Integration of the DCF, WACC, and NPV with the EMH: The Role of No-Arbitrage Condition

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Abstract

The purpose of this paper is to integrate the discounted cash flow method (DCF), the weighted average cost of capital (WACC), and the net present value (NPV) rule of capital budgeting, with the efficient markets hypothesis (EMH). The paper accomplishes this by noting that in efficient markets the no-arbitrage condition is satisfied. The paper, therefore, starts with a state preference model and shows when "states" are replaced with "time periods" then the state preference model is replaced with the DCF model. It then uses the state preference model to define the no-arbitrage condition and shows again when "states" are replaced with "time periods" then every exchange would constitute a zero NPV transaction. It then emphasizes that in finding the component costs of capital and the WACC, zero NPV calculations are performed. That is, the WACC is based on the EMH. Finally, the paper discusses how the use of the WACC in a positive NPV project might be reconciled by recourse to the micro-economic partial equilibrium analysis. At the end, the paper also reflects on the coverage of current introductory finance textbooks with respect to the issues discussed.

1. Introduction

The purpose of this paper is to integrate the discounted cash flow method (DCF), the weighted average cost of capital (WACC), and the net present value (NPV) rule of capital budgeting, with the efficient markets hypothesis (EMH). The paper accomplishes this by noting that in efficient markets the no-arbitrage (NA) condition is satisfied. The paper, therefore, starts with a state preference model and shows when "states" are replaced with "time periods" then the state preference model is replaced with the DCF model. It then uses the state preference model to define the NA condition and shows again when "states" are replaced with "time periods" then every exchange would constitute a zero NPV transaction. As is well known, the component costs of capital are calculated and used to find the WACC. To calculate the component costs of capital, the market value of the capital component is set equal to the present value of the expected cash flows to be exchanged with respect to that capital component and, therefore, all the component costs of capital calculations constitute zero NPV calculations. Therefore, the WACC in based on the EMH. Finally, this WACC is applied to the NPV rule of capital budgeting to decide with respect to the investment in real assets. A positive NPV is inconsistent with the EMH, as noted above, and, therefore, the application of the WACC within the positive NPV framework may seem inconsistent. However, this might be reconciled in the micro-economic partial equilibrium framework. That is, where all markets are efficient except for the real asset under consideration.

Over the years, introductory finance textbooks have improved their exposition of a variety of topics and are currently in good standing. This paper also reflects on the current textbook integration of the DCF, WACC, and NPV with the EMH.

Section 2 defines the state preference model and shows its relationship with the DCF model. Section 3 defines the NA condition and shows its relationship with zero NPV transactions. Section 4 examines the relationship between the EMH and the WACC. Section 5 discusses the relationship between the EMH and the NPV rule of capital budgeting. Section 6 reflects on the current textbook treatment of the issues discussed. Section 7 concludes the discussion.

2. The State Preference Model and the DCF

Consider a single-period, state-contingent-claims model with M risky securities and S states. Let A be the (MxS) security payoff matrix, where element a_{ms} is the payoff of security m in state s, $A_s^{'} = (a_{1s},..., a_{Ms})$ is the M-vector of claims paid on the M securities if state s occurs, $x^l = (x_1^l, \dots, x_M^l)$ is the investor l's portfolio order, and $p = (p_1,..., p_M)$ is the vector of security prices.

The NA condition is equivalent to:

there exists a vector $d>0$ *such that* $Ad = p$

That is, any element in d acts as a one-period discount rate and gives the present value of $1 in a particular state, s, which is in one period from now. They are, in fact, the Arrow-Debreu securities.

In the above state preference model, if "states" are replaced by "time periods" then the state preference model is replaced by the DCF model. That is, any element in d acts as a one- or multi-period discount rate and gives the present value of $1 which is in some specific future period.

3. The NA condition and the NPV

A significant literature now exists that examines the role of the NA condition. This section reviews the standard definition of the NA condition. It notes that it is defined in efficient markets. It also emphasizes the role of the NA condition for the existence of equilibrium.

