# SIMULATION STUDY OF A TWO-LEVEL WAREHOUSE INVENTORY REPLENISHMENT SYSTEM

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**Abstract**

This paper presents a simulation study of a multi-product, two-echelon inventory replenishment system. The simulation allows the comparison of a one-warehouse N-retailer replenishment scenario to a two-warehouse, N-retailer system with cost per unit of distribution and delivery lead times as performance measures. We consider Periodic review (s,S) inventory policies by varying the reorder point and order up to levels. A benchmark one-warehouse (plant) N-retailer system is used to develop the simulation model, and the demand distribution parameters are estimated from historical data. It is found that the distribution costs do not vary significantly between the one-warehouse and the two-warehouse replenishment system; however, implementation of a two-level warehouse system significantly lowers delivery lead times. Under conditions of low reorder levels, and provided the company can control freight costs, a higher service level can be achieved under a two-level warehouse replenishment system with no additional cost.

**Keywords:** Multi-Echelon Inventory System, Warehouse Replenishment, Simulation, One-Warehouse N-Retailer Distribution System.

## Introduction

Wholesale inventory systems often exhibit erratic demand histories whose underlying process is difficult to characterize. Wholesale facilities frequently experience consecutive periods of low demand followed by periods of large and varied demand. Ehrhardt [5] explained this behavior as due to wholesaler's demands originating from other facilities (retailers) which employ (s,S) type replenishment policies. This paper presents a wholesale warehouse system which has a two-echelon inventory system. The system consists of a number of lower-echelon facilities (retail stores) filling customer demand and themselves acting as customers to a single upper-echelon facility (manufacturing plant). The use of a regional warehouse between the retail stores and the manufacturing plant is to improve service levels. This paper presents a simulation model for a two-level warehouse distribution system supporting multiple retail outlets. In particular, the system service and cost levels are examined as a function of the presence of a warehouse, warehouse stock levels, and the demand level. System performance is measured by distribution costs and customer delivery times.

According to Schwarz [12], inventory policy decisions addressed by researchers include multi-echelon manufacturing systems or a multi-location distribution system. He suggests that the optimal policy for any one stocking point in a multi-echelon inventory system depends to some extent upon the cost constraints and policies imposed upon it by the other stocking points in the system.

Muckstadt [10] examined multi-echelon inventory systems and found multi-echelon systems that maintain low inventories achieve similar performance results to a single-level system, which suggests that multiple level systems may be worthwhile in numerous situations.

Deuermeyer and Schwarz [4] used a model to approximate N-identical-retailer systems to determine where system inventory should be held. In their study, the warehouse and retailers follow (Q, r) inventory replenishment policies. Schwarz examined the fill-rate maximizing position of a fixed quantity of safety stock between the warehouse and retailers, and found that the best policies involve very small warehouse on-hand inventories. Badinelli and Schwarz [2] found similar results when minimizing backorders subject to a constraint on average system-wide inventory.

Multi-level distribution systems are found in practice and frequently modeled in the management science literature, however, little is known about the service-level performance of such systems. Schwarz, et al. [13] examined the system fill rate of a one-warehouse, N-identical retailer distribution system as a function of warehouse and retailer safety stock. They investigated the intersection of a fill-rate policy line and the safety stock budget line which suggested some near optimal heuristic policies.

Bergmann [3] stated that the customer service aspects of inventories are not easy to handle using traditional methods of analysis, causing many model builders to ignore them. Bergmann used computer simulation to look at the interactions of a firm with the customers. The simulation looked at rules firms apply to inventory management that avoid the loss of customers due to stock outs while incurring low costs.

Schwarz [14] examined N-identical retailers supplying normally distributed demand, using a periodic review demand-replenishment system to examine the value of warehouse risk pooling in high service-level systems. This investigation looked at the alternative of the retailers acting independently to order the product from plant or using an intermediate warehouse to distribute product. Schwarz concluded that the pipeline inventory costs significantly influence the overall value of using the warehouse.

McGavin, et al. [9] investigated the inventory allocation for a one-warehouse, N-identical retailer distribution system facing stochastic demand for a single product. The policies are intended to minimize lost sales per retailer between system replenishment. McGavin et al. used simulation to test an allocation heuristic.

