# CONTINUOUS-TIME CLSP WITH SEQUENCE-DEPENDENT SETUP COSTS AND SETUP TIMES

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## ABSTRACT

This paper presents a new continuous-time model on lot-sizing and scheduling problem in a single capacitated machine with sequence-dependent setup costs and setup times. Under the finite planning horizon the model determines the production quantity and timing of multiple products with deterministic dynamic demands minimizing the sum of inventory costs and setup costs. Backlogs are not allowed. Assuming continuous time, setups or productions can continue to the next period without interruptions. Setup states are preserved through idle periods. The sequence-splitting model splits an entire schedule into subsequences, leading to tractable subproblems. A column generation/branch and bound methodology is applied to randomly generated problem instances.

**KEYWAORDS**: Lotsizng, Scheduling, Sequence-Dependent Setup Costs, Sequence-Dependent Setup Times, Column Generation

## INTRODUCTION

In production systems, often a number of similar items are produced on a few facilities. The timing and run lengths for the items are scheduled to meet customer demands. Lot-sizing and scheduling, the problem of determining production quantities and orders so as to minimize costs for product changeovers and inventory, has been an intensive research topic over decades. A realistic lot-sizing model deals with a finite production capacity for dynamic demands on a finite planning horizon. Sequence-dependent setup costs add a sequencing dimension to the problem requiring an integration of sequencing and lot-sizing. Despite its relevance little research has been conducted in the area of lot-sizing and scheduling with sequence-dependent setup costs (Haase 1996).

Customer orders are transformed to production orders so as to minimize setup and holding costs with respect to the limited availability of the resources in the planning phase of a jobshop environment. Complex multi-level product structures and different routes for different items through the shop are typical. Sequencing decisions therefore are considered in a short-term level.

Tempelmeier and Derstroff (1996) propose a heuristic approach for the dynamic multi-level multi-item lot-sizing problem in general product structures with multiple constrained facilities and setup times. Upper bounds are generated by a heuristic scheduling procedure, and lower bounds are obtained from decomposition of the problem into several uncapacitated single-item lot-sizing problems by Lagrangean relaxation. New solution techniques have been developed recently for the lot-sizing part of the problem (Helber 1995) while sequencing is supported by a wide range of job shop scheduling literature.

In a flow shop environment, however, a large amount of items has to be scheduled on one or a few highly utilized parallel production line. Since routing is the same for all items, each line may be planned as a single unit. Changeovers often cause significant sequence-dependent setup times and setup costs. In such a situation, lot-sizing and scheduling has to be done simultaneously in a single step of planning (Fleischmann and Meyr 1997).

The discrete lot-sizing and scheduling problem (DLSP) is the problem of determining the sequence and size of production batches for multiple items on a single machine. Fleischmann (1994) considers a single-machine DLSP with sequence-dependent setup costs (DLSPSD). Formulated as a traveling salesman problem with time windows, problems up to 10 items and 150 periods are solved using Lagrangean relaxation. The DLSPSD is a single machine model and considers only three possible states for the machine in a period: idle, set up for an item, production of an item. Another drawback of the model is that the setup costs through idle periods are not dependent on the last production before the interruption. Hasse (1996) describes the capacitated lot-sizing and scheduling problem with sequence-dependent setup costs (CLSD). Unlike DLSPSD, CLSD preserves setup states over idle periods and allows continuous lot-sizes. Kang et al.~(1994) formulate a multi-machine model including sales revenue suggesting an optimization-based heuristic solution approach that is applied to a collection of practical problems, termed CHES problems. In the model, setup states are preserved over idle periods and early ending is permitted.

