{w_j}))^2$ Subject to: $a_{ij} \ge 0$,

where a_{ij} denotes the evaluator’s preference between objects i and j, n is the number of objects, the ratio w_i / w_j represents the comparison between each pair of objects ‘ i ’ and ‘ j ’, and $a_{ij} = 1 / a_{ji}$ in a positive reciprocal matrix.

Previous studies [5-6] have described how to employ a goal programming method (GPM) to generate the priority vector from the reciprocal matrix of pairwise comparison. The underlying notion of using GPM to treat AHP problems is that GPM can easily treat both the interval and point estimates. Furthermore, using GPM for priority derivation not only fulfills the four axioms proposed by Fichtner [18], but also fulfills another axiom called single outlier neutralization [5-6] and can be combined with other decision tools to deal with complex real-world problems [12,16].

3.1 The proposed decision-making model

Building on the above discussion, this work utilizes the LLSM and GP properties to minimize variance from pairwise comparisons and maximize the group consensus and/or the subjective preference of each individual evaluator, as discussed in the next section. Hence, to treat the problem addressed in Section 2, the following model is presented.

Model 2

Minimize
$$\sum_{q=1}^{Q}\sum_{q'>q}^{Q}(\log \delta^{+}_{eqq'} + \log \delta^{-}_{eqq'}) + \sum_{q=1}^{Q}\sum_{c=1}^{c_q}\sum_{c'>c}^{c_q}(\log \delta^{+}_{eqcc'} + \log \delta^{-}_{eqcc'}) +$$

$$\sum_{q=1}^{Q}\sum_{c=1}^{c_q}\sum_{i=1}^{N}\sum_{i'>i}^{N}(\log \delta^{+}_{eqcii'} + \log \delta^{-}_{eqcii'})$$

Subject to:

$$\log a_{eqq'} - (\log w_{eq} - \log w_{eq'}) - \log \delta^{+}_{eqq'} + \log \delta^{-}_{eqq'} = 0, \tag{3.1}$$

$$\log a_{eqcc'} - (\log w_{eqc} - \log w_{eqc'}) - \log \delta^{+}_{eqcc'} + \log \delta^{-}_{eqcc'} = 0, \tag{3.2}$$

$$\log a_{eqcii'} - (\log w_{eqci} - \log w_{eqci'}) - \log \delta^{+}_{eqcii'} + \log \delta^{-}_{eqcii'} = 0, \tag{3.3}$$

$$\text{for all } (q, q') \in \{(q, q'): 1 \le q < q' \le Q\}, \tag{3.4}$$

$$\text{for all } (c, c') \in \{(c, c'): 1 \le c < c' \le c_q\}, \tag{3.5}$$

$$\text{for all } (i, i') \in \{(i, i'): 1 \le i < i' \le N\}, \tag{3.6}$$

where $q = 1, 2, \ldots, Q$ denotes the g'th dimension (in this case $Q = 3$), $c = 1, 2, \ldots, c_q$ represents the c'th criterion under the q'th dimension (in this case $c_1 = 3$, $c_2 = 4$, and $c_3 = 3$), $i = 1, 2, \ldots, N$ expresses the i'th marketing strategy (in this case $N = 5$), e represents the e'th evaluator, $a_{eqq'}$ denotes the e'th evaluator's preference between dimensions q and q', $a_{eqcc'}$ means the e'th evaluator's comparison value to which criterion c is preferred to criterion c' under the q'th dimension, and $a_{eqcii'}$ represents the e'th evaluator's comparison value to which marketing strategy i is preferable to marketing strategy i' under the c'th criterion of the q'th dimension.

3.2 To treat a fuzzy assessment

AHP tends to reflect an ideal decision making environment in which decision makers can rationally consider all aspects of the problem, think through them, obtain accurate and relevant information, and then provide a consensus solution. However, in real world cases nosy, incomplete, and uncertain information, as well as different opinions, make it difficult to give a precise assessment when evaluating marketing strategies.

Fuzzy set theory was initially applied in decision making by Bellman et al. [12, 16, 19, 44], who noted that much actual decision making happens in an environment where the goals, constraints and consequences of possible actions are unknown, and fuzzy set theory can be used to deal with uncertainties in human decision making. Since Bellman et al., various applications of fuzzy theory to decision-making problems have been implemented [12, 16, 19, 34, 44]. The early work on fuzzy AHP of Graan [22], late Lootsma [29] and van Laarhoven et al. [27] proposed some additional methods of treating Satty's AHP with fuzzy ratings. Since then, the literature has developed numerous methods [2, 8-11, 13-15, 23, 28-29, 31, 35, 40-42] to solve AHP problems in a fuzzy environment.

