Reliability Analysis of Supply Chain Networks

William J. Cosgrove[1)]

[1)] California Polytechnic University-Pomona, Professor of Technology and Operations Management (wcosgrove@csupomona.edu)

Abstract

This paper proposes the use of reliability theory as a performance measure for supply chains, and includes the consideration of backup elements (e.g., backup vendors, subcontractors, and transportation links) to improve the supply chain's reliability. Performance is defined in terms of the success or failure in meeting predetermined levels for the on-time delivery of an acceptable quantity of product, which is applied to outcomes for (1) the entire supply chain, from the beginning of the chain to the ultimate customer or market that it serves, and for (2) each element in the chain, including suppliers, vendors, plants, distribution centers, and transportation links.

The supply chain is represented as a network model, where nodes represent vendors, plants, distribution centers, and markets, and arcs represent transportation links. Expressions for the reliabilities of the entire supply chain and its elements (i.e., arcs and nodes) are derived employing reliability theory concepts for series and parallel systems, where the latter considers additional "backup" elements to be added to the original supply chain, thus improving the overall reliability of the supply chain. In a similar manner, consideration is given to adding parallel subnetworks to the original supply chain. Elements of the supply chain network are assumed to be independent, and the chain is treated as a series system based on the assumption that a failure in the predetermined performance of any element in the chain results in a failure of the entire chain.

1. Introduction

The typical "supply chain" for tangible goods can be physically represented as a network model [4] employing arcs and nodes to correspond to supply chain components and links (e.g., nodes to represent vendors/suppliers, plants, and distribution centers; and arcs to represent transportation links). In this manner, existing techniques based on reliability networks are suitable for (1) determining the reliability of the entire supply chain, and (2) determining the reliabilities of the various key components that make up the supply chain (i.e., transportation links, vendors, distribution centers, etc.). Note that the importance of the latter (item 2) follows in that weak elements in key supply chain components could be identified and studied to determine the extent that they contribute to the overall reliability of the entire supply chain.

The purpose of this study is to model supply chains as reliability networks, employing methodologies from reliability theory [3] which apply to mechanical systems. An application of reliability theory which is of particular interest in supply chains is lead time, which is defined as the time it takes for a tangible good to be transferred to the final customer after the order by the final customer is released [1]. Since several previous studies have discussed issues related to lead times and delays involving order processing, manufacturing, and transportation (e.g., [1][2][5][6]), it follows that the reliability of the entire supply chain can be measured as the probability that the agreed number of tangible goods of acceptable quality are delivered to the final customer within the agreed time frame (i.e., not too late and not too early, where the latter could be viewed by the final customer as a shift in the inventory burden from the vendor to them). The reliability of the entire supply chain refers to the "high-level" aggregated network model, which depends on the reliabilities of its components as determined from the reliabilities of the more detailed network models of the components. Reliability measures can provide a very comprehensive measure of the overall performance of supply chains, and very specific measures of the reliabilities of supply chain components. A detailed analysis of reliabilities from supply chain components could lead to clues as to which components would be critical in improving or hampering overall performance, perhaps leading to alternative designs of the supply chain.

This study will derive mathematical expressions for several configurations of reliability systems based on network models. This includes simple series and parallel systems, and systems consisting of configurations with reliability subnetworks. Consideration will be given to supply chain components that represent the availability of independent vendors/suppliers or transportation links in a re-design of the supply chain network (e.g., such as the addition of these independent parties as parallel network elements which would increase the reliability of the supply chain).

2. Methodology

Assume that a supply chain network (Figure 1) can be described by the pair $\mathbf{A}\cup\mathbf{N}$, where $\mathbf{A}$ and $\mathbf{N}$ represent non-empty sets of arcs and nodes, respectively, with the activities at all arcs and nodes being independent. The following nomenclature is employed to describe the network as a supply chain model:

