The Joint Replenishment Inventory Model with Uncertain Demand

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Abstract

This paper presents a joint replenishment inventory model for raw materials from outside sources where demand is uncertain. The distributions of demand are estimated in accompany with the activity-based costing which is employed to estimate the relevant costs of the system. The multiple criteria decision-making, nonlinear programming, and simulation techniques are employed to determine the efficient economic ordering quantity and a reorder point where total inventory cost is minimized.

Key words: Inventory control, Probabilistic inventory model, Joint replenishment inventory model, and Multiple criteria decision-making

1. Introduction

The efficient inventory management can be considered as the business strategies for cost reduction as well as increasing profitability and competitiveness. Inventory system provides the useful information for inventory management and control. Since there are various kinds of raw materials used for a particular production, in which some raw materials may be supplied by a particular supplier, the concept of combining the ordering cost among various materials leads to the joint replenishment inventory models. The earlier studies mostly focus on the deterministic system to determine the economic ordering quantity (EOQ) for the single raw material where demand is known with certain value and no quantity discount, shortage cost, and lead time [5], [9]. The deterministic inventory systems are proposed and extended to the joint replenishment inventory models as in [1], [2], [3], [4], [7], and [8]. However, in many situations decision-makers are faced with the problem of uncertain demand. The probabilistic inventory models are caused an interest and are developed as in [6]. He proposes the (Q,r) model to determine the economic ordering quantity and a reorder point for a single product where random variable of demand, no quantity discount, constant lead time, and shortage cost are assumed. Such a model is modified by [10] with an additional of the normal distribution of demand. Obviously, in the studies on the probabilistic model, the benefits from the joint replenishment concepts are still not taken into account.

The purpose of this paper is to develop the model for the joint replenishment inventory system with uncertain demand. It examines the context of a continuous review system, where an ordering quantity and a reorder point are utilized to control the inventory cost of the system and considered as decision variables. The multiple objectives decision-making via lexicographic maximization approach, nonlinear programming and simulation techniques are employed. The paper is organized as follows: In Section 2, the mathematical model including assumptions and notations is presented. Section 3 is concerned with the application of the model and the sensitivity analysis results. Finally, in Section 4, the conclusions are drawn.

2. The Mathematical Model

The inventory model presented in this paper is based on the following assumptions:

1. The single-period and continuous review model is assumed.
2. Demand over a given period of time is uncertain. It is a continuous random variable and occurs at a constant rate, which equals to its expected value.
3. The lead time is constant with small value and less than the planning horizon.
4. No shortages are allowed.
5. Purchasing cost is an irrelevant cost.
6. The raw materials are procured from outside sources.
7. The number of order for each raw material is less than or equals to the number of joint replenishment from the particular supplier.
8. Safety stock is constant.

In addition, the following notations are used for the mathematical model.

$T_1(Q_{ij}, r_{ij}, k_{ij})$ = Expected joint ordering cost per unit time

$T_2(Q_{ij}, r_{ij}, k_{ij})$ = Expected holding cost per unit time

D = Random variable demand with mean $E(D)$, variance var D, and standard deviation σ_D

$E[D_{ij}]$ = Expected demand for raw material j per unit time from supplier i

L_i = Lead time for supplier i

C_i = Fixed ordering cost per order from supplier i per unit time

c_{ij} = Variable ordering cost per order for raw material j from supplier i .
h_{ij} = Holding cost for raw material j from supplier i per unit per unit time
A = Available space area in the warehouse
SS_{ij} = Safety stock for raw material j from supplier i
S_{ij} = Area at the bottom of raw material's container for raw material j from supplier i
I_{ij} = Number of rows of raw material's container placed overlapped in the warehouse for raw material j from supplier i
Q_{ij} = Ordering quantity each time an order takes place for raw material j from supplier i
r_{ij} = Inventory level at which order is placed (reorder point) for raw material j from supplier i
k_{ij} = Proportion of N_i / N_{ij}

where
N_{ij} = Number of replenishment for raw material j from supplier i
N_i = Number of joint replenishment for some particular raw materials from supplier i

The model is developed and classified into 2 models. In the first model, the objective functions are composed of equation (1) and (2) with constraints as shown below. In this case the decision-maker will trade off between ordering cost and holding cost where ordering cost is weighted more than holding cost. Therefore, the multiple objectives nonlinear programming via lexicographic maximization approach is employed to determine the efficient inventory policy. In the second, the objective function is equation (1) plus (2) while constraints are the same as the first one. It means that both costs are treated with equal weight for controlling in the system. Then the nonlinear programming technique is employed.