Traditional fuzzy AHP methods generally only treat triangular membership functions, but in practical cases, fuzzy ratings obtained from the decision maker(s) should not be limited in a triangular format. Accordingly, this study describes an efficient means of interpreting general concave membership functions. First, consider the following proposition:

Proposition 1

Let $\mu(a)$ be a membership function of a fuzzy value a, as depicted in Fig. 1, where a_k (for k = 1, 2, ..., m) are the break points of $\mu(a)$, s_k (k = 1, 2, ..., m-1) are the slopes of line segments between a_k and a_{k+1}, and $s_k = [\,(\mu(a_{k+1})-\mu(a_k)) / (a_{k+1} - a_k)]$.

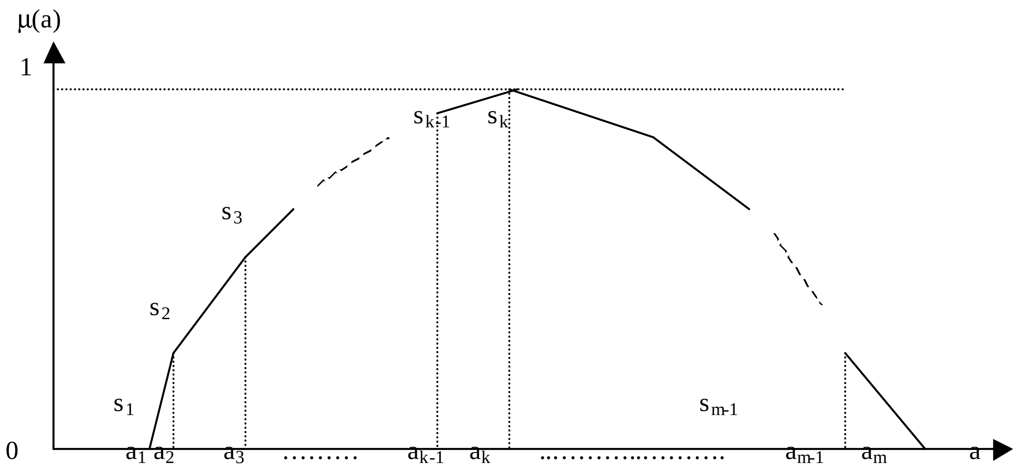

Fig. 1 A general concave piecewise membership function

Then, $\mu(a)$ can be expressed below:

$$\mu(a) = \mu(a_1) + s_1(a - a_1) + \sum_{k=2}^{m-1} \frac{s_k - s_{k-1}}{2}\left(|a - a_k| + a - a_k\right)$$

where $\mu(a_1) = 0$, $\mu(a_k) = 1$, $\mu(a_m) = 0$, and |*| is the absolute value of *.

This proposition can be verified as follows:

(i) If $a_1 \le a \le a_2$ then

$$\mu(a) = \mu(a_1) + s_1(a - a_1) + \frac{s_2 - s_1}{2}\left(|a - a_2| + a - a_2\right) = \mu(a_1) + s_1(a - a_1)$$

(ii) If $a_2 \le a \le a_3$ then

$$\mu(a) = \mu(a_1) + s_1(a - a_1) + \frac{s_2 - s_1}{2}\left(|a - a_2| + a - a_2\right) + \frac{s_3 - s_2}{2}\left(|a - a_3| + a - a_3\right)$$

$$= \mu(a_1) + s_1(a_2 - a_1) + (s_2 - s_1)(a - a_2)$$

(iii) If $a_{k'-1} \le a \le a_{k'}$ then $\sum_{k=k'}^{m-1} (|a - a_k| + a - a_k) = 0$ and $\mu(a)$ becomes

$$\mu(a_1) + s_1(a_2 - a_1) + \sum_{k=2}^{k'-1} (s_k - s_{k-1})(a - a_k).$$

Consider a fuzzy number "a" as depicted in Fig. 2. Based on Proposition 1, $\mu(a)$ can be expressed below:

$\mu(a) = 1.6(a-3)-(1.2/2)(|a-3.5|+a-3.5)-(0.8/2)(|a-4|+a-4)-(1.2/2)(|a-4.5|+a-4.5)$