Consider the state preference model defined in Section 1. The NA condition may be defined by the satisfaction of:

$$\max_{x^{l}} \left\{ -x^{l'}(p) \quad s.t. \quad x^{l'}A \quad 0 \right\} = 0 \quad and$$

$$x^{l'}A = 0 \quad for \ every \ optimal \ solution \qquad (1)$$

The economic interpretation of (1) is straightforward and intuitive: every portfolio that generates a positive cash flow in one future state, and a non-negative cash flow in every other state, must have a positive price; and every portfolio that generates a zero cash flow in every future state must have a zero price.

Condition (1) is equivalent to the above definition. It requires that:

every portfolio x^{l} *satisfying* $x^{l'}p \quad 0$ *and* $x^{l'}A \quad 0$, *must also satisfy* $x^{l'}p = 0$ *and* $x^{l'}A = 0$

Or, the absence of a trade x for which:

$$x^{l'}p \quad 0, \ x^{l'}A_s \quad 0, \ for \ all \ s, \ and \ at \ least \ one \ inequality \ is \ strict$$

Alternatively, the optimal value of the following maximization problem is zero:

$$\max_{x}\{-x^{l'}p\} \qquad subject \ to \qquad x^{l'}A \quad 0 \qquad (2)$$

The NA condition, as defined in (2), is equivalent to:

$$there \ exists \ a \ vector \qquad d > 0 \qquad such \ that \qquad Ad = p \qquad (3)$$

The duality theory of linear programming provides a method of proving the existence of the desired price vector p and importantly, allows the properties of p to be revealed more explicitly. Employing Farkas' Lemma, condition (1) holds if and only if (iff):

$$there \ exists \ a \ vector \qquad d > 0 \qquad such \ that \qquad Ad = p$$

Thus, every vector which is a strictly positive linear combination of the columns of A, satisfies (1). Note that, since $a_{ms} \quad 0 \ \forall m, s,$ and no row of A is zero, for every vector $d>0$, each element of the vector p is strictly positive (i.e., $p>0$) if $p = Ad$. Condition (1) is therefore informative regarding the equilibrium price vector that may prevail in a financial market.

The NA condition is a necessary condition for the existence of equilibrium. This is because as long as there are arbitrage opportunities, equilibrium cannot exist. Under certain conditions, the NA condition is both necessary and sufficient for the existence of equilibrium.

The NA condition may be expressed in terms of present values:

$$there\ is\ a \quad d > 0 \quad such\ that \quad \max_x \{-x^{l'} p + x^{l'} Ad\} = 0 \qquad (4)$$

Any element in d in (4) can be regarded as the present value of a $1 in state s. The sum of terms in braces in (4) shows the net present value of portfolio x^l. Where, the net present value is composed of the algebraic sum of the cost of x^l, and the discounted value of the payoffs to x^l. According to (4) the net present value is zero. It should be noted that this net present value is defined in a one-period multi-state state preference framework.

In the above state preference model, if "states" are replaced by "time periods" then the zero net present value in the state preference framework is replaced by the zero NPV in the multi-period framework. In this framework, there are S time periods and the vector d translates into the vector of S discount factors for which the NA condition is satisfied, i.e., condition (4) is met, and the NPV is zero. Therefore, in efficient markets, where the NA condition is satisfied, all exchanges are zero NPV transactions.

4. The EMH and the WACC

Capital is customarily divided into capital components: debt, common equity, and preferred equity. The component costs of capital are calculated and used to find the WACC. The calculation of the component costs of capital is based on the EMH and its consequent zero NPV transactions. That is, in order to calculate the component costs of capital, the market value of the capital component is set equal to the sum of the present value of the expected cash flows to be exchanged with respect to that capital component. Therefore, all the component costs of capital calculations constitute zero NPV calculations. In turn, the calculation of the WACC in based on the EMH.

