Mixed Acceptance Sampling Plans for Multiple Products Indexed by Cost of Inspection

Paitoon Howying[1], Prapaisri Sudasna – Na - Ayudthya[2]

[1] Dhurakijpundit University, Faculty of Engineering (howying@yahoo.com)
[2] Kasetsart University, Faculty of Engineering (fengpsa@nontri.ku.ac.th)

Abstract

Acceptance sampling plan is a tool that used for controlling the quality of materials that put into production processes or deliver to the consumer. The selection of sampling plan depends on a characteristic of the data. There are two types of lot for lot acceptance sampling plans: the former is the attribute sampling plan that is used for sampling the products with accept or reject characteristic of quality, the latter is variable sampling plan that is used for measuring variable data such as weight, length or volume. The others acceptance sampling plan, in generally, Dodge and Romig's sampling plan which is used for protecting the consumer's risk, the ANSI/ASQC Z1.4 and the ANSI/ASQC Z1.9 which are used for protecting the producer's risk. They do not concern with sampling cost which occur in practice such as a unit cost, salary of workers, materials or tools for inspection and the budget for inspection are limited. This article proposes the method to design the acceptance sampling plans for multiple products that are restricted by the budget. They can be designed by applied the dynamic programming with objective of the plan is to minimize the maximum consumer's risk.

Introduction

Normally, acceptance sampling plan can be categorized into two groups; attribute sampling plan and variable sampling plan. For attribute sampling plan, it can be divided into single sampling plan which decision can be made by taking one sample. Double sampling plan that the decision can be made by taking at most two samples. The another attribute plan which complicate to use is multiple sampling plan that the decision usually can be made by taking at most seven samples. For variable sampling plan, it can be divided into two groups, the former is the plan that used for controlling the process mean which are single and double specification limit, the latter is the plan that used for controlling the percent defective. These sampling plans can be designed by following.

1. Design single attribute sampling plans by using the operating characteristic curve (OC- Curve) passes through the two points $(1 - \alpha,\ AQL)$ and $(\beta,\ LTPD)$ as shown in figure 1.

where as

α = The probability of rejecting a lot that meets the stated quality level (The producer's risk)

β = The probability of accepting an undesirable lot (The consumer's risk)

AQL = The average quality level

LTPD = The lot tolerance percent defective

X = The number of defective units that are found in inspection which has a binomial distribution

Ac = The acceptance number

Re = The rejection number

n = The sample size.

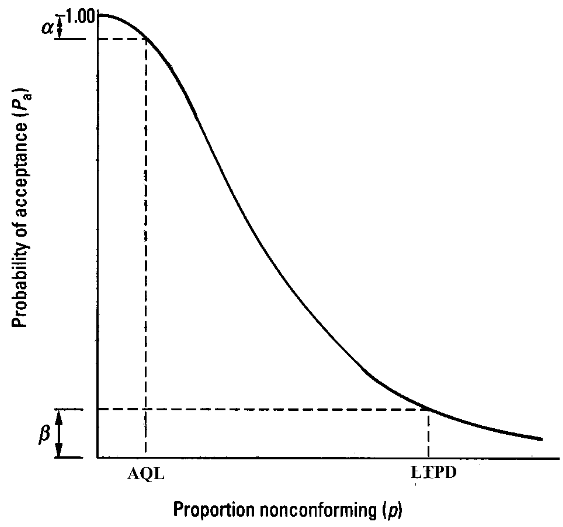


Fig. 1 The operating characteristic curve passes through the two points $(1 - \alpha,\ AQL)$ and $(\beta,\ LTPD)$.

It can be written as equation 1 and 2.

$$\Pr(X \le Ac) = \Pr(X \le Ac \mid p = AQL) \quad \ge \quad 1 - \alpha \qquad (1)$$

$$\Pr(X \le Ac) = \Pr(X \le Ac \mid p = LTPD) \quad \le \quad \beta \qquad (2)$$

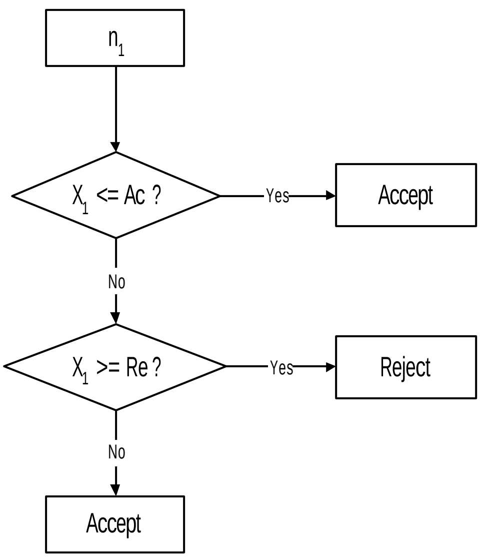


Fig. 2 General procedure for single sampling plans.