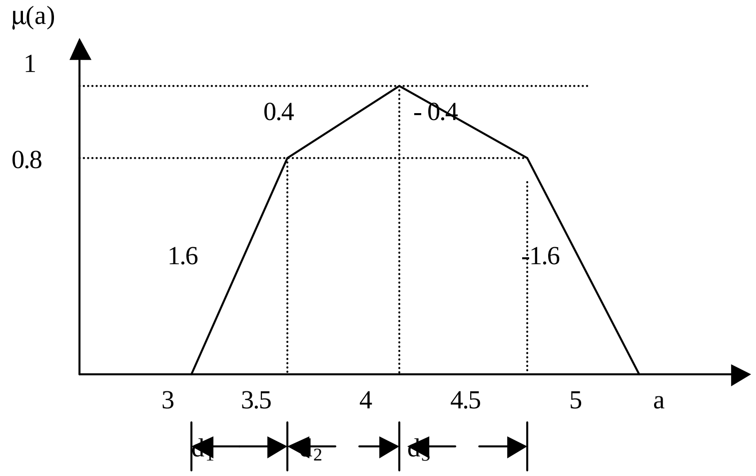

Fig. 2 A fuzzy number "a"

To linearize an absolute term, the proposition 2 is introduced below:

<u>Proposition 2</u>

Consider an absolute term program with a negative coefficient expressed as follows:

<u>PP1:</u> Maximize $Z = (s/2)(|a - a_k| + a - a_k)$ Subject to: $a \ge 0$, s is a negative coefficient (that is, $s < 0$), where a_k is a given constant.

can be transformed into following linear program

<u>PP2:</u> Maximize $ZZ = s(a - a_k + d)$ Subject to: $a + d \ge a_k$, $d \ge 0$, $s < 0$, where a_k is a given constant.

Proof. This proposition can be examined as follows:

(i) Case 1: If $a - a_k \ge 0$, $Z = s(a - a_k)$.
At the optimal solution d will be forced as $d = 0$, resulting in $ZZ = s(a-a_k) = Z$.

(ii) Case 2: If $a - a_k < 0$, $Z = 0$.
At the optimal solution d will be forced as $d = a_k - a$, resulting in $ZZ = 0 = Z$.

This proposition is then proved. Consider the following example with absolute terms:

<u>Example 1</u>

Maximize $\mu(a) = 1.6(a-3)-(1.2/2)(|a-3.5|+a-3.5)-(0.8/2)(|a-4|+a-4)-(1.2/2)(|a-4.5|+a-4.5)$

Subject to: $a \ge 0$,

where $\mu(a)$ is depicted in Fig. 2.

By using Proposition 2, Example 1 can be linearized as follows:

Maximize $\mu(a) = 1.6(a-3)-1.2(a-3.5+d_1) - 0.8(a-4+d_2)-1.2(a-4.5+d_3) = -1.4a - 1.2d_1 - 0.8d_2 - 1.2d_3 + 8$

Subject to: $a+d_1 \geq 3.5$, $a+d_2 \geq 4$, $a+d_3 \geq 4.5$, $a \geq 0$.

Using Excel [17] or LINGO [39] obtains the solution set as ($\mu(a) = 1$, $a = 4$, $d_1 = 0$, $d_2 = 0$, and $d_3 = 0.5$). Assume that an addition constraint $a \leq 3.6$ is added to Example 1, after computing this modified example on Excel or LINGO, the computed solution set becomes ($\mu(a) = 0.84$, $a = 3.6$, $d_1 = 0$, $d_2 = 0.4$, and $d_3 = 0.5$). Based on Propositions 1 and 2, Corollary 1 is stated as follows:

Corollary 1

A generalized concave fuzzy number loga illustrated in Fig. 3 can be interpreted below:

$$\log a = \frac{1}{-s_{m-1}} \left[-\mu(\log a) + \sum_{kk=1}^{m-2} (s_{m-1} - s_{kk}) d_{kk} + \sum_{k=2}^{m-1} (s_k - s_{k-1}) \log a_k \right]$$

where $\log a + d_k \geq \log a_k$, $\log a \geq 0$, $s_0 = 0$, $kk = 1, 2, \ldots, m-2$ and $k = 1, 2, \ldots, m-1$.