1. The network consists of I nodes and J arcs where i=1,2,…I and j=1,2,…J
2. Set element n_i represents the *i*th node in $\mathbf{N}$ (i.e., $n_i\in\mathbf{N}$)
3. Set element a_j represents the *j*th arc in $\mathbf{A}$ linking nodes n_i and $n_{i,i+1}$ (i.e., $a_j\in\mathbf{A}$)
4. Associated with each n_i and a_j are reliability probabilities $r(n_i)$ and $r(a_j)$
5. Associated with any n_i or a_j are K parallel network elements $n_{k,i}\in\mathbf{N_k}$ and $a_{k,j}\in\mathbf{A_k}$ with reliability probabilities $r(n_{k,i})$ and $r(a_{k,j})$, where k=1,2,…K and $\mathbf{N_k}\cap\mathbf{N}\cup\mathbf{A}=$**Æ**, and $\mathbf{A_k}\cap\mathbf{N}\cup\mathbf{A}=$**Æ**
6. The set $\mathbf{N_s}\cup\mathbf{A_s}$ is the sth subset (or subnetwork) in $\mathbf{N}\cup\mathbf{A}$ (i.e., $\mathbf{N_s}\cup\mathbf{A_s}\subset\mathbf{N}\cup\mathbf{A}$; $n_{s,j},a_{s,j}\in\mathbf{N_s}\cup\mathbf{A_s}$; s=1,2,…S)
7. Associated with each set $\mathbf{N_s}\cup\mathbf{A_s}$ is the *s*th set of parallel network elements described by $\boldsymbol{N}_\mathbf{s}\cup\boldsymbol{A}_\mathbf{s}$ ($n_{s,j},a_{s,j}\in\boldsymbol{N}_\mathbf{s}\cup\boldsymbol{A}_\mathbf{s}$) with reliability probabilities $r(\boldsymbol{N}_\mathbf{s}\cup\boldsymbol{A}_\mathbf{s})$, where $\boldsymbol{N}_\mathbf{s}\cup\boldsymbol{A}_\mathbf{s}\cap\mathbf{N}\cup\mathbf{A}=$**Æ**

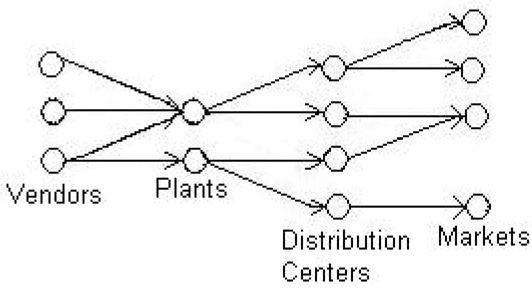

Fig. 1 Supply Chain Network

Since independence is assumed, and *all* elements in the supply chain must meet their separate performance expectations, then the elements in the chain can be treated as a series reliability system. This follows because a failure of one or more elements in the chain results in the chain failing to meet its performance expectations for all the markets it serves (Figure 1). Hence, for a series system, the reliability of the chain is given by [3]

$$R(\mathbf{N}\cup\mathbf{A}) = \prod_{i,j\,|\,n_i,a_j\in\mathbf{N}\cup\mathbf{A}} [r(n_i) * r(a_j)] \ . \tag{1}$$

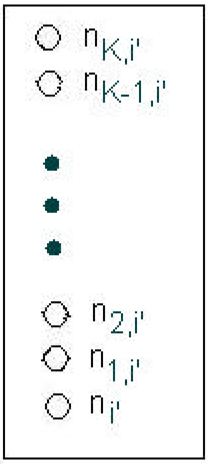

Fig. 2 Parallel Nodes

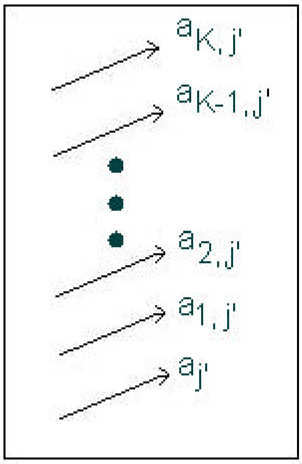

Fig. 3 Parallel Arcs

To improve the reliability of the chain, assume that the i'th node includes one or more backup systems (e.g., backup vendors or subcontractors who are available to expedite orders on short notice, covering for a potential failure of the activity represented by the i'th node). The backup nodes can be viewed as part of a parallel reliability system (Figure 2) which improves the performance expectations of the task represented by the i'th node. From reliability theory, the improved reliability at the i'th node for a parallel system is given by [3]

$$R[r(n_{i'}),\mathbf{N_k}] = 1 - r(n_{i'}) * \prod_{k \mid n_{k,i'} \in \mathbf{N_k}} [1 - r(n_{k,j'})] \ . \tag{2}$$

The reliability of the chain can be expressed by

$$R(\mathbf{N} \cup \mathbf{A} \cup \mathbf{N_k}) = R[r(n_{i'}),\mathbf{N_k}] * \prod_{i,j \mid n_i, a_j \in \mathbf{N} \cup \mathbf{A},\, i \neq i'} [r(n_i) * r(a_j)]$$

$$= \left\{ 1 - r(n_{i'}) * \prod_{k \mid n_{k,i'} \in \mathbf{N_k}} [1 - r(n_{k,j'})] \right\} \cdot \left\{ \prod_{i,j \mid n_i, a_j \in \mathbf{N} \cup \mathbf{A},\, i \neq i'} [r(n_i) * r(a_j)] \right\} , \tag{3}$$

where $R[r(n_{i'}),\mathbf{N_k}] > r(n_{i'})$ and $R(\mathbf{N} \cup \mathbf{A} \cup \mathbf{N_k}) > R(\mathbf{N} \cup \mathbf{A})$.

