# Fuzzy Production Planning and Its Application to Decision Making

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## ABSTRACT

This paper presents a new methods for solving a production planning problem. First the modified s-curve membership function as a methodology is constructed. Then fuzzy production planning problems with vagueness parameters alpha and fuzzy objective coefficients, fuzzy technical coefficients and fuzzy resource variables are outlined. The objective of this paper is to find a satisfactory solution for optimal profit in which vagueness is playing major factor in selecting the solution. Finally a practical applications of decision making in production planning is illustrated.

## INTRODUCTION

Many problems in operations research, decision science, engineering and management have mainly been studied from optimizing points of view. As the decision making is much influenced by the disturbances of a social and economical circumstances, optimization approach is not always the best. It is because under such influences, many problems are ill-structured. Therefore, a satisfaction approach may be much better than an optimization one. In this regards, it is acceptable that the aspiration level on the treated problem is resolved on the base of past experiences and current knowledge possessed by a decision maker, in the case where the aspiration level of a decision maker should be considered to solve a problem from the perspective of satisfaction strategy. Therefore, it is more natural that the vagueness in the fuzzy system denoted by fuzzy numbers by decision maker.

In this paper, the new methodology of modified s-curve membership function using fuzzy linear programming in production planning and their applications to decision making are carried out. Especially, fuzzy linear programming based on a vagueness in the fuzzy parameters such as objective coefficients, technical coefficients and resource variables given by a decision maker is analyzed.

Various types of membership functions were used in fuzzy linear programming problem and its application such as a linear membership function (Zimmermann, 1976; 1978), a tangent type of a membership function (Leberling, 1981), an interval linear membership function (Hannan, 1981), an exponential membership function (Carlsson and Korhonen, 1986), inverse tangent membership function (Sakawa, 1983), logistic type of membership function (Watada, 1997), concave piecewise linear membership function (Inuiguchi, Ichihachi and Kume, 1990) and piecewise linear membership function (Hu and Fang, 1999). As a tangent type, of a membership function, an exponential membership function, and hyperbolic membership function are non-linear function, a fuzzy mathematical programming defined with a non-linear membership function results in a non-linear programming. Usually a linear membership function is employed in order to avoid non-linearity. Nevertheless, there are some difficulties in selecting the solution of a problem written in a linear membership function. Therefore, in this paper a modified s-curve membership function is employed to overcome such deficits which a linear membership function has. Furthermore, S-curve membership function is more flexible enough to describe the vagueness in the fuzzy parameters for the production planning problems.

Due to limitations in resources for manufacturing a product and the need to satisfy certain conditions in manufacturing and demand, a problem of fuzziness occurs in industrial production planning systems. This problem occurs also in chocolate manufacturing when deciding a mixed selection of raw materials to produce varieties of products. This is referred here to as the Product- mix Selection Problem (Tabucanon, 1996). The objective of the company is to maximize its profit, which is, alternatively, equivalent to maximizing the gross contribution to the company in terms of US$. That is to find the optimal product mix under uncertain constraints in the technical, raw material and market consideration. Furthermore, it is possible to show the relationship between the optimal profits and the corresponding membership values (Zimmermann, 1978). According to this relationship, the decision maker can then obtain his satisfactory solution with a trade-off under a pre-determined allowable imprecision.

## MODIFIED S-CURVE MEMBERSHIP FUNCTION

The modified S-curve membership function is a particular case of the logistic function with specific values of B, C and α. This logistic function as given by equation (1) and depicted in Figure 1 is indicated as S-shaped membership function by Gonguen (1969) and Zadeh (1971).

We define, here, a modified S-curve membership function as follows:

$$\mu(x) = \begin{cases} 1 & x < x^a \\ 0.999 & x = x^a \\ \dfrac{B}{1 + Ce^{\alpha x}} & x^a < x < x^b \\ 0.001 & x = x^b \\ 0 & x > x^b \end{cases} \tag{1}$$

where $\mu$ is the degree of membership function. Notation α determine the shapes of membership function μ(x), where α > 0. The larger parameter α get, the less their vagueness becomes. It is necessary that parameter $\alpha$, which determine the figures of membership functions, should be heuristically and experientially decided by experts.

Figure 1 shows the modified S-curve. The membership function is redefined as $0.001 \leq \mu(x) \leq 0.999$. This range is selected because in production system the work force need not be always 100% of the requirement. At the same time the work force will not be 0%. Therefore there is a range between $x^a$ and $x^b$ with $0.001 \leq \mu(x) \leq 0.999$ . This concept of range of $\mu(x)$ is used in this research paper.

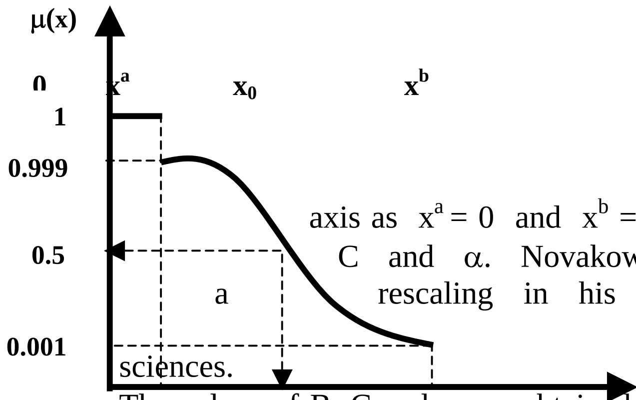


Figure 1 Modified S-Curve Membership Function

We rescale the x axis as $x^a = 0$ and $x^b = 1$ in order to find the values of B, C and $\alpha$. Novakowska (1977) has performed such a rescaling in his work of social sciences.