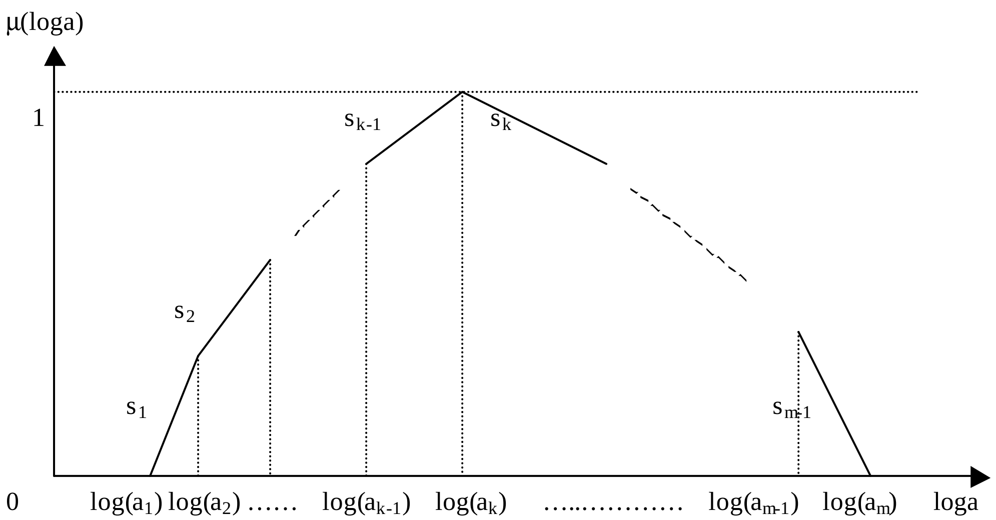

Fig. 3 A fuzzy value loga's generalized membership function

Notably, in case of $\log a < 0$, the scale switching approach can be used to solve this problem. Taking $b = 1/3$ as an example, then $\log b = -0.47712$. Consider the example presented in Fig. 4(a). By using the scale switching approach, loga shown in Fig. 4(a) can be replaced by loga' from Fig. 4(b) where loga' = loga + 1.

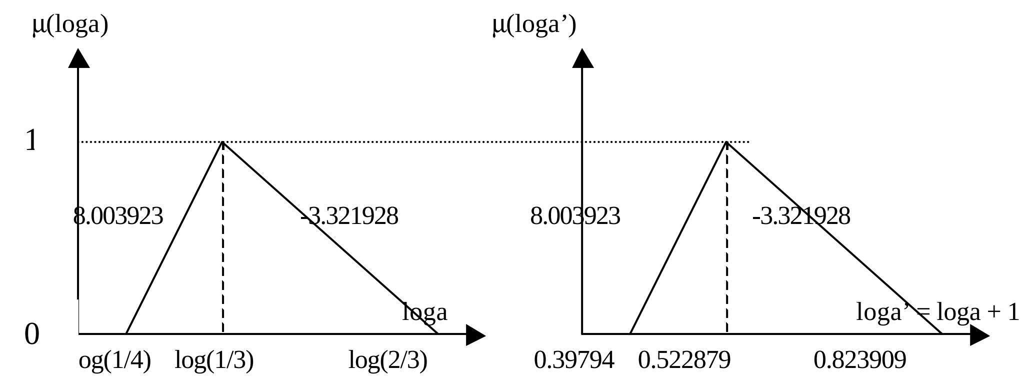

Fig. 4(a) a negative fuzzy value **Fig. 4(b) a scaled positive fuzzy value**

3.3 Proposed fuzzy decision-making model

Based on Corollary 1, the fuzzy AHP model is developed below:

Model 3

$$\text{Minimize} \quad \text{Goal A} = \sum_{q=1}^{Q}\sum_{q'>q}^{Q}(\log \delta^{+}_{eqq'} + \log \delta^{-}_{eqq'}) + \sum_{q=1}^{Q}\sum_{c=1}^{c_q}\sum_{c'>c}^{c_q}(\log \delta^{+}_{eqcc'} + \log \delta^{-}_{eqcc'}) +$$

$$\sum_{q=1}^{Q}\sum_{c=1}^{c_q}\sum_{i=1}^{N}\sum_{i'>i}^{N}(\log \delta^{+}_{eqcii'} + \log \delta^{-}_{eqcii'})$$