2. Design double attribute sampling plans by using the operating characteristic curve (OC - Curve) passes through the two points $(1 - \alpha,\ AQL)$ and $(\beta,\ LTPD)$ can be calculated sampling plans by solving equation 3 and 4.

$$\Pr(X_1 \le Ac_1) + \sum_{i=Ac_1}^{Ac_2} \Pr(X_1 = i) * \Pr(X_2 \le Ac_2 - i) \ge 1 - \alpha \qquad \text{where } p = AQL. \qquad (3)$$

$$\Pr(X_1 \le Ac_1) + \sum_{i=Ac_1}^{Ac_2} \Pr(X_1 = i) * \Pr(X_2 \le Ac_2 - i) \le \beta \qquad \text{where } p = LTPD. \qquad (4)$$

Where as X_1 = The number of defective in first sample
X_2 = The number of defective in second sample
Ac_1 = The acceptance number in first sample
Re_1 = The rejection number in first sample
Ac_2 = The acceptance number in second sample
Re_2 = The rejection number in second sample
n_1 = The sample size in first sample
n_2 = The sample size in second sample ($n_2 = n_1$).

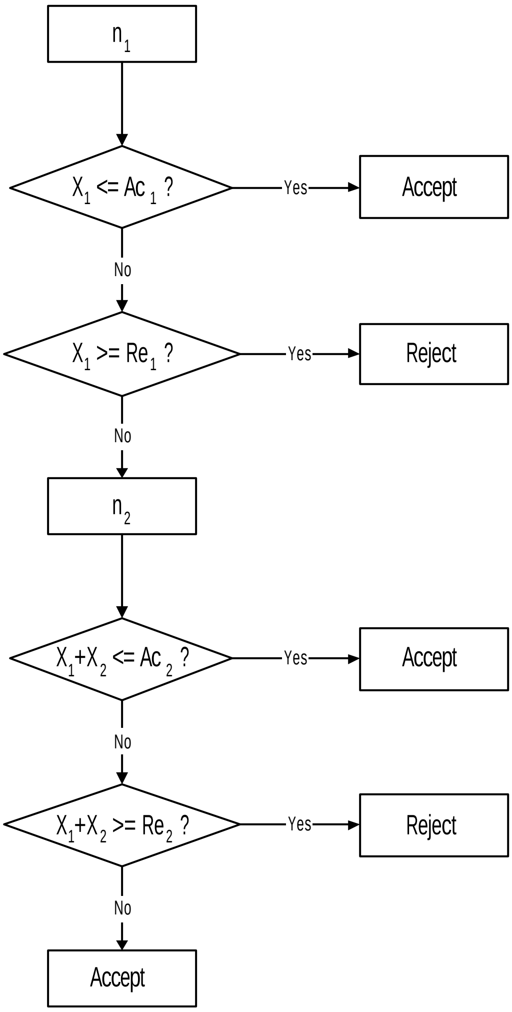


Fig.3 General procedure for Double sampling plans.

3. Design variable acceptance sampling plans to control a process mean.

3.1 Single (one sided) specification limit. Design variable acceptance sampling plans when single specification limit is used and the operating characteristic curve passes through the two points ($1-\alpha$, $\bar{X}_1$) , (β, $\bar{X}_2$) where as

α = The probability of rejecting a lot that meets the specified quality level

β = The probability of rejecting a lot that does not meet the specified quality level

$\bar{X}_1$ = The average value of the quality characteristics for that the probability of acceptance is high

$\bar{X}_2$ = The average value of the quality characteristics for that the probability of acceptance is low

$\bar{X}_a$ = The acceptance limit.