Lu and Posner [8] proposed two heuristic procedures for a one-warehouse, multi-retailer system. They stated that procedures to find optimal policies are very complex, and that most researchers have concentrated on developing good heuristics for special policies.

Greis's [6] research in assessing service level targets in production and inventory planning suggests that establishing service level targets consistent with the firm's strategic orientation must be done in consideration with both the characteristics of the demand process and the capacities of the production and inventory system. Greis provides a tool using service reliability curves for estimating the premium above unit costs that must be paid to provide a designated service level.

In this paper we shall develop a cost model for a two-level, multi-product, one-plant, one-warehouse, and N-retailer inventory replenishment system. The unit cost, along with delivery lead time as a measure for service levels can be used to compare the performance of the plant- retailer replenishment system with or without a second-level warehouse. A simulation model is developed, using a benchmark one-warehouse N-retailer inventory replenishment system, to compare the two systems, and historical data from this system is used to estimate the demand distribution parameters.

## Two-Level Warehouse Inventory Model

The model considered in this paper is similar to Ehrhardt, Shultz, and Wagner [5] with an added regional warehouse proposed, which provides improved service levels. The system consists of a two level wholesale warehouse system represented in figure 1. Independent demand, $Q_r$, from retailers, who need to replenish their stocks, are filled by a single warehouse following a ($Q_w$, $r_w$) policy of replenishment. $Q_w$ and $r_w$ are integer multiples of $Q_r$. The warehouse receives its supply after a lead time, $L_w$, from an unlimited-supply manufacturing plant. The retailers receive their orders from the warehouse after a lead time, $L_r$, provided the warehouse has sufficient on-hand inventory. If the warehouse inventory is not sufficient, the entire retailer order is instead filled from the manufacturing plant with a lead time, $L_p$. Backorders are not allowed. Thus, the lead time retailers experience is either $L_r$ or $L_p$ depending upon inventory levels at the warehouse. Lead times $L_w$, $L_r$, and $L_p$ are stochastic, rather than fixed, and assumed to be poisson. Consequently, the service level performance of the system depends upon stock levels at the warehouse.

Items kept in inventory are assumed to be conserved, there being no losses due to deterioration, obsolescence, or pilferage. Inventory on hand at the end of a given period is the inventory from the previous period plus any replenishment that arrives, less demand in the given period. Inventory on hand cannot be negative, since backorders are not allowed.

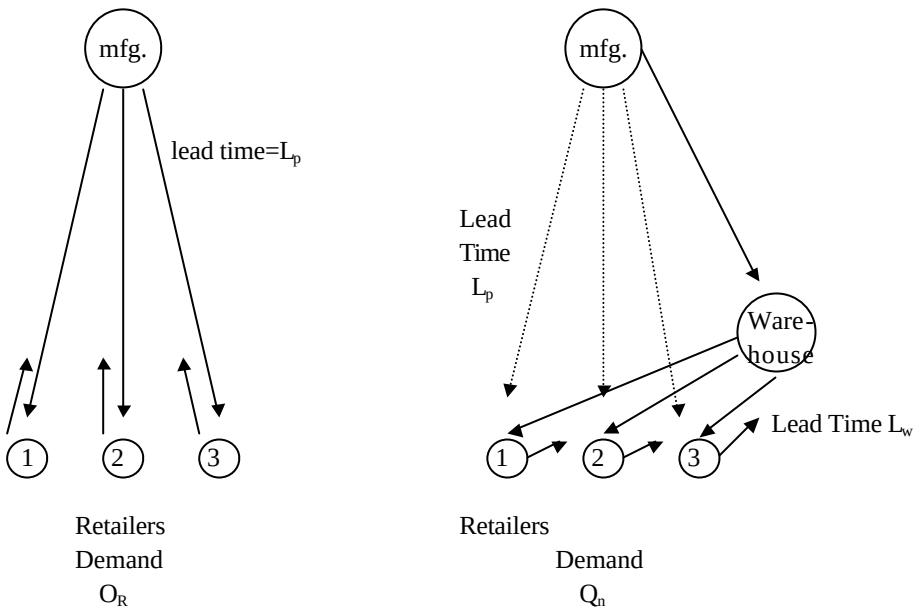


**Figure 1 Single-Level vs. Two-Level Warehouse Inventory Replenishment System**

## Retailer Demand

Demand by the retailers is determined by: (1) when an order is placed; and (2) what is ordered. The time-between-orders demand distribution is based on historical records of retail. The items-ordered demand distribution is also based on the same period. For any order, all or none of the items might be ordered, depending on the item demand distribution. Historical demand from retailers is better represented by actual conditions rather than a theoretical distribution. This distribution includes demand for the items under consideration, anywhere between 1 and n.

