# Stock Index Futures Real-time Long and Short Decision Support Systems 

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#### Abstract

This study presents a real-time long and short decision-making technique for stock index futures. The simple linear regression model and partial SPRT are applied to construct the real-time decision support systems (RTDSS) and the effectiveness of the proposed RTDSS is verified by applying it to four types of stock index futures, TX, TE, TF and MTX in Taiwan.


Key word: Stock index futures, TSE, TX, TE, TF, regression model, partial SPRT

## 1. Introduction

Taiwan Stock Exchange (TSE) Capitalization Weighted Stock Index Futures (TX), TSE Electronic Sector Stock index Futures (TE), TSE Banking and Insurance Sector Stock index Futures (TF), and stock index Mini-TSE Futures (MTX) provide investment instruments to domestic and international investors and make the Taiwan stock market more attractive to foreign capital by promoting internationalization, liquidity and volume. The current stock index futures market includes three types of trading strategies, speculation, hedge and arbitrage. Although various types of financial models can describe market mechanisms, most are based on hypothesis and suffer many limitations (Brock et al., 1992; Chou, 2000; Gencay and Stengos, 1998; Hsu, 1999; Wu and Lee; 2000). Speculation in stock index futures is popular strategy among many investors (Johnson, 1960). Most investors hope to obtain large profits from trading in stock index futures, but investors must consider numerous factors, including capital, political and economic variables and psychology. Owing to the difficulty faced by investors in making appropriate and timely decisions regarding trading in stock index futures, this study attempts to develop a decision support system to help investors in making real-time trading decisions. To provide real-time suggestions for investors, this work applies the slope of the simple linear regression model and partial sequential probability ratio test (SPRT) method to construct the real-time decision support system (RTDSS) for stock index long (buying) and short (selling). SPRT or partial SPRT have been widely applied in manufacturing engineering and medical engineering due to their ability to achieve patient safety, trial efficiency, and cost reduction (Chen, 1989; Chen and Wei, 1997, Chen and Wei, 1998; Lia and Hall, 1999), but applications in financial engineering remain novel. Consequently, this work aims to:
(1) construct a RTDSS according to the slope of the simple linear regression model and partial SPRT method to allow investors to obtain significant profits from the trading of stock index futures; and
(2) examine the effectiveness of applying RTDSS to trade four types index futures.

## 2. Partial Sequential Probability Ratio Test on the Slope of the Simple Linear Regression Model

The partial SPRT (Arghami and Billard, 1987) is used to test the slope, $\hat{b}_{1}$, of the simple linear regression model.
Let $\left(t_{i}, \Delta p_{i}\right), i=1,2, \ldots, n$, be a data pair representing the transaction number, $t_{i}$, and the difference between the stock index value at $t_{i}$ and the settlement price on the previous day. Assume the null hypothesis,

$$
\begin{equation*}
H_{0}: \theta=\theta_{0}=\hat{b}_{1}, \text { and } \tag{1}
\end{equation*}
$$

the alternative hypothesis,

$$
\begin{align*}
& H_{1}: \theta=\theta_{1}=\hat{b}_{1}+t o l,  \tag{2-1}\\
& \text { or } H_{1}: \theta=\theta_{1}=\hat{b}_{1}-t o l, \tag{2-2}
\end{align*}
$$

where $t o l$ denotes a given tolerance of slope, $\hat{b}_{1} ; \Delta p_{i}$ is independent and

$$
\begin{equation*}
\Delta p_{i} \approx N\left(\hat{b}_{0}+\hat{b}_{1} t_{i}, \sigma^{2}\right), i=1,2, \ldots, n \tag{3}
\end{equation*}
$$

and $\hat{b}_{1}=\frac{n \sum_{i=1}^{n} t_{i} \Delta p_{i}-\sum_{i=1}^{n} t_{i} \sum_{i=1}^{n} \Delta p_{i}}{n \sum_{i=1}^{n} t_{i}^{2}-\left(\sum_{i=1}^{n} t_{i}\right)^{2}}$ is the parameter of interest, where $\hat{b}_{0}$ represents the intercept of the simple linear regression model and $\sigma^{2}$ is the variance of $\Delta p_{i}, i=1,2, \ldots, n$. A transformation similar to that used in Arghami and Billard (1987), can eliminate the nuisance parameters $\hat{b}_{0}$ and $\sigma^{2}$. The partial SPRT based on the transformed variables can then be performed as follows.