$$\text{Maximize} \quad \text{Goal B} = \mu_e = \frac{2}{Q(Q-1)}\sum_{q=1}^{Q}\sum_{q'>q}^{Q}\mu(\log a_{eqq'}) \; +$$

$$\frac{1}{Q}\sum_{q=1}^{Q}\Big(\frac{2}{c_q(c_q-1)}\sum_{c=1}^{c_q}\sum_{c'>c}^{c_q}(\mu(\log a_{eqcc'}))\Big) + \frac{1}{Q}\Big(\sum_{q=1}^{Q}\Big(\frac{2}{c_q(c_q-1)}\sum_{c=1}^{c_q}\Big(\frac{2}{N(N-1)}\Big(\sum_{i=1}^{N}\sum_{i'>i}^{N}(\mu(\log a_{eqcii'}))\Big)\Big)\Big)\Big)$$

Subject to: (3.1) – (3.6)

$$\log a_{eqq'} = (1/\text{-}s_{eqq',m\text{-}1})[-\mu(\log a_{eqq'}) + \mu(\log a_{eqq',1}) + \sum_{k=1}^{m_{eqq'}-2}(s_{eqq',m-1} - s_{eqq',k})d_{eqq',k} + \sum_{k=1}^{m_{eqq'}-1}(s_{eqq',k} - s_{eqq',k-1})\log a_{eqq',k}\;] \tag{3.7}$$

$$\log a_{eqq'} + \sum_{k=1}^{m_{eqq'}-2} d_{eqq',k} \geq \log a_{eqq',m\text{-}1},\; \log a_{eqq'} \geq 0,\; s_{eqq',0} = 0, \tag{3.8}$$

$$\log a_{eqcc'} = (1/\text{-}s_{eqcc',m\text{-}1})[-\mu(\log a_{eqcc'}) + \mu(\log a_{eqcc',1}) + \sum_{k=1}^{m_{eqcc'}-2}(s_{eqcc',m-1} - s_{eqcc',k})d_{eqcc',k} + \sum_{k=1}^{m_{eqcc'}-1}(s_{eqcc',k} - s_{eqcc',k-1})\log a_{eqcc',k}\;] \tag{3.9}$$

$$\log a_{eqcc'} + \sum_{k=1}^{m_{eqcc'}-2} d_{eqcc',k} \geq \log a_{eqcc',m\text{-}1},\; \log a_{eqcc'} \geq 0,\; s_{eqcc',0} = 0, \tag{3.10}$$

$$\log a_{eqcii'} = (1/\text{-}s_{eqcii',m\text{-}1})[-\mu(\log a_{eqcii'}) + \mu(\log a_{eqcii',1}) + \sum_{k=1}^{m_{eqcii'}-2}(s_{eqcii',m-1} - s_{eqcii',k})d_{eqcii',k} + \sum_{k=1}^{m_{eqcii'}-1}(s_{eqcii',k} - s_{eqcii',k-1})\log a_{eqcii',k}\;] \tag{3.11}$$

$$\log a_{eqcii'} + \sum_{k=1}^{m_{eqcii'}-2} d_{eqcii',k} \geq \log a_{eqcii',m\text{-}1},\; \log a_{eqcii'} \geq 0,\; s_{eqcii',0} = 0, \tag{3.12}$$

where μ_e denotes the e'th evaluator's average overall preference within all membership functions, $k = 1, 2, \ldots, m_{eqq'}-1$ or $m_{eqcc'}-1$ or $m_{eqcii'}-1$ represent the break points of each membership function as specified by each evaluator.

4. Treating the group consensus

The teacher recruiting committee in this case comprises six senior faculty, and each evaluator's preferred weights for five candidates can be obtained by entering each evaluator preferred judgment into Model 3 when ignoring Goal B.

However, an interesting problem created is how much disagreement space can be allowed within a group decision environment. The process of treating the group consensus may not be complicated, but a reasonable, sensible and practical process is required to convince the minority opinions and prevent conflicts among the reviewing committee or decision makers. Any selected alternative or course of action without considering the group consensus may operate in a hostile or unhelpful working environment that hampers its future performance. Consequently, a solution derived by considering the trade-off between a group of decision-makers' consensus and individual decision-maker's preference will be less vulnerable to conflicting judgments, and will also becomes easier to persuasive the entire committee to accept the selected alternative.