Two stocking levels for the three product A, B, and C, are considered. The lower stocking level triggers an order quantity of 20, 25, and 5 for A, B, and C, respectively, with a variable reorder point x. At the higher stocking level, the order quantities are the same, except that the reorder point is set at 5, 5 and 1 for products A, B, and C, respectively. A periodic review interval of 5 days is set to study the performance of the system under condition of with or without a regional warehouse.

## Design Of Experiment

The criterion for evaluating the performance is expected cost per square foot for the various scenarios. The factors of the experiments are: (1) whether a regional warehouse exists or not (two levels); (2) order quantities, Q and order point, r, for the warehouse (two levels); and (3) historical demand and a ± 10% change in demand (three levels). These three factors are varied in the simulation, for a total of 12 combinations. Three combinations were eliminated due to not being meaningful (changes in Q, r when warehouse is not present). Table 1 lists the factors and levels.

**Table 1: Factors and Levels**

| **Factor** | **Levels** | | |
|---|---|---|---|
| Regional Warehouse | no | yes | |
| (Q,r) Policy | Q,rA = 20,x<br>Q,rB = 25,x<br>Q,rC = 5,x | Q,rA = 20,5<br>Q,rB = 25,5<br>Q,rC = 5,1 | |
| Demand | 90% | 100% | 110% |

**Table 2: Scenarios**

| **Trial** | **Warehouse** | **(Q,r)** | **Demand** |
|---|---|---|---|
| Scenario-1 | No | N/A | 100 |

| | | | |
|---|---|---|---|
| Scenario-2 | No | N/A | 90 |
| Scenario-3 | Yes | Q,r | 100 |
| Scenario-4 | Yes | Q,r | 90 |
| Scenario-5 | Yes | Q,x | 100 |
| Scenario-6 | Yes | Q,x | 90 |
| Scenario-7 | No | N/A | 110 |
| Scenario-8 | Yes | Q,r | 110 |
| Scenario-9 | Yes | Q,x | 110 |

The costs for scenarios 1, 2, and 7 are different from the rest since the warehouse does not exist and all shipments to retailers are made directly from the plant. The scenarios considered are presented in table 2.

### Simulation

The complexity of the model having demand that is a convolution of two historical distributions, makes the computation of a truly optimal policy difficult. Simulation provides the ability to conduct experiments under the constraints of the historical distributions and the operations of an actual system. Schwarz et al. [13] used simulation to examine a similar system with a single stock item to maximize system fill-rate subject to a constraint in system safety stock.

Others have seen the need for simulation to analyze inventory problems:

> *The gap between theory and practice (in inventory problems) does not seem to have been appreciably diminished. This is due to the fact that advanced models have often been developed in a theoretical vacuum, far from real-world situations in which their decision rules have to operate. Simulation allows the possibility of analyzing which decision rules work best - also in situations where analytical results cannot be deduced. Simulation makes it possible to construct models with a greater degree of similarity between the model and reality. At the same time, the use of simulation models gives the user a deeper insight into relations which have a considerable impact on the mode of operation and inventory control systems* (Alstrom and Madsen 1992).

SLAM II [11] was used for the simulation analysis. Since the system under study is non-terminating, steady state results were collected for data analysis. In absence of a statistical procedure to do this, Law and Kelton [7] suggest the rule of thumb that initial observations be thrown away as long as they seem to increase or decrease steadily. Theoretically, before steady state is reached, the mean first difference, between successive daily averages of inventory, costs, etc., should be positive values, and that mean first differences should converge to zero at the steady state. This can be statistically verified by performing a “t” test. Analysis of the simulation output showed that steady state condition is reached after 150 time periods. Sixteen runs of the 150 time periods were performed for each trial. Results from the first run were not used in the analysis, which resulted in 15 runs of 150 time periods for each scenario. Each scenario run contained from 80 to 120 orders, depending on the random number stream to generate demand.. Average square foot costs and delivery times from each run were therefore averages of 80+ observations. The central limit theorem can be used to justify that these averages represent a normal population.