To do the partial SPRT, take $n_{0}(\geq 3)$ pairs from the initial data pair $\left(t_{1}, \Delta p_{1}\right), \ldots,\left(t_{n_{0}}, \Delta p_{n_{0}}\right)$ and calculate the unbiased estimator of minimum variance, $S_{0}{ }^{2}$, of $\sigma^{2}$,

$$
\begin{equation*}
S_{0}^{2}=\frac{\left(1-r_{0}^{2}\right) \sum_{i=1}^{n_{0}}\left(\Delta p_{i}-\Delta \bar{p}\right)^{2}}{n_{0}-2} \tag{4}
\end{equation*}
$$

where $\quad r_{0}^{2}=\frac{S_{t \Delta p}}{S_{t t} S_{\Delta p \Delta p}}, \quad \Delta \bar{p}=\frac{\sum_{i=1}^{n_{0}} p_{i}}{n_{0}} \quad S_{t \Delta p}=\sum_{i=1}^{n_{0}}\left(t_{i}-\bar{t}\right)\left(\Delta p_{i}-\Delta \bar{p}\right), \quad S_{t t}=\sum_{i=1}^{n_{0}}\left(t_{i}-\overline{t_{0}}\right)^{2}, \quad S_{\Delta p \Delta p}=\sum_{i=1}^{n_{0}}\left(\Delta p_{i}-\Delta \bar{p}\right)^{2}$ and $\bar{t}_{0}=\frac{\sum_{i=1}^{n_{0}} t_{i}}{n_{0}}$.
$r_{0}$ denotes the correlation coefficient of $t$ and $\Delta p$ based on the $n_{0}$ initial data pair, $\Delta \bar{p}$ and $\bar{t}_{0}$. Take $n_{1}$ additional data pair, where $n_{1}$ is the smallest integer ( $n_{1} \geq 2$ ) such that

$$
\begin{equation*}
\sum_{i=1}^{n^{*}} w_{i}^{2}>\frac{S_{0}^{2}}{z_{1}} \tag{5}
\end{equation*}
$$

where $n^{*}=n_{0}+n_{1}$,

$$
w_{i}= \begin{cases}t_{i}-\bar{t}_{0}, & i=1,2, \ldots, n_{0} \\ t_{i}-\bar{t}_{1}, & i=n_{0}+1, n_{0}+2, \ldots, n^{*}\end{cases}
$$

$\bar{t}_{0}=\frac{\sum_{i=1}^{n_{0}} t_{i}}{n_{0}}, \bar{t}_{1}=\frac{\sum_{i=n_{0}+1}^{n^{*}} t_{i}}{n_{1}}$ and $z_{1}$ denotes a positive number and is independent of $\Delta p_{i}, i=1,2, \ldots, n^{*}$, which may depend on $t_{i}, i=1,2, \ldots, n^{*}$. However, a set of real numbers $g_{1}, \ldots, g_{n^{*}}$ can be found in a way in the following,

1. $g_{i}$ is proportional to $w_{i} / \sum_{i=1}^{n^{*}} w_{i}^{2}, i=1,2, \ldots, n^{*}$,
2. $\sum_{i=1}^{n^{*}} g_{i}=0$,
3. $\sum_{i=1}^{n^{*}} g_{i} t_{i}=1$, and
4. $\sum_{i=1}^{n^{*}} g_{i}^{2}=\frac{z_{1}}{S_{1}^{2}}$.
where $S_{1}^{2}=\frac{\left(1-r_{1}^{2}\right) \sum_{i=1}^{n^{*}}\left(\Delta p_{i}-\Delta \bar{p}\right)^{2}}{n^{*}-2}, r_{1}^{2}=\frac{S_{t \Delta p}}{S_{t t} S_{\Delta p \Delta p}}$.
Then, let

$$
\begin{equation*}
U_{1}=\sum_{i=1}^{n^{*}} \frac{g_{i} \Delta p_{i}}{\sqrt{z_{1}}} . \tag{6}
\end{equation*}
$$

Next, take $n_{2}$ pairs, where $n_{2}$ is the smallest integer ( $>2$ ) such that

$$
\begin{equation*}
\sum_{i=n^{*}+1}^{n^{*}+n_{2}} w_{i}^{2}>\frac{S_{1}^{2}}{z_{2}}, \tag{7}
\end{equation*}
$$

where $w_{i}=t_{i}-\bar{t}_{2}, i=n^{*}+1, \ldots, n^{*}+n_{2}$ and $z_{2}$ is a positive number and is independent of $\Delta p_{i}, i=1,2, \ldots$, which may depend on $t_{i}, i=1,2, \ldots$. Similarly, a set of real numbers $h_{1}, \ldots, h_{n_{2}}$ can be found in a wayin the following,

1. $\sum_{i=n^{*}+1}^{n^{*}+n_{2}} h_{i-n^{*}}=0$,
2. $\sum_{i=n^{*}+1}^{n^{*}+n_{2}} h_{i-n^{*}} \cdot t_{i}=1$, and
3. $\sum_{i=n^{n}+1}^{n^{*}+n_{2}} h_{i-n^{*}}^{2}=\frac{z_{2}}{S^{2}}$.