At this juncture, it is worthwhile to note that most often the after-tax cost of debt is calculated as $k_d(1\text{-}t)$, where k_d is the before-tax cost of debt and t is the tax rate. This is typically explained by the tax-deductibility of the interest expense. This might be true for perpetual bonds. However, in practice, most bonds have finite maturities. For these bonds the cost of debt should be found by calculating the sum of the present value of the expected after-tax cash outflows, which are to be paid to the bondholders, and setting this sum equal to the market value of the bond. In other words, the cost of debt should be obtained, as in the case of any of the other capital component referred to in the previous paragraph, by setting the NPV of the bond transaction equal to zero. This is based on the EMH and is consistent with the NA condition, as defined in (4) above.

The cost of common equity can also be calculated by the application of the capital asset pricing model (CAPM). This model is also based on the EMH and is consistent with the NA condition.

To summarize, calculations of the component costs of capital and the WACC are all based on the EMH. The WACC is then used in the application of the NPV rule of capital budgeting.

5. The EMH and the NPV Rule of Capital Budgeting

Among different capital budgeting decision rules, the state of the art in finance favors the NPV rule. It is favored because the NPV rule has desirable properties. One of its properties is that the total value of the firm shares changes by the amount of the NPV. Therefore, only projects with positive NPV should be undertaken. In other words, the NPV rule is more compatible, than other rules, with the share value maximization as the goal of the firm.

However, note should be taken of the relationship between a positive NPV project and the EMH. Essentially, the two concepts do not combine well. In efficient markets there are no arbitrage opportunities and that there are no positive NPV projects. Conversely, the existence of a positive NPV project means that there are arbitrage opportunities in the market, that markets are inefficient, that the law of one price does not hold, and that equilibrium does not prevail.

More specifically, in deciding on a capital budgeting problem, the application of the NPV rule of capital budgeting uses the WACC. As noted in Section 4, the use of the market implied interest rate as the opportunity cost of the component costs of capital and the WACC is based on the existence of the EMH in all the capital components markets. However, a positive NPV capital budgeting project is based on the lack of the existence of the EMH. Therefore, the use of the WACC in calculating the positive NPV of a capital budgeting project might seem incompatible. This might be resolved by the standard partial equilibrium logic of micro-economic analysis. Applied to the problem in hand, it states that, in fact, all other markets, including those of capital components, are in the state of equilibrium except the market for the real asset which is the subject of the capital budgeting analysis.

6. Textbook Treatment of the Issues

Over the years, introductory finance textbooks have improved their exposition of a variety of topics and are currently in good standing. This section reflects on the current textbook integration of the DCF, WACC, and NPV with the EMH.

This study examines twenty-four current introductory finance textbooks published by major finance textbook publishers, i.e., those who were present at the latest annual Financial Management Association meeting. All textbooks, but three, discuss the efficient markets hypothesis. All twenty-four textbooks discuss both the NPV rule of the capital budgeting and the WACC. However, none of them discusses their underlying assumptions and relationships.

To further improve current introductory textbooks, the paper recommends a fuller integration of different topics discussed in the same textbooks. Of course, textbooks should improve their exposition not by the use of sophisticated techniques such as the NA condition, but by simpler and more intuitive approaches. Following the recommendation would result in both a better understanding of the topics and a more comprehensive grasp of the subject matter by students.

7. Conclusion

The purpose of this paper was to integrate the DCF, the WACC, and the NPV rule of capital budgeting, with the EMH. The paper accomplished this by noting that in efficient markets the NA condition is satisfied. The paper, therefore, started with a state preference model and showed when "states" are replaced with "time periods" then the state preference model is replaced with the DCF model. It then used the state preference model to define the NA condition and showed again when "states" are replaced with "time periods" then every exchange would constitute a zero NPV transaction. It then emphasized that in finding the component costs of capital and the WACC, zero NPV calculations are performed. That is, the WACC is based on the EMH. Finally, the paper discussed how the use of the WACC in a positive NPV project might be reconciled by recourse to the micro-economic partial equilibrium analysis. At the end, the paper also reflected on the coverage of current introductory finance textbooks with respect to the issues discussed.