So far a capacitated dynamic lot-sizing and scheduling problem (CLSP) in which setup costs and non-trivial setup times are sequence-dependent, multiple setups are allowed in a period, setup states are preserved over idle periods, and continuous lot-sizes are allowed has not been considered in the literature. Its time structure is restricted by the assumption that a lot never continues over two successive periods. A new setup at the beginning of each non-idle period is thus necessary. The continuous-time assumption lifts this limitation. In this paper, we consider an extended form of the lot-sizing and scheduling problem, termed the continuous-time capacitated lot-sizing and scheduling problem with sequence-dependent setup costs and times (CTCLSD), which consists of determining the quantities and timings of the productions for several items in a finite number of periods so as to satisfy a known demand in each period and minimize the sum of the production and inventory costs without backlogs. A changeover of items incurs both of a sequence-dependent setup cost and time. The limited availability of the machine may be further reduced by changeover time. The costs and time vary for each sequence of the items that take turns. The problem is known to be NP-hard (Dixon 1979) even without the existence of sequence-dependent setup costs. Naturally recent researches on the topic have been focused on the development of heuristic algorithms. A known typical approach is to obtain lower bounds on the basis of generalized duality theory, by viewing the capacity limits as complicating constraints. Smith-Daniels (1986) suggests a formulation that is designed to solve small problem instances to optimality.

Hasse (1994) introduces a model called the proportional lot-sizing and scheduling problem (PLSP). The PLSP is based on the assumption that at most one setup may occur within a period. Hence at most two items are producible per period. It allows continuous lot-sizes and preserves setup states over idle periods.

## FORMULATION

Let j=1,2,…,J be a set of items scheduled over a finite planning horizon consisting of time periods t = 1,…,T. A changeover from item i to item j incurs a sequence-dependent setup cost $c_{ij}$ and a sequence-dependent setup time $f_{ij}$. The machine is initially set up for item $i_o$ at the beginning of the planning horizon. Closing the machine with the final item j incurs ending cost $EC_j$. *Dummy* product $e_o$ is defined for the purpose of modeling the endings of production sequences with the following setup cost structure: the transition cost to $e_o$ from product j is $EC_j$ and the transition cost from $e_o$ to any product is infinity. $K_t$ is the capacity available in period t and one unit of item j consumes $a_j$ of the capacity. $p_{jt}$ is the quantity of item j produced in period t. Demand of item j in period t, $D_{jt}$, must be satisfied from the inventory (that includes the current production) without backlogs. The initial inventory level for item j, $I_{j0}$, is given, and nd-of-period stock, $I_{jt}$, incurs a per-unit cost of $h_j$. Note that $I_{jt}$ is non-negative from the no-backlogs assumption.

We assume *continuous-time;* that is, a production lot can be carried over to the following period and also a setup can span two consecutive periods consuming a continuous time from the capacities of both periods. The objective of the continuous-time capacitated lot-sizing and scheduling problem with sequence-dependent setup costs and times (CTCLSD) is to find a continuous-time production schedule that minimizes the sum of sequence-dependent setup costs and holding costs on the condition that each item demand is satisfied and the sum of production time and sequence-dependent setup time does not exceed the facility capacity in each period.

We define a *production-sequence* to be a sequence of items produced over the planning horizon - the duration of time on which a scheduling is made. Kang et al. (1994) introduced an unconventional model where a production sequence is modeled as a collection of sub-sequences. Let a *period-sequence* t be a sequence of items produced in period t. A production-sequence is naturally split into period-sequences. We consider a further splitting of a period-sequence into smaller segments, termed *split-sequence*. With a pre-determined parameter Lt, the model treats a period-sequence as to be made up of Lt consecutive split-sequences. The split-sequences are indexed by s during the entire planning horizon, with s ranging from 1 to L = $\sum_{t=1}^{T}$ Lt = T*Lt. In sum, Lt split-sequences are patched together to define a period-sequence, and T period-sequences are patched together to construct a production-sequence.

A non-empty split-sequence r is composed of two exclusive parts: B(r) and the last item of the sequence, e(r). The first item of the split-sequence is b(r) $\in$ B(r). Let $c_r$ be the setup cost of split-sequence r. $c_r$ is the length of the minimum-weight Hamiltonian path in traversing nodes of B(r)$\cup$e(r) beginning at b(r) and ending at e(r). A valid split-sequence r is such that

(1) neither B(r) nor e(r) is empty,
(2) there is no repetition of an item in B(r).
(3) $c_r < \infty$.

Examples of valid split-sequences are (j1, j2, j3), (j1, j2), (j1, $e_o$), (j1, j2, j1), (j2,j1,j1). Examples of non-valid split-sequences: (j_1), ($e_o$), (j1, $e_o$, j2), and (j1, j2, j1, j2).