Suppose that a judgment group contains three decision-makers, who give comparison values between objects 1 and 2 of 3.7, 3 and 5, respectively. The average group assessment value for comparing objects 1 and 2 is then calculated as (3.7 + 3 + 5) / 3 = 3.9. Accordingly, the degree of agreement of the first decision-maker' opinion to the average rating of the group is 0.9487, calculated by 1 - | (3.7 – 3.9) / 3.9 |. Consequently, the consensus of the first decision-maker to the group average assessment value is 94.87%. Similarly, the consensuses of the second and third decision-makers to the average assessment value of the group are found to be 76.92% and 71.76%, respectively. Hence, the group consensus is 81.18%, computed by (94.87% + 76.92% + 71.76%) / 3. When all three evaluators give identical comparison values when comparing objects 1 and 2, then the group consensus becomes 100%.

Therefore, the following model is constructed for tackling the group consensus.

Model 4

Maximize $\quad G = 1 - GG$

Subject to:
$$GG = \frac{2}{Q(Q-1)}\left(\sum_{q=1}^{Q}\sum_{q'>q}^{Q}\left(\sum_{e=1}^{E}\left(\left|\frac{a_{eqq'}-\bar{a}_{qq'}}{\bar{a}_{qq'}}\right|\right)\right)/E\right)+$$

$$\frac{1}{Q}\left(\sum_{q=1}^{Q}\left(\frac{2}{c_q(c_q-1)}\left(\sum_{c=1}^{C_q}\sum_{c'>c}^{c_q}\left(\sum_{e=1}^{E}\left(\left|\frac{a_{eqcc'}-\bar{a}_{qcc'}}{\bar{a}_{qcc'}}\right|\right)\right)/E\right)\right)\right)+$$

$$\frac{1}{Q}\left(\sum_{q=1}^{Q}\left(\frac{2}{c_q(c_q-1)}\sum_{c=1}^{c_q}\left(\frac{2}{N(N-1)}\left(\sum_{i>1}^{N}\sum_{i'>i}^{N}\left(\sum_{e=1}^{E}\left(\left|\frac{a_{eqcii'}-\bar{a}_{qcii'}}{\bar{a}_{qcii'}}\right|\right)\right)/E\right)\right)\right)\right)$$

$$\bar{a}_{qq'} = \frac{1}{E}\left(\sum_{e=1}^{E} a_{eqq'}\right),\quad \bar{a}_{qcc'} = \frac{1}{E}\left(\sum_{e=1}^{E} a_{eqcc'}\right),\quad \bar{a}_{qcii'} = \frac{1}{E}\left(\sum_{e=1}^{E} a_{eqcii'}\right) \tag{4.1}$$

where G denotes the group consensus, q = 1, 2, …, Q is the g'th dimension (in this case study Q = 3), e = 1, 2, …, E represents the e'th evaluator (here E = 9), c = 1, 2, …, c_q expresses the c'th criterion under the q'th dimension (here c_1 = 3, c_2 =4, and c_3=3), i = 1, 2, … N stands for the i'th marketing strategy (here N = 5), $\bar{a}_{qq'}$ is the average assessment value of the group to which dimension q is preferred to dimension q', $\bar{a}_{qcc'}$ represents the average judgment value of the group to which criterion c is preferred to criterion c' under the q'th dimension, and $\bar{a}_{qcii'}$ denotes the average evaluation value of the group to which marketing strategy i is preferred to marketing strategy i' under the c'th criterion of the q'th dimension.

Taking group consensus into account and treating the fractional forms in Model 4 with the logarithmic and goal programming properties, a fuzzy group decision-making model can be formulated as follows:

Model 5

Minimize Goal 1 = $\sum_{e=1}^{E}\sum_{q=1}^{Q}\sum_{q'>q}^{Q}(\log \delta^{+}_{eqq'} + \log \delta^{-}_{eqq'}) +$

$$\sum_{e=1}^{E}\sum_{q=1}^{Q}\sum_{c=1}^{c_q}\sum_{c'>c}^{c_q}(\log \delta^{+}_{eqcc'} + \log \delta^{-}_{eqcc'}) + \sum_{e=1}^{E}\sum_{q=1}^{Q}\sum_{c=1}^{C_q}\sum_{i=1}^{N}\sum_{i'>i}^{N}(\log \delta^{+}_{eqcii'} + \log \delta^{-}_{eqcii'})$$