## Data Analysis And Results

Table 5 provides a summary of the costs. SPSSPC+ Version 5.0 was used for the data analysis. The output data is summarized and presented in table 3 and summary data is used in the statistical analysis.

The EXAMINE procedure was used to examine the data prior to analysis. All points identified as extremes or outliers were verified as correct data points. The Levene statistic, a test for homogeneity of variance, indicated there was a significant difference between the variances of cost and changes in demand and of cost as grouped by the presence of the warehouse and order levels. These results suggest that the assumption of equal variances among populations is unrealistic.

Non-parametric tests make minimal assumptions about the underlying distribution. The NPAR Tests in SPSSPC+ were used to test for a difference among the group means. The MANN-WHITNEY indicated a significant difference

among the two stock levels; lower stock level, higher stock level, and no warehouse. The same test noted no significant difference when comparing the lower stock level and no warehouse. The Kruskal-Wallis (K-W) test indicated a significant difference between the stock levels and no warehouse. The K-W also indicated a significant difference between the cost levels by whether a warehouse was present or not. See Tables 5 and 6 for the results summary. The summary results from table 6 .

**Table 3: Summary of Simulation Output**

| **Trial** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | |
|---|---|---|---|---|---|---|---|---|---|---|
| Run 2 | 0.0150 | 0.0156 | 0.0188 | 0.0196 | 0.0152 | 0.0142 | 0.0161 | 0.0181 | 0.0155 | |
| 3 | 0.0153 | 0.0151 | 0.0159 | 0.0210 | 0.0154 | 0.0140 | 0.0151 | 0.0186 | 0.0148 | |
| 4 | 0.0154 | 0.0151 | 0.0188 | 0.0208 | 0.0153 | 0.0153 | 0.0153 | 0.0163 | 0.0154 | |
| 5 | 0.0157 | 0.0151 | 0.0190 | 0.0149 | 0.0163 | 0.0165 | 0.0161 | 0.0190 | 0.0165 | |
| 6 | 0.0152 | 0.0159 | 0.0190 | 0.0164 | 0.0159 | 0.0143 | 0.0164 | 0.0174 | 0.0184 | |
| 7 | 0.0153 | 0.0150 | 0.0160 | 0.0190 | 0.0173 | 0.0171 | 0.0165 | 0.0175 | 0.0162 | |
| 8 | 0.0154 | 0.0163 | 0.0205 | 0.0162 | 0.0152 | 0.0153 | 0.0153 | 0.0172 | 0.0149 | |
| 9 | 0.0154 | 0.0155 | 0.0155 | 0.0205 | 0.0158 | 0.0166 | 0.0153 | 0.0177 | 0.0155 | |
| 10 | 0.0154 | 0.0160 | 0.0187 | 0.0163 | 0.0155 | 0.0150 | 0.0150 | 0.0152 | 0.0150 | |
| 11 | 0.0161 | 0.0153 | 0.0172 | 0.0271 | 0.0155 | 0.0155 | 0.0150 | 0.0199 | 0.0147 | |
| 12 | 0.0150 | 0.0161 | 0.0162 | 0.0177 | 0.0158 | 0.0147 | 0.0152 | 0.0151 | 0.0163 | |
| 13 | 0.0153 | 0.0153 | 0.0200 | 0.0246 | 0.0146 | 0.0157 | 0.0181 | 0.0171 | 0.0179 | |
| 14 | 0.0154 | 0.0151 | 0.0182 | 0.0190 | 0.0153 | 0.0147 | 0.0158 | 0.0160 | 0.0159 | |