When variance is known and process mean has normal distribution. We can be calculated sampling plans by solving equation 5 and 6 simultaneously.

$$Z_\alpha = \frac{\bar{X}_a - \bar{X}_1}{\sigma/\sqrt{n}} \quad (5)$$

and $$Z_\beta = \frac{\bar{X}_a - \bar{X}_2}{\sigma/\sqrt{n}} \tag{6}$$

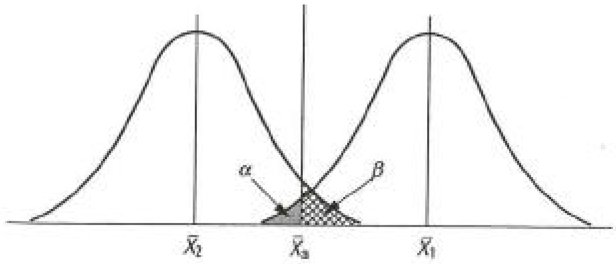


Fig. 4 Relationship of $\bar{X}_1$ and $\bar{X}_2$ to $\bar{X}_a$ in the sampling distribution of $\bar{X}$.

3.2 Double (two-sided) specification limit. For design of variable sampling plans when double specification limits are obtained and OC Curve passes two points ($1-\alpha$, $\bar{X}_1$) , ($\beta, \bar{X}_{2L}$) and ($\beta, \bar{X}_{2U}$). The sample size (n) , the upper acceptance limit $(\bar{X}_U)$ and the lower acceptance limit $(\bar{X}_L)$ are calculated by solving equation (7), (8), (9) and (10) . Where as $\bar{X}_1 = \frac{\bar{X}_{2L} + \bar{X}_{2U}}{2}$.

$$Z_{\alpha/2} = \frac{\bar{X}_{Ua} - \bar{X}_1}{\sigma/\sqrt{n}} \tag{7}$$

$$-Z_{\alpha/2} = \frac{\bar{X}_{La} - \bar{X}_1}{\sigma/\sqrt{n}} \tag{8}$$

$$Z_\beta = \frac{\bar{X}_{La} - \bar{X}_{2L}}{\sigma/\sqrt{n}} \tag{9}$$

$$-Z_\beta = \frac{\bar{X}_{Ua} - \bar{X}_{2U}}{\sigma/\sqrt{n}} \tag{10}$$

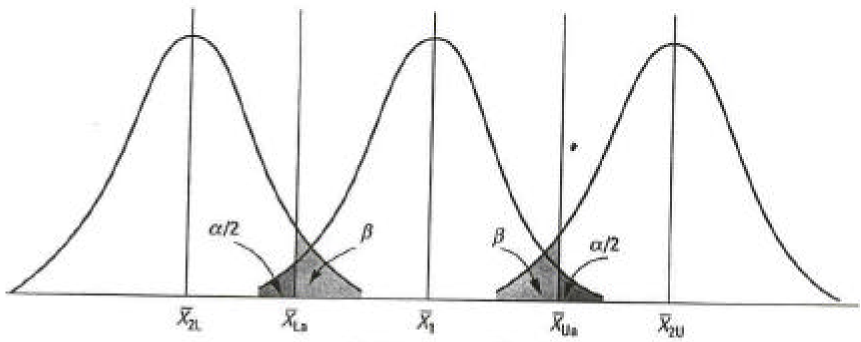


Fig. 5 Relationship between the various parameters for a plan using double specification limits.

4. Design variable acceptance sampling plans to control the percent defective.

4.1 Variance known. Design variable sampling plans by using the operating characteristic curve (OC Curve) passes through the two points (1 - α , AQL) and (β , LTPD). Where as

- α = The probability of rejecting a lot that meets the stated quality level.
- β = The probability of accepting an undesirable lot.
- AQL = The average quality level.
- LTPD = The lot tolerance percent defective.
- k = The critical value.
- L = The single lower limit.