The values of B, C and $\alpha$ are obtained from equation (1) as

$$B = 0.999(1+C) \quad (2)$$

$$\frac{B}{1+Ce^{\alpha}} = 0.001 \quad (3)$$

By substituting equation (2) into equation (3) :

$$\alpha = \ln \frac{1}{0.001}\left(\frac{0.998}{C} + 0.999\right) \quad (4)$$

Since, B and $\alpha$ depend on C , we require one more condition to get the values for B, C and $\alpha$ . $\mu(x_0) = 0.5$ ; Therefore

$$\alpha = 2\ln\left(\frac{2B-1}{C}\right) \quad (5)$$

Substituting equation (4) in to equation (5), we obtain

$$2\ln\left(\frac{2(0.999)(1+C)-1}{C}\right) = \ln\frac{1}{0.001}\left(\frac{0.998}{C} + 0.999\right) \quad (6)$$

Since $C$ has to be positive, equation (6) gives $C = 0.001001001$ and from equation (2) and (4), B = 1 and $\alpha = 13.81350956$.

**THE FUZZY PRODUCT – MIX SELECTION PROBLEM (FPSP)**

There are 8 products to be manufactured by mixing 8 raw materials with different proportion and by using 9 varieties of processing. There are limitations in resources of raw materials. There are also 10 constraints imposed by marketing department such as product – mix requirement, main product line requirement and lower and upper limit of demand for each product. All the above requirements and conditions are fuzzy. It is necessary to obtain maximum profit with certain degree of satisfaction by using fuzzy linear programming.

The problem illustrated in this paper is only three cases of FPSP problems which occur in real life applications. All cases are listed as :
Case 1 FLP with fuzzy objective coefficients and the other two are non fuzzy
Case 2 FLP with fuzzy resources and the other two are non fuzzy
Case 3 FLP with fuzzy technical coefficients and the other two are non fuzzy

The FLP model and fuzzy formulation for the above three cases are given as :

Model 1

$$\text{Maximize} \quad \sum_{j=1}^{8} \tilde{c}_j x_j$$

$$s.t. \quad \sum_{i=1}^{29}\sum_{j=1}^{8} a_{ij} x_j \le b_i$$

Model 2

$$\text{Maximize} \quad \sum_{j=1}^{8} c_j x_j$$

$$s.t. \quad \sum_{i=1}^{29}\sum_{j=1}^{8} a_{ij} x_j \le \tilde{b}_i$$

Model 3

$$\text{Maximize} \quad \sum_{j=1}^{8} c_j x_j$$

$$s.t. \quad \sum_{i=1}^{29}\sum_{j=1}^{8} \tilde{a}_{ij} x_j \le b_i$$

where $x_j > 0$ is decision variable and the range for all the fuzzy variables are given as in Figure 1.The following fuzzy variables are derived from equation (1) and substituted in the respected model with $C = 0.001001001$,B=1and $\alpha = 13.81350956$.

$$\tilde{c}_j\Big|_{\mu=\mu_{C_J}} = c_j^a + \left(\frac{c_j^b - c_j^a}{\alpha}\right)\ln\frac{1}{C}\left(\frac{B}{\mu_{c_j}} - 1\right), \quad \tilde{a}_{ij}\Big|_{\mu=\mu_{A_{IJ}}} = a_{ij}^a + \left(\frac{a_{ij}^b - a_{ij}^a}{\alpha}\right)\ln\frac{1}{C}\left(\frac{B}{\mu_{a_{ij}}} - 1\right)$$

and

$$\tilde{b}_i\Big|_{\mu=\mu_{B_I}} = b_i^a + \left(\frac{b_i^b - b_i^a}{\alpha}\right)\ln\frac{1}{C}\left(\frac{B}{\mu_{b_i}} - 1\right).$$

By using linear programming technique we will be able to solve the above FLP models and the individual best fuzzy solution for constraints and objective functions could be obtained. The obtained result are summarized in Table 1. Detail solution and computational procedure for each model for varies vagueness and degree of satisfaction is available in Pandian (2002).

**FRONTIER FUZZY SOLUTION**

The range and maximum value of $z^*$ for case 1 to case 3 vary from case to case. We require the largest $z^*$ with less range ($\Delta z^*$) as vagueness varies. Smaller range $\Delta z^*$ means that the maximum value of $z^*$ is more or less guaranteed even at varied vagueness.

Table 1 : Optimal $z^*$ Range of $z^*$ ($\Delta z^*$) for $2 \le \alpha \le 20$

| Case | Fuzzy Coefficient | $z^*_{max}$ | $\Delta z^* = z^*_{max} - z^*_{min}$ |
|---|---|---|---|
| 1 | Objective ($c_j$) | $3.1800 \times 10^5$ | $1.2721 \times 10^5$ |
| 2 | Technical ($a_{ij}$) | $3.1801 \times 10^5$ | $1.2039 \times 10^5$ |
| 3 | Resources ($b_i$) | $3.2902 \times 10^5$ | $1.1948 \times 10^5$ |

Case 3 gives the largest $z^*$ and a guaranteed profit of $3.2902 \times 10^5$. Table 1 is very useful for the decision maker to decide upon changing the constraint and the fuzzy intervals of material, the processing and the demand in order to make $\Delta z^*$ smaller and $z^*$ larger. This result explains that the optimum and satisfactory solutions has been achieved.

**CONCLUSION**

Decision making processes that exist in production planning, being subject to imprecise data and subjective judgments, can be explained by the fuzzy linear programming approach. The results clearly indicate the superiority of the FLP approach in terms of best solutions for the three models and degree of satisfaction. Furthermore , for the problem considered, the optimal solution respect to degree of satisfaction helps to infer that by incorporating fuzziness in a linear programming model in objective function and constraints, provides a better level of satisfaction for obtained results compared to non-fuzzy linear programming.

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