Then, let

$$
\begin{equation*}
U_{2}=\sum_{i=n^{*}+1}^{n^{*}+n_{2}} \frac{h_{i-n^{*}} \Delta p_{i}}{\sqrt{z_{2}}} \tag{8}
\end{equation*}
$$

Similarly, calculate $U_{j}, j=3,4, \ldots$, using positive numbers $z_{j}, j=3,4, \ldots$, which are independent of the $\Delta p_{i}$ but may depend on the $t_{i}$.

The joint density of $U_{1}, \ldots, U_{m}$ can then be shown to be $f\left(u_{1}, u_{2}, \ldots \ldots, u_{m} ; \delta_{1}, \delta_{2}, \ldots . ., \delta_{m}\right)$
$=\prod_{i=1}^{m} f\left(u_{j} ; \delta_{j}\right)=\left(n_{0}-2\right)^{\left(n_{0}-2\right) / 2} \Gamma\left[\left(n_{0}+m-2\right) / 2\right] \Pi^{-m / 2}\left[\Gamma\left(\frac{n_{0}-2}{2}\right)\right]^{-1}\left\{\left(n_{0}-2\right)+\sum_{j=1}^{m}\left(u_{j}-\delta_{j}\right)^{2}\right\}^{-\left(n_{0}+m-2\right) / 2},-\infty<U_{j}<\infty$,
$j=1,2, \ldots, m$, where $\delta_{j}=\hat{\beta}_{1} / \sqrt{z_{j}}$.
To perform a partial SPRT with two probability risks, $\alpha$ (Type I error) and $\beta$ (Type II error), the hypothesis testing in Eq. (1) is proceeded as follows:
(1) If $\lambda \leq\left(\frac{\beta}{1-\alpha}\right)^{2 /\left(n_{0}+m-2\right)}$, then accept $H_{0}$;
(2) If $\lambda \geq\left(\frac{1-\beta}{\alpha}\right)^{2 /\left(n_{0}+m-2\right)}$, then accept $H_{1}$; and
(3) If $\left(\frac{\beta}{1-\alpha}\right)^{2 /\left(n_{0}+m-2\right)}<\lambda<\left(\frac{1-\beta}{\alpha}\right)^{2 /\left(n_{0}+m-2\right)}$ make an additional observation;
where the test value,

$$
\begin{aligned}
& \lambda=\frac{n_{0}-2+\sum_{j=1}^{m}\left(U_{j}-\delta_{j 0}\right)^{2}}{n_{0}-2+\sum_{j=1}^{m}\left(U_{j}-\delta_{j 1}\right)^{2}}, \quad \text { and } \\
& \delta_{j 0}=\frac{\hat{b}_{1}}{\sqrt{z_{j}}}, \quad j=1,2, \cdots, m \text {, and } \\
& \delta_{j 1}=\frac{\hat{b}_{1} \pm t o l}{\sqrt{z_{j}}}, \quad j=1,2, \cdots, m .
\end{aligned}
$$

## 3. Trading Decision-Making Flow Chart of Partial SPRT for RTDSS

In the RTDSS, the slope of the simple linear regression model is sequentially detected in real-time until the stock index futures market crashes. Figure 1 illustrates the computer program flow chart of trading decision- making. When $\hat{b}_{1}$ is positive and the test value, $\lambda$, exceeds the boundary of $\left(\frac{1-\beta}{\alpha}\right)^{2 /\left(n_{0}+m-2\right)}$, the alternative hypothesis is now, $H_{1}: \theta=\hat{b}_{1}-$ tol (Eq. (2-2), accepted, then the latest data of $\Delta p$ make the slope declining. Thus, RTDSS sends out the message "Short". As the test value, $\lambda$ is less than $\left(\frac{\beta}{1-\alpha}\right)^{2 /\left(n_{0}+m-2\right)}$, the null hypothesis is now, $H_{0}: \theta=$ $\hat{b}_{1}$, accepted and the slope is not declined by $\Delta p$. Thus, the "Stop" messages of short will be sent out from the RTDSS.

Meanwhile, when $\hat{b}_{1}$ is negative and the test value, $\lambda$, exceeds the boundary of $\left(\frac{1-\beta}{\alpha}\right)^{2 /\left(n_{0}+m-2\right)}$, the alternative hypothesis is now, $H_{1}: \theta=\hat{b}_{1}+t o l$ (Eq. 2-1), accepted, then the newest data of $\Delta p$ make the slope increasing.

RTDSS then sends out the message "Long". As the test value, $\lambda$ is less than $\left(\frac{\beta}{1-\alpha}\right)^{2 /\left(n_{0}+m-2\right)}$ the null hypothesis is now, $H_{0}: \theta=\hat{b}_{1}$, accepted and the slope is not increased by $\Delta p$. Thus, the "Stop" messages of long will be sent out.