The derivation of reliabilities for arcs is similar to that for nodes, leading to expressions for arcs that parallel expressions (2) and (3) for nodes. Hence, it follows that the improved reliability from backup systems at the j'th arc (Figure 3) is given by

$$R[r(a_{j'}),\mathbf{A_k}] = 1 - r(a_{j'}) * \prod_{k \mid a_{k,j} \in \mathbf{A_k}} [1 - r(a_{k,j'})] \ . \tag{4}$$

The reliability of the chain becomes

$$R(\mathbf{N} \cup \mathbf{A} \cup \mathbf{A_k}) = R[r(a_{j'}),\mathbf{A_k}] * \prod_{i,j \mid n_i, a_j \in \mathbf{N} \cup \mathbf{A},\, j \neq j'} [r(n_i) * r(a_j)]$$

$$= \left\{ 1 - r(a_{j'}) * \prod_{k \mid a_{k,j} \in \mathbf{A_k}} [1 - r(a_{k,j'})] \right\} \cdot \left\{ \prod_{i,j \mid n_i, a_j \in \mathbf{N} \cup \mathbf{A},\, j \neq j'} [r(n_i) * r(a_j)] \right\} , \tag{5}$$

where $R[r(a_{j'}),\mathbf{A_k}] > r(a_{j'})$ and $R(\mathbf{N} \cup \mathbf{A} \cup \mathbf{A_k}) > R(\mathbf{N} \cup \mathbf{A})$.

Note that expressions (3) and (5) can be employed to derive more complex expressions for reliability calculations for numerous combinations of backup nodes and arcs.

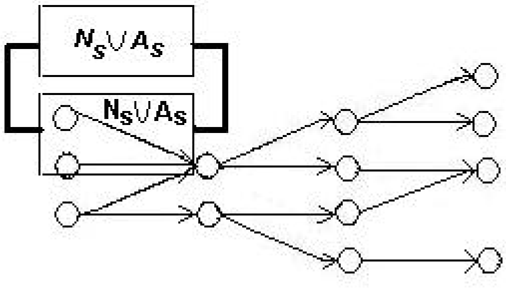

Fig. 4 Parallel Subnetwork

Another consideration is the use of external networks to backup several activities of a subset on a supply chain, such as a subnetwork in the original chain (Figure 4). For the sth subnetwork in the original chain, the *s*th external subnetwork is added in parallel to the sth network to improve reliability. The reliability of the chain with the external subnetwork added is given by

$$R(\boldsymbol{N_s} \cup \boldsymbol{A_s} \cap \mathbf{N} \cup \mathbf{A}) = \{ 1 - [1 - R(\mathbf{N_s} \cup \mathbf{A_s})] \cdot [1 - R(\boldsymbol{N_s} \cup \boldsymbol{A_s})] \} \times \{ \prod_{i,j \,|\, n_i, a_j \in (\mathbf{N} \cup \mathbf{A} - \mathbf{N_s} \cup \mathbf{A_s})} [r(n_i) \times r(a_j)] \}, \tag{6}$$

where $R(\mathbf{N_s} \cup \mathbf{A_s})$ and $R(\boldsymbol{N_s} \cup \boldsymbol{A_s})$ are the reliabilities of the original subnetwork and the external subnetwork, respectively. Expression (6) can be employed repeatedly for other values of s and *s*.

3. Conclusion

While the methodology developed in this study provides a means for finding reliabilities of supply chains and some of their components, the assumption of independence and the simplicity of the configurations presented in the paper may not always be applicable in real world problems. However, relaxing such assumptions would extend the complexity of analytical procedures to a level that would make it impractical to develop them. Such problems would be more suitable for network simulation, which could be extended not only to address dependencies among elements of the supply chain, but also to accommodate additional categories of performance that would go beyond the “two” categories considered in this study (i.e., success or failure in meeting performance levels for on-time delivery of an acceptable quantity of product).

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