When variance is known and random variable has normal distribution can calculate sampling plan from solving equation 11 and 12. If variance is unknown, we can calculate sampling plans from equation (13) and (14).

$$n = \left(\frac{Z_\alpha + Z_\beta}{Z_1 - Z_2}\right)^2 \qquad (11)$$

$$k_1 = Z_1 - \frac{Z_\alpha}{\sqrt{n}} \quad , \quad k_2 = Z_2 - \frac{Z_\beta}{\sqrt{n}} \qquad \text{where} \quad k = \frac{k_1 + k_2}{2} \qquad (12)$$

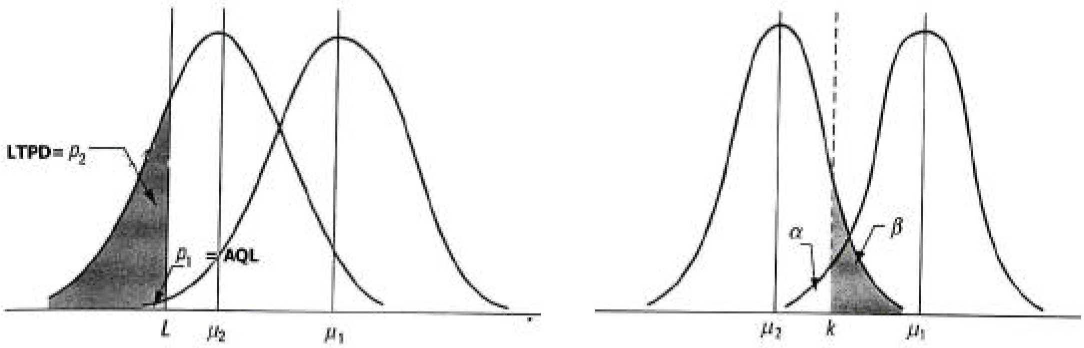
Accept lot if $Z_L \quad k$, otherwise reject it.

4.2 Variance unknown.

$$n = \left(1 + \frac{k^2}{2}\right)\left(\frac{Z_\alpha + Z_\beta}{Z_1 - Z_2}\right)^2 \qquad (13)$$

$$k = \frac{Z_\alpha Z_2 + Z_\beta Z_1}{Z_\alpha + Z_\beta} \qquad (14)$$

Accept lot if $Z_L \quad k$, otherwise reject it.



a. Distribution of quality characteristic, X

b. Distribution of the sample average, $\bar{X}$

Fig. 6

Method

When we use attribute or variable sampling plans for multiple products with the objective is minimize the maximum consumer's risk and subject to budget cost constraint. It can be written in form of the mathematical model as follow.

Objective Function : Minimize T

Subject to : $T \quad \beta_i$

$$\sum_{i=1}^{m} n_i c_i \quad B \qquad \text{and} \quad \forall\, n_i \geq 1 \; ; \quad i = 1, 2, \ldots, m.$$

where as

β_i = The consumer's risk that occurs with product i.

c_i = The cost of sampling for product i (Baht / unit).

n_i = The sample size for sampling product i (unit).

B = The budget which can be available for sampling m products (Baht).

m = The number of products which is inspected.

j = $\begin{cases} 1 & \text{When used single sampling plans or variable sampling plans} \\ 2 & \text{When used double attribute sampling plans } (n_2 = n_1). \end{cases}$

The algorithm for designs the multiple products sampling plans is shown below.

1. Calculate sample sizes that can possibly be in the plan regarding of the budget constraints as of equation (15).

$$1 \quad n_i \quad \frac{B - j\sum_{i=1}^{m} c_i + j\, c_i}{j\, c_i} \qquad i = 1, 2, \ldots, m \qquad (15)$$

2. Calculate the consumer's risk for all possible sample sizes from equation (15) for every product as follows.

2.1 Single and double acceptance sampling plans for attribute.

For each interval of the product sample sizes, calculate the acceptance number (Ac) for single acceptance sampling plan and double acceptance sampling plan, regarding of equation (1) and (3), respectively. If the feasible sample size of product 1 is $1 \le n_1 \le 6$ and specific the AQL at 1% , LTPD at 10% and the producer's risk is 5%

When $n_1 = 1$: varies the acceptance number until equation 1 is satisfied.

with Ac = 0 : $\sum_{x=0}^{Ac}\binom{1}{x} 0.01^{x}(1-0.01)^{1-x} > 0.95$ then, $n_1 = 1$ and Ac = 0.

Calculate the consumer's risk for product 1 when use the sample size(n_1) = 1 and the acceptance number (Ac) = 0. From equation (12), suppose that the LTPD = 10%, we got the consumer's risk as follow.

$$\sum_{x=0}^{0}\binom{1}{0} 0.1^{0}(1-0.1)^{1-0} = 0.90 .$$

When $n_1 = 2$: varies the acceptance number until equation 1 is satisfied.