Maximize Goal 2 = $\frac{1}{E}\sum_{e=1}^{E}(\mu_e)$

Minimize Goal 3 = $1 - \{\frac{2}{Q(Q-1)}(\sum_{q=1}^{Q}\sum_{q'>q}^{Q}(\sum_{e=1}^{E}(\delta^{-}_{eqq'} + \delta^{+}_{eqq'}))/E) +$

$$\frac{1}{Q}(\sum_{q=1}^{Q}(\frac{2}{c_q(c_q-1)}(\sum_{c=1}^{C_q}\sum_{c'>c}^{c_q}(\sum_{e=1}^{E}(\delta^{-}_{eqcc'} + \delta^{+}_{eqcc'}))/E))) +$$

$$\frac{1}{Q}(\sum_{q=1}^{Q}(\frac{2}{c_q(c_q-1)}\sum_{c=1}^{c_q}(\frac{2}{N(N-1)}(\sum_{i>1}^{N}\sum_{i'>i}^{N}(\sum_{e=1}^{E}(\delta^{-}_{eqcii'} + \delta^{+}_{eqcii'}))/E)))) \}$$

Subject to: (3.1) – (3.12), (4.1),

$$\mu_e = \frac{2}{Q(Q-1)}\sum_{q=1}^{Q}\sum_{q'>q}^{Q}\mu(\log a_{eqq'}) + \frac{1}{Q}\sum_{q=1}^{Q}(\frac{2}{c_q(c_q-1)}\sum_{c=1}^{c_q}\sum_{c'>c}^{c_q}(\mu(\log a_{eqcc'}))) +$$

$$\frac{1}{Q}(\sum_{q=1}^{Q}(\frac{2}{c_q(c_q-1)}\sum_{c=1}^{c_q}(\frac{2}{N(N-1)}(\sum_{i=1}^{N}\sum_{i'>i}^{N}(\mu(\log a_{eqcii'})))))),$$

$$\log(a_{eqq'} - \bar{a}_{qq'}) - \log \bar{a}_{qq'} + \delta^{-}_{eqq'} - \delta^{+}_{eqq'} = 0,$$

$$\log(a_{eqcc'} - \bar{a}_{qcc'}) - \log \bar{a}_{qcc'} + \delta^{-}_{eqcc'} - \delta^{+}_{eqcc'} = 0,$$

$$\log(a_{eqcii'} - \bar{a}_{qcii'}) - \log \bar{a}_{qcii'} + \delta^{-}_{eqcii'} - \delta^{+}_{eqcii'} = 0,$$

where Goal 2 is the expected average overall individual preference degree and Goal 3 is the expected group consensus.

When multiple goals exist, Model 5 can be solved by a preemptive goal programming method or a trade-off weighting method. This investigation employs an interactive approach to treat Model 5. Initially, a threshold can be set for the group consensus, such as Goal 3 ≥ 75%, and also for Goal 2, such as $\frac{1}{E}\sum_{e=1}^{E}(\mu_e) \geq 0.4$. Then, using Excel [17] or LINGO [39] solve Model 5, the most consistency priority vector can be generated under the constraints Goal 3 ≥ 75% and $\frac{1}{E}\sum_{e=1}^{E}(\mu_e) \geq 0.4$. When the generated solution is infeasible, the group consensus threshold can decrease by 1% or 0.5% each time until a feasible solution is reached. When the generated solution is feasible, the group consensus threshold can increase by 1% or 0.5% each time until the computed solution becomes infeasible. The expected average overall preference degree can be adjusted in the same manner.

From the above discussion, an interactive solution flowchart is presented next page. In this case study, the teacher recruiting committee wishes to raise the group consensus as much as possible with Model 5 under the constraint Goal 2 ≥ 0.5, which means that the expected average overall individual preference degree must exceed 0.5. Following the proposed interactive solution flowchart, the derived highest group consensus is 83.5% with final weightings of 0.2025, 0.2375, 0.1926, 0.1638, and 0.2036 for prospective candidates A, B, C, D, and E, respectively. By taking USC as a case study, the selection of Candidate B reflects the majority preferences of a group of senior scholars as measured by pair-to-pair comparison of a collection of criteria under an uncertain assessment environment.