Fig. 1 Trading Decision-making Flow Chart

## 4. Case Implementation

This study applies the proposed RTDSS to the four types of index futures, namely, TX, TE, TF and MTX. Table 1 displays the origin margins and the minimum price fluctuations of the four types of stock index futures.

Table 1 Origin Margins of the Four Types of Stock Index Futures and Their Minimum Price Fluctuations

| Type of Stock Index Futures | Origin Margins | Minimum Price Fluctuation |
| :---: | :---: | :---: |
| TX | NT\$120,000 | NT\$ 200 per tick (one stock index point) |
| TE | NT\$165,000 | NT\$ 200 per tick (0.05 tock index point) |
| TF | NT\$90,000 | NT\$ 200 per tick ( 0.2 tock index point) |
| MTX | NT\$30,000 | NT\$ 50 per tick (one stock index point) |

### 4.1 Source Data and Parameter Settings

This study used historical data of TX, TE, TF and MTX between March 4, 2002 and March 15, 2002 (ten days) were used to this study. The number of collected data pairs of TX, TE, TF and MTX within this ten days are 15,618 , $11,609,5,843$, and 13,649 , separately.

The parameters used in this study are: (1) the number of initial data pair to construct the slope of the simple linear regression model, $n_{0}$, (2) the Type I error, $\alpha$, (3) the Type II error, $\beta$, and (4) the alternative hypothesis $\hat{b}_{1} \pm$ tol, where tol $=0.1 \hat{b}_{1}$. Table 2 illustrates the parameter settings for the RTDSS.

Table 2 Parameter Settings for RTDSS

| $\alpha$ | $\beta$ | Hypothesis of $H_{1}\left(\right.$ i.e, $\left.\hat{b}_{1} \pm t o l\right)$ | Initial Sample Pairs $n_{0}$ |
| :---: | :---: | :---: | :---: |
| 0.05 | 0.1 | $0.1 \hat{b}_{1}$ | 30 |

### 4.2 Implemental Results

Table 3 lists the implementation results, applying RTDSS, from testing the four types of stock index futures over ten transaction days and clearing trading every day without open interest (that is, the trading will be cleaning daily.) Table 3 displays to the gain/loss per transaction day as well as the gain/loss ratio. On Table 3, the higher values of mean of gain/loss and gain/loss ratio show that the RTDSS yields a very good trading performance. The data on Table 3 are plotted in Fig. 2. In Figure 2, TX and MTX displays higher dependence because of the same underlying of Taiwan Stock Exchange Capitalization Weighted Stock Index. In Table 3, TE illustrates that TE is the most suitable stock index futures by using the proposed RTDSS for speculation since it has the highest value of mean of gains among the four types of stock index futures.

## 4. Conclusion

This study presents the speculation of the four types stock index futures, TX, TE, TF and MTX. The proposed RTDSS performs well in real-time long (or buying) and short (or selling) trading decision-making. The values of mean of gains/ losses of the TX, TE, TF and MTX, are NT\$21,980, NT\$24,980, NT\$11,580 and NT\$6,395, respectively. Since gains were achieved in all cases, the proposed RTDSS appears to be very effective. However, space still exists to improving the total gain in the future, since the four types of stock index futures still display serious losses. The next consideration may be a hypothesis testing of the intercept, $\hat{b}_{0}$, of a simple linear regression model by partial SPRT to correct loss causing trades.

Table 3 Implementation Results of the Mean of Gain/loss and Gain/loss Ratio
per Transaction Day

| $\qquad$ | TX | TE | TF | MTX |
| :---: | :---: | :---: | :---: | :---: |
| 1 | - NT\$ 60,800 | NT\$ 33,800 | NT\$ 1,800 | - NT\$ 15,250 |
| 2 | 15,400 | 5,200 | 21,000 | 1,950 |
| 3 | 51,600 | 6400 | -21,800 | 9,900 |
| 4 | -39,200 | -4,800 | 1,400 | 50 |
| 5 | 29,600 | 60,800 | 11,600 | 12,900 |
| 6 | 35,400 | -37,600 | 8,800 | 14,750 |
| 7 | -47,200 | 12,600 | 28,800 | -4,900 |
| 8 | 55,800 | 54,400 | 22,200 | 6,500 |
| 9 | 50,800 | 14,600 | -8,000 | 21,650 |
| 10 | 128,400 | 104,400 | 50,000 | 16,400 |
| Mean of Gain/loss | 21,980 | 24,980 | 11,580 | 6,395 |
|  |  |  |  |  |



Fig. 2 Implementation Results of Gain/loss over Ten Transaction Days

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