Note that the last node e(r) of each split-sequence is in fact only a token to designate the first item of the following first non-empty split-sequence; it is just for connecting two successive non-empty split-sequences. The dummy item $e_o$ is the last node of every production-sequence. Define:

$R$ = the set of all valid split-sequences,
$R^s$ = {r $\in$ $R$ | r belongs to the s-th split-sequence},
$x_r$ = a binary variable; if $x_r$ = 1, then r is selected for production (0, otherwise),
$y_{js}$ = a binary variable; if $y_{js}$ = 1, then the machine is set up for item j in s (0, otherwise),
$\delta_{js}$ = a binary variable; if $\delta_{js}$ = 1, then the s-th split-sequence is empty and the machine is set up for item j for j $\in \{1,\ldots,J\} \cup \{ e_o \}$ (0, otherwise).

Then the sequence-splitting model (P) is:

$$\text{(P) min} \quad \sum_{r \in R} c_r x_r + \sum_t \sum_j h_j I_{jt}$$

subject to

$$\sum_{r \in R(s) | j \in B(r)} x_r = y_{js} \quad \forall j \in \{1,\ldots,J\}, s \tag{1}$$

$$\sum_{r \in R(1)} x_r = 1 \tag{2}$$

$$\sum_{r \in R(s) | b(r)=j} x_r - \sum_{r \in R(s-1) | e(r)=j} x_r = \delta_{j,s-1} - j_{js} \quad \forall j \in \{1,\ldots,J\} \cup \{e_o\}, \quad s \geq 2 \tag{3}$$

$$Q \sum_{s \in S(t)} y_{js} \geq p_{jt} \quad \forall j \in \{1,\ldots,J\}, t \tag{4}$$

$$I_{j,t-1} + p_{jt} - D_{jt} = I_{jt} \tag{5}$$

$$\sum_j a_j p_{jt} + \sum_{s \in S(t)} \sum_{r \in R(s)} f_r x_r - \sum_{r \in R(S(t,e))} f_r(e) x_r \leq K_t - O_{t-1} \quad \forall t \leq L-1 \tag{6}$$

$$\sum_{r \in R(S(t,e))} f_r(e) x_r \leq O_t \quad \forall t \leq L-1 \tag{7}$$

$$\sum_j a_j p_{jt} + \sum_{s \in S(t)} \sum_{r \in R(s)} f_r x_r \leq K_t - O_{t-1} \quad \forall t = L \tag{8}$$

$$p_{jt}, I_{jt}, O_t \geq 0 \quad \forall j, t \tag{9}$$

In the model, $f_r$ denotes the total setup time of the split-sequence r, which is the sum of the setup time of all setups in the split sequence indicated by r, not including the setup time for the first item, but including the setup time for the last item. S(t) denotes the set of split-sequences belonging to period t; that is, S(t) = {(t-1)×Lt +1,…, t×Lt} for t=1,2,…,T. In (4), Q is the maximum lot size. S(t,e) is the last split-sequence in period t, and $f_r(e)$ is the last setup time of split-sequence r.

Note that since continuous time is ssumed, a setup process may span two consecutive periods, starting in period t and finishing in period t+1. To take care of such cases, the model defines $O_t$ to be the unfinished part of the setup time for the setup in period t that spans two periods; that is, $O_t$ is a portion of the capacity of period t+1 that is used to finish the setup (for the first item in period t+1) that starts but does not finishes in period t.

The objective function minimizes inventory costs, and the constraints can be interpreted as follows: Constraints (1) defines variable $y_{js}$ by $x_r$. Constraints (5) and (9) assure that the individual demand in each period is fulfilled without backlogging. Constraints (6) – (8) enforce machine capacity constraints with setup time considered. Finally, integrality conditions for $x_r$ and $\delta_{js}$ are not required, as shown in the following theorem.

Theorem 1.
*In extremal feasible solutions of (P), $x_r$ and $\delta_{js}$ are integral for all r,j,s.*

Proof: See Kang et al. (1999).

Since time is assumed to be continuous, as well as the setups, production runs can span two consecutive periods. The model deals with this possibility by allowing the production of the last item of a period again as the first item in the subsequent period. The necessary conditions are zero recurrent (transition to self) setup costs and zero recurrent setup times.

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