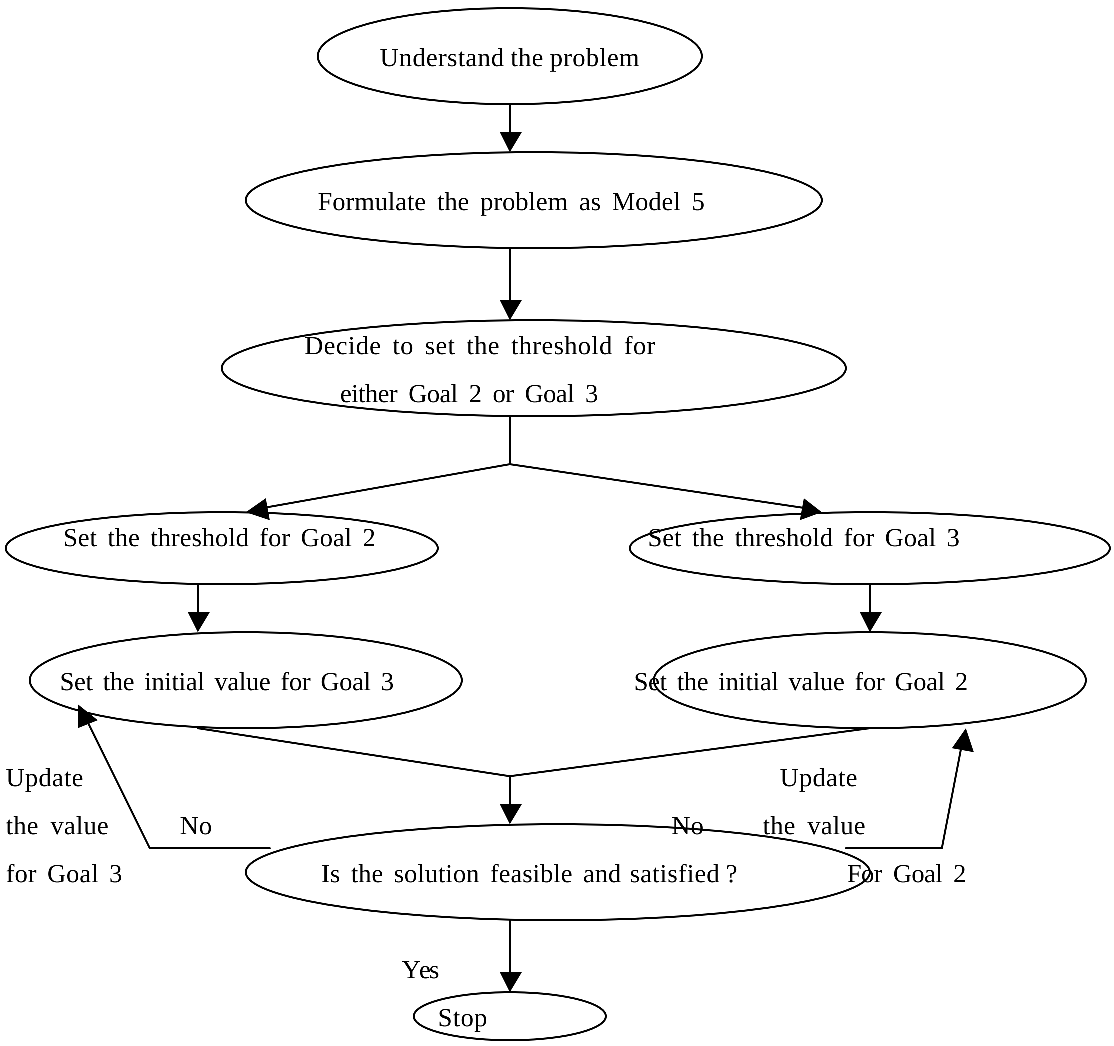

Fig. 5 An interactive solution flow chart

5. Concluding Remarks

Compared with prevailing methods of treating group consensus such as geometric means method, average means method, and weighted arithmetic mean method [3-4, 7, 20-21, 24, 26, 30, 32-33, 38, 43], this study proposes an interactive fuzzy group decision-making model to treat group consensus, individual preference and judgment consistency in the same decision framework. Regardless of whether in business, education, governmental or other organizational activities, a decision-making problem is frequently tackled via a group evaluation by committees or expert representatives, for example in planning priority setting, portfolio selection, personnel performance assessment, environmental hazards judgment, energy resource allocation and marketing strategy evaluation. Therefore, The interactive fuzzy group decision-making model presented herein can also be applied to other economic, social, political and management sciences problems.

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