Objective function

$$\min T_1(Q_{ij}, r_{ij}, k_{ij}) = \sum_{i=1}^{n} \frac{\sum_{j=1}^{m} E[D_{ij}]}{\sum_{j=1}^{m} Q_{ij}} \left(C_i + \sum_{j=1}^{m} \frac{c_{ij}}{k_{ij}} \right) \quad ; \quad \text{Expected joint ordering cost} \qquad (1)$$

$$\min T_2(Q_{ij}, r_{ij}, k_{ij}) = \sum_{i=1}^{n} \sum_{j=1}^{m} \left(\frac{Q_{ij}}{2} + r_{ij} - L_i.E[D_{ij}] \right) k_{ij} h_{ij} \quad ; \quad \text{Expected holding cost} \qquad (2)$$

Constraints:

$$\frac{\sum_{j=1}^{m} E[D_{ij}]}{\sum_{j=1}^{m} Q_{ij}} = N_i \text{ for } \forall\, i; \quad \text{Number of joint replenishment from supplier i} \qquad (3)$$

$$\frac{E[D_{ij}]}{Q_{ij}} = \frac{N_i}{k_{ij}} \text{ for } \forall i, j; \quad \text{Number of replenishment for raw material j from supplier i} \qquad (4)$$

$$\frac{E[D_{ij}]}{Q_{ij}} \qquad N_i \qquad \text{for } \forall i, j \qquad (5)$$

$$k_{ij} \qquad N_i \qquad \text{for } \forall i, j \qquad (6)$$

$$L_i \qquad \frac{Q_{ij}}{E[D_{ij}]} \qquad \text{for } \forall i, j; \quad \text{Lead time from supplier i} \qquad (7)$$

$$r_{ij} - L_i.E[D_{ij}] \qquad SS_{ij} \qquad \text{for } \forall i, j; \quad \text{On-hand inventory level at the end of period} \qquad (8)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \left[Q_{ij} + r_{ij} - L_i.E[D_{ij}] \right] \frac{S_{ij}}{I_{ij}} \leq A \text{; Available area in the warehouse.} \qquad (9)$$

$$Q_{ij}, r_{ij}, N_i, k_{ij} \geq 0 \tag{10}$$

where

$$SS_{ij} = \sqrt{L_i E[D_{ij}]} \tag{11}$$

i = 1, 2, …, n

j = 1, 2, …, m

3. An Application of the Model and Sensitivity Analysis

Both models are applied to a selected thread-dyeing factory as a case study. The activity-based costing approach is utilized to estimate all relevant costs. Other relevant data according to the model are collected from the past data. Due to the limitation of demand data for each raw material, the existing data combined with the simulation technique are used to estimate their distributions. They are normal, lognormal, beta, exponential, erlang, uniform, and triangular distributions with their specific means and standard deviations. Since lead time has a small value as mentioned in assumption 3, the difference in the distributions of demand has little effect to the cost of the inventory system [10]. As mentioned earlier, the multiple objectives nonlinear programming and nonlinear programming techniques are employed to determine the efficient and optimal inventory policies for model 1 and 2, respectively. The results reveal that both models can reduce the expected cost of the inventory system significantly about 48-65 percent as compared to the existing system due to the joint replenishment of some raw materials from the particular suppliers.

In accordance with the nature of the cost system of the case study where ordering cost is generally much higher than holding cost, the sensitivity analysis is conducted on the difference between expected ordering cost and expected holding cost when other factors are assumed constant. It indicates that when the difference between these two costs trend to increase, in order to trade off between these two costs the first model provides the expected total cost of the system lower than the second one. Because the ordering cost has the first priority on the cost control, the number of order will decrease whereas the amount of ordering quantity for each raw material will increase. In turn, the difference between these costs trend to decrease, the second model will provide the lower expected total cost of the system than the first one. Since ordering cost and holding cost in the second model are treated as an equal priority on the cost control, the number of replenishment will increase whereas the amount of ordering quantity will decrease. These lead to the increasing in ordering cost which is slower than the decreasing in holding cost as a result. Moreover, when the expected demand for each raw material is increased, the second model will provide the lower expected total cost than the first one. In contrast, if the expected demand is decreased, the first model will be more appropriate than the second one.

4. Conclusions

Both models can be implemented in the factories where the inventory system is the continuous review system, an average ordering cost per unit time being higher than an average holding cost per unit time, and a small value of lead time. Which models will be appropriate depending upon the decision-maker's strategy for controlling the inventory cost of the system. In the case that the system has an average holding cost per unit time being higher than an average ordering cost per unit time or backorder cost to be allowed, the model needs to be modified.

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