With Ac = 0 : $\sum_{x=0}^{Ac}\binom{2}{x} 0.01^{x}(1-0.01)^{2-x} > 0.95$ so, $n_1 = 2$ and Ac = 0.

Calculate the consumer's risk for product item 1 when the sample size is 2 and the acceptance number is 0. Supposed that the lot tolerance percent defective is 10% then put into equation 2 ,the consumer's risk will be 81.0%

$$\sum_{x=0}^{0}\binom{2}{0} 0.1^{0}(1-0.1)^{2-0} = 0.81 .$$

Calculate the consumer's risk until $n_1 = 6$.

when $n_1 = 6$: The Acceptance number is varied so the calculated probability are investigated as follows:

Ac = 0 $\sum_{x=0}^{Ac}\binom{6}{x} 0.01^{x}(1-0.01)^{6-x} < 0.95$ (not valid)

Ac = 1 $\sum_{x=0}^{1}\binom{6}{x} 0.01^{x}(1-0.01)^{6-x} > 0.95$ (valid) , therefore,$n_1 = 6$ and Ac = 1.

Calculate the consumer's risk for product item 1 when the sample size is 6 and the acceptance number is 1. Supposed that the lot tolerance percent defective is 10% and put into equation 2 ,the consumer's risk will be 88.5%

$$\sum_{x=0}^{1}\binom{6}{x} 0.1^{x}(1-0.1)^{6-x} = 0.885 .$$

Variable sampling plans for single specification limit to control process mean. We can calculate the acceptance limit ($\bar{X}_a$) from equation (16) and the consumer's risk (β) from equation (17) and (18).

$$\bar{X}_a = \bar{X}_1 + Z_{\alpha}\frac{\sigma}{\sqrt{n}} \qquad (16)$$

$$Z_{\beta} = \frac{\bar{X}_a - \bar{X}_2}{\sigma/\sqrt{n}} \qquad (17)$$

$$\beta = 1 - \Pr(Z < Z_{\beta}) \qquad (18)$$

2.3 Variable sampling plans for double specification limits to control process mean. We can calculate the upper acceptance limit ($\bar{X}_{Ua}$) and the lower acceptance limit ($\bar{X}_{La}$) as

$$\bar{X}_{La} = \bar{X}_1 - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \qquad (19)$$

and

$$\bar{X}_{Ua} = \bar{X}_1 + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} . \qquad (20)$$

So that, the consumer's risk can be obtained from equation (21) or (22) .

$$Z_{\beta} = \frac{\bar{X}_{La} - \bar{X}_{2L}}{\sigma/\sqrt{n}} \qquad (21)$$

$$Z_\beta = \frac{\overline{X}_{Ua} - \overline{X}_{2U}}{\sigma / \sqrt{n}} \qquad (22)$$

2.4 Calculate the consumer's risk from equation (23) .

$$Z_\beta = \sqrt{n}\ \left(Z_1 - Z_2\right) - Z_\alpha \qquad (23)$$

2.5 Variable sampling plans in case of variance unknown to control percent defective. We apply equation (24) for obtaining the consumer's risk.

$$(Z_1^2 + 2)\ Z_\beta^2 + (4\ Z_\alpha + (2\ Z_\alpha\ Z_2\ Z_1))\ Z_\beta + Z_\alpha^2\ (2 + Z_2^2) - 2\ n\ (Z_1 - Z_2)^2 = 0 \qquad (24)$$

3. The minimum maximum consumer's risk for product 1, 2, … , m which was ordered in step 2. These are indexed stage for calculating sampling plans . The product which has maximum consumer's risk will be ordered following the consumer's risk until the product which yields the minimum consumer's risk will be assigned to the last stage or stage m.

The sample size which was calculated from step 1 , get these values to calculate the path which give minimum maximum consumer's risk by backward dynamic programming and the recursive function from equation (25) and (26).

$$f_m(i) = \beta_m(i) \qquad (25)$$

$$f_t(i) = \min_{\forall j} \{ \max\ [\ \beta_t(i)\ ,\ f_{t+1}(j)\ ]\ \} \qquad (26)$$

$\beta_t(i)$ = the consumer's risk for stage t when use i sample size.