| 15 | 0.0153 | 0.0155 | 0.0173 | 0.0221 | 0.0147 | 0.0159 | 0.0152 | 0.0178 | 0.0151 | |
| 16 | 0.0152 | 0.0156 | 0.0193 | 0.0163 | 0.0152 | 0.0147 | 0.0156 | 0.0178 | 0.0155 | |
| $n_i$ | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 | $\sum n=$ 135 |
| SUM y | 0.2306 | 0.2325 | 0.2704 | 0.2915 | 0.2330 | 0.2295 | 0.2395 | 0.2607 | 0.2376 | c $\sum y=2.221687$ 1 |
| Sum of Squares | 0 | | | | | | | | | |
| Run 2 | 0.00023 | 0.00024 | 0.00036 | 0.00038 | 0.00023 | 0.00020 | 0.00026 | 0.00033 | 0.00024 | |
| 3 | 0.00023 | 0.00023 | 0.00025 | 0.00044 | 0.00024 | 0.00020 | 0.00023 | 0.00035 | 0.00022 | |
| 4 | 0.00024 | 0.00023 | 0.00035 | 0.00043 | 0.00023 | 0.00023 | 0.00023 | 0.00027 | 0.00024 | |
| 5 | 0.00025 | 0.00023 | 0.00036 | 0.00022 | 0.00027 | 0.00027 | 0.00026 | 0.00036 | 0.00027 | |
| 6 | 0.00023 | 0.00025 | 0.00036 | 0.00027 | 0.00025 | 0.00021 | 0.00027 | 0.00030 | 0.00034 | |
| 7 | 0.00023 | 0.00023 | 0.00026 | 0.00036 | 0.00030 | 0.00029 | 0.00027 | 0.00031 | 0.00026 | |
| 8 | 0.00024 | 0.00027 | 0.00042 | 0.00026 | 0.00023 | 0.00023 | 0.00023 | 0.00030 | 0.00022 | |
| 9 | 0.00024 | 0.00024 | 0.00024 | 0.00042 | 0.00025 | 0.00028 | 0.00023 | 0.00031 | 0.00024 | |
| 10 | 0.00024 | 0.00026 | 0.00035 | 0.00027 | 0.00024 | 0.00022 | 0.00023 | 0.00023 | 0.00022 | |
| 11 | 0.00026 | 0.00023 | 0.00030 | 0.00074 | 0.00024 | 0.00024 | 0.00023 | 0.00039 | 0.00022 | |
| 12 | 0.00023 | 0.00026 | 0.00026 | 0.00031 | 0.00025 | 0.00022 | 0.00023 | 0.00023 | 0.00027 | |
| 13 | 0.00023 | 0.00024 | 0.00040 | 0.00060 | 0.00021 | 0.00025 | 0.00033 | 0.00029 | 0.00032 | |
| 14 | 0.00024 | 0.00023 | 0.00033 | 0.00036 | 0.00023 | 0.00022 | 0.00025 | 0.00025 | 0.00025 | |
| 15 | 0.00024 | 0.00024 | 0.00030 | 0.00049 | 0.00021 | 0.00025 | 0.00023 | 0.00032 | 0.00023 | |
| 16 | 0.00023 | 0.00024 | 0.00037 | 0.00027 | 0.00023 | 0.00022 | 0.00024 | 0.00032 | 0.00024 | |
| Sum of Squares | 0.00355 | 0.00361 | 0.00491 | 0.00582 | 0.00362 | 0.00352 | 0.00372 | 0.00456 | 0.00378 | 9 $\sum y^2=0.03701$ |
| | ysq/n | Ysq/n | | | | | | | | |
| F-num | 0.03682 | 0.01646 | 0.03477 | | | | | | | |
| F-den | 0.03709 | 0.03682 | 0.00267 | | | | | | | |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| F-STAT | | | 130.052 | | | | | | | |

$$\sum_{1}^{9} \frac{Y_j^2/nj - Y^2/n}{9-1} = .039765$$

$$\sum_{1}^{c} \sum_{1}^{15} Y_j^2 - \sum(Y^2/n) = .000267$$

F = 130.519

$F_{95, 9, 100} = 1096 > F_{95, 9, 120} < F = 130.5$

| Table 4: Scenario Cost Results | | |
|---|---|---|
| Scenario | Average SF Cost | Delivery Time |
| 1 | .0154 | 5.01 |
| 2 | .0155 | 5.02 |
| 3 | .0180 | 2.01 |
| 4 | .0194 | 2.05 |
| 5 | .0155 | 2.19 |
| 6 | .0153 | 2.20 |
| 7 | .0157 | 5.04 |
| 8 | .0174 | 2.03 |
| 9 | .0158 | 2.20 |