5. Backtracking to find the path which has the minimum maximum consumer's risk . Then the sample size will be checked that $\overset{m}{\underset{i=1}{}} n_i c_i \quad B$. If the total cost is greater than limited budget, decrease the sample size in stage 1 by one and assign consumer's risk equal to infinity then repeat step 5 until the total cost is less than the budget.

An example

The Five Stars manufacturing has 6 products to inspect before sending the products to the consumers. The products will be inspected by the attribute or variable sampling plans depended on its characteristics.

1. The products that will be inspected by single attribute sampling plans are shown as follow.
   1.1 Product A001: AQL= 5%, LTPD = 20%, α= 5% and the sampling cost is $10 per unit.
   1.2 Product A002: AQL= 1%, LTPD = 10%, α= 5% and the sampling cost is $50 per unit.
2. The products that will be inspected by variable sampling plans to control process mean are follow.
   2.1 Product A003: $\overline{X}_{2L} = 180{,}000\ N/m^2$, $\overline{X}_{2U} = 200{,}000\ N/m^2$ and the standard deviation is 5,000 N/m^2, α= 5%.The sampling cost is $500 per unit.
   2.2 Product A004: $\overline{X}_1 = 0.1675$ Kg. , $\overline{X}_2 = 0.1525$ kg, and the standard deviation is 0.015 kg , α= 5% with the sampling cost is $50 per unit.
3. The products that will be inspected by variable sampling plans to control percent defective are follow.
   3.1 Product A005: AQL= 2%, LTPD = 12%, α= 5% and the sampling cost is $30 per unit.
   3.2 Product A006: AQL= 5%, LTPD = 20%, α= 5% and the sampling cost is $10 per unit.

The budget for sampling inspection is not greater than $50,000.What is the acceptance sampling plans for these products with minimize maximum consumer's risk?

To solve this problem, we employed the applied dynamic programming algorithm that was shown previously.The results of the calculation are shown in table 1:

Table1: The results of the calculation.

Product number	sample size	decision criteria	consumer's risk	sampling cost
A001	139	c = 1	0.007 %	$1,390

A002	174	c = 4	0.008 %	$8,700
A003	8	$\bar{X}_{Ua}$ = 186,534.479 N/m^2 $\bar{X}_{La}$ = 193,465.521 N/m^2	0.011 %	$4,000
A004	28	$\bar{X}_a$ = 0.163 Kg.	0.013 % *	$1,400
A005	38	k = 1.175	0.008 %	$1,140
A006	88	k = 1.399	0.010 %	$880

All of the consumer's risk that occur, the minimum maximum consumer's risk is 0.013 %. The total cost that will be used in inspection activity is $17,510.

Conclusion

The attribute and variable acceptance sampling plans for sampling multiple products that are indexed by cost of inspection and yieldly the minimum maximum consumer's risk so that the producer and the consumer can be both appreciated. The producer can obtain the sampling plans under the budget cost which are restricted and the consumer will have the least chance of accept the worse products. Further studies may be solved as same as the previous algorithm such as restrict in time of inspection. It can be calculated by changing the inspection cost per unit by the time of inspection per unit.

References

[1] Bank, J. : Principles of Quality Control. John Wiley and Son, Inc., New York. 643 p., 1989.

[2] Dodge, H.F. and H.G. Romig. : Sampling Inspection Tables. John Wiley and Son, Inc., New York. 224 p., 1998.

[3] Duncan, A.J. : Quality Control and Industrial Statistics. Richard D. Irwin, Inc., Homewood, Illinois . 1123p. 1986.

[4] Grant,E.L. and R.W. Leavenworth. : Statistical Quality Control. McGraw Hill,Inc., New York. 720p. 1980.

[5] Mitra, A. : Fundamental of Qualitty Control and Improvement. Mac millan, Inc., New York. 690p. 1993.

[6] Montgomery, D.C. : Introduction to Statistical Quality Control. John Wiley and Son, Inc., New York. 677 p., 1996.

[7] Paitoon Howying and Prapaisri Sudusna – Na - Ayudthaya; The attribute sampling plans for multiple products, First National Symposium on Graduate Research, pp66-76, 2000.

[8] Paitoon Howying and Prapaisri Sudusna – Na - Ayudthaya; The variable sampling plans for multiple products, IE Network National Conference, pp655-662, 2001.

[9] Winston, W.L. : Operation Research. International Thomson Publishing. 1378p. 1994.