| Table 5: Statistical Tests for Significance | | |
|---|---|---|
| ONE-WAY ANOVA | F-Test | p |
| Cost by Stock Level | 45.7 | <.001 |
| Cost by Demand Level | .8 | .43 |
| Cost by Warehouse | 15.6 | <.001 |
| Levene Test for Homogeneity of Variance | Statistic | p |
| Cost by Stock Level | 23.5 | <.001 |
| Cost by Warehouse | 35.1 | <.001 |
| Duncan Procedure | | p |
| High Stock Level versus Low Stock Level and No Warehouse | | <.001 |
| MANN-WHITNEY | | p |
| Cost by Lower Stock Level vs. | | |
| Higher Stock Level and | | |
| No Warehouse | | <.001 |
| Cost by Lower Stock Level and | | |
| No Warehouse | | <.82 |
| Kruskal-Wallis | | p |
| Cost by Warehouse vs. No Warehouse | | <.001 |
| Cost by Lower Stock Level vs. | | |
| Higher Stock Level | | |
| No Warehouse | | <.001 |

| Table 6: Summary of Trials | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Trial | | | | Unit Cost Avg | Cost SD | Cost Range | Deliver Time Avg | Time SD | Time Range |
| Scenario | Conditions | | | | | | | | |
| | $O_R$ | Demand | WH | | | | | | |
| 1 | — | 100 | NO | 0.0154 | 0.0003 | 0.0011 | 5.006 | 0.239 | 1.02 |
| 2 | — | 90 | NO | 0.0155 | 0.0004 | 0.0013 | 5.023 | 0.255 | 0.82 |
| 3 | H | 100 | YES | 0.0180 | 0.0016 | 0.0049 | 2.010 | 0.116 | 0.45 |
| 4 | H | 90 | YES | 0.0194 | 0.0034 | 0.0122 | 2.052 | 0.095 | 0.33 |
| 5 | L | 100 | YES | 0.0155 | 0.0007 | 0.0027 | 2.189 | 0.174 | 0.74 |
| 6 | L | 90 | YES | 0.0153 | 0.0009 | 0.0031 | 2.195 | 0.173 | 0.64 |
| 7 | — | 110 | NO | 0.0157 | 0.0008 | 0.0031 | 5.037 | 0.163 | 0.56 |
| 8 | H | 110 | YES | 0.0174 | 0.0013 | 0.0047 | 2.031 | 0.169 | 0.62 |
| 9 | L | 110 | YES | 0.0158 | 0.0037 | 0.0037 | 2.202 | 0.132 | 0.49 |

## Conclusions

Two implicit assumptions occur in this analysis. First, the warehouse is geographically closer to retail location than is the plant. In this analysis this assumption is supported by reality by the cost structure supplied by the company. Second, it is the assumption that the proportion of time that the warehouse is out of stock is small, resulting in shorter lead times from the warehouse. This is offset by the assumption that if the warehouse is even one unit shy of the order, the entire order is filled from the manufacturing plant.

From the results of the simulation analysis, we conclude the following regarding the two-level warehouse inventory system versus no second-level warehouse:

1. Under conditions of low stocking levels, the average unit cost of operating a second-level warehouse is not significantly different from operating without a second-level warehouse. This does not hold for high stocking levels, which can be attributed to the higher warehouse storage costs associated with the higher stocking levels.
2. As one would expect, implementation of the two-level warehouse system significantly reduces delivery lead times. Provided the order levels are not high, the higher service level can be achieved without appreciable increase in cost.
3. At high order levels, the extra cost of holding inventory offsets the improved service level from the two-level warehouse. Therefore, the replenishment of stock levels at the warehouse should be implemented only when warehouse inventory levels cannot meet immediate orders.
4. The lack of significant difference in performance with demand variation indicates that the decision is not affected by a ten percent change in nominal demand. This invariance to demand provides additional support to the two-level warehouse system at low reorder levels.
5. In any warehouse model examining the addition of warehouses outside the plant, there is an inherent trade-off between freight and warehousing costs. The model examined in this paper addresses a specific case of one company. While the freight costs and warehousing costs will vary across companies, the costs represented here may be used as a gage for evaluating systems with costs structures in the vicinity of those for the company represented in this paper. Additionally, the model executed here is amenable to substitution of other firms' cost structures.

Further research should consider the effects of variable shipping costs between the plant, warehouse, and the retailer. Future research should also consider multiple second-level warehouses. Additionally, exploration of the relationship between distribution costs, replenishment heuristics, and customer service is desirable.

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