

# A CROSS-EFFICIENCY PROFILING FOR INCREASING DISCRIMINATION IN DATA ENVELOPMENT ANALYSIS

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## ABSTRACT

Data Envelopment Analysis (DEA) cannot provide adequate discrimination among efficient decision making units (DMUs). To discriminate these efficient DMUs is an interesting research subject. The purpose of this paper is to present a Cross-Efficiency Profiling (CEP) model which can be used to improve discrimination power of DEA and conduct a methodological comparison of CEP and the other developed methods without a priori information. CEP retains the original spirit of DEA in trying to extract as much information as possible from the data without weight restrictions. We propose that inputs which are not substitutes for each other be assessed separately and only with respect to outputs which consume them or to which they are otherwise related. In this way input-specific ratings based on the concept of cross-efficiency measure are derived giving a profile for each DMU. We will demonstrate that CEP is more discriminating through an example taken from Baker and Talluri [*Computer and Industrial Engineering*, 32(1), 101-108 (1997)].

**KEYWORDS:** data envelopment analysis, cross-efficiency, super-efficiency, profiling

## INTRODUCTION

Data Envelopment Analysis (DEA), as developed by Charnes et al. (1978) and extended by Banker et al. (1984) is a linear programming procedure for a frontier analysis of inputs and outputs. The procedure does not require *a priori* weights on inputs and outputs. Whilst DEA has generated a good deal of attention it does have revealed some drawbacks. One of drawbacks is that it produces plural decision making units (DMUs) having the full efficient status denoted by unity (or 100%). To discriminate between these efficient DMUs is an interesting research subject. Tone (2002) calls this problem the “super-efficiency problem”. This problem becomes more serious if the number of inputs or outputs is increased. This lack of discrimination is because specialized DMUs may have the efficient status due to a single input or output, even though that input or output may be seen as relatively unimportant.

Previously, various efforts have been devoted to develop methods without a priori information to improve discrimination in DEA. Sexton *et al.* (1986) first introduce the concept of cross-efficiency in DEA by using peer evaluation instead of a self-evaluation. Andersen and Petersen (1993) present the procedure referred to SE-CCR model for ranking efficient units. Their basic idea is to compare the unit under evaluation with all other units in the sample, i.e., the DMU itself is excluded. Doyle and Green (1994) further extend the work by Sexton *et al.* (1986) by introducing aggressive and benevolent cross-efficiency referred to CEM. Tofalls (1996) addresses the discrimination problem by presenting the profiling method. He uses the original DEA but taking one input at a time and only with related outputs. Seiford and Zhu (1999) develop a super-efficiency DEA model referred to SE-BCC model. Li and Reeves (1999) propose a multiple criteria approach to DEA referred to MCDEA. Recently, Tone (2002) proposes the super-efficiency model (referred to SE-SBM model) using the slacks-based measure of efficiency.

However, none of research works in DEA literature has been done to examine which of these

developed methods is more discriminating. The purpose of this paper is two-fold: (1) to develop a cross-efficiency profiling model for improving discrimination power of DEA; and (2). to conduct a methodological comparison of CEP and these previously developed methods for the supper-efficiency problem. CEP is based upon a modified version of the profiling model by Tofalls (1996) and the cross-efficiency measure by Doyle and Green (1994). We propose that inputs which are not substitutes for each other be assessed separately and only with respect to outputs which consume them or to which they are otherwise related. In this way input-specific ratings based on cross-efficiency ratings are derived giving a profile for each DMU. For the purpose of comparison, we will use the same illustrative sample employed by Banker and Talluri (1997).

The paper unfolds as follows. Section 2 introduces the methods without a priori information: CEM, Profiling, SE-CCR, SE-BCC, MCDEA and SE-SBM: Section 3 describes the development of CEP. Section 4 presents a methodology comparison of CEM, three supper-efficiency models, MCDEA and CEP. Section 5 provides discussions on the advantages and disadvantages of the seven methods. Conclusions are given in Section 6.

## LITERATURE REVIEW

In this section, we introduce six methods developed for the supper-efficiency problem. These methods are CEM, Profiling, SE-CCR, SE-BCC, MCDEA and SE-SBM.

### CEM

Sexton *et al.* (1986) first introduces the concept of cross-efficiencies in DEA. The basic idea is to use DEA in a peer-evaluation instead of a self-evaluation which is calculated by the classic DEA models. A peer-evaluation means that the efficiency score of a DMU achieves when evaluated with the optimal weights (input and output weights obtained by the classic DEA models) of other DMUs. Thus, for each DMU there will be  $n-1$  cross-efficiencies where  $n$  represents the total number of DMUs. The cross-efficiency of DMU  $k$ , using the weighting scheme of DMU  $h$ , is then:

$$\begin{aligned}
 \text{Max } \theta_{hk} &= \sum_{s=1}^p v_{sh} y_{sk} \\
 \text{subject to} & \\
 \sum_{j=1}^m u_{jh} x_{jk} &= 1 \\
 \sum_s v_s y_{si} - \sum_j u_j x_{ji} &\leq 0 \quad \forall i \\
 v_s, u_j &\geq 0 \quad \forall s \text{ and } j
 \end{aligned} \tag{1}$$

where  $s = 1$  through  $p$ ,  $j = 1$  through  $m$ ,  $i = 1$  through  $n$ ,  $\theta_{hk}$  = the cross efficiency of DMU  $k$  using the weighting scheme of DMU  $h$ ,  $y_{si}$  = amount of output  $s$  produced by DMU  $i$ ,  $x_{ji}$  = amount of input  $j$  used by DMU  $i$ ,  $u_j$  = weight attached to input  $j$ , and  $v_s$  = weight attached to output  $s$ .

Averaging the cross efficiencies of DMU  $k$  using the weighting scheme of other DMUs, we can compute the mean cross efficiencies of DMU  $k$  by the following formulation:

$$\theta_k = 1/(n-1) \sum_{h \neq k} \theta_{hk}. \tag{2}$$

$\theta_k$  then becomes an index for effectively differentiating between good and poor performers.

Thus, the performer of the DMUs can be ranked based on their mean cross-efficiency scores.

As indicated by Baker and Talluri (1997), a limitation with the CEM evaluated from formulation (2) is that input/output weights (optimal weights) obtained from this formulation may not be unique. This condition occurs if multiple optimum solutions exist, because one scheme can be favorable to one DMU and not favorable to another, or vice versa. Doyle and Green (1994) propose the aggressive and benevolent formulations to solve this ambiguity. Doyle and Green not only maximize the efficiency of the target DMU, but they also take a second goal into account. This second goal, in the case of aggressive formulation, is to minimize the efficiency of the composite DMU constructed from n-1 DMUs. The outputs and inputs of a composite DMU are obtained by summing the corresponding outputs and inputs of all the other DMUs except the target DMU. The weights obtained from this formulation make the efficiency of the target DMU the best that it can be, and all other DMUs worst. Thus, the CEM evaluated from these weights is more meaningful. The aggressive formulation is generally used when relative dominance among the DMUs is to be identified. The formulation is shown below:

$$\begin{aligned}
 & \text{Min } \sum_s (v_s \sum_{i \neq k} y_{si}) \\
 & \text{subject to} \\
 & \sum_j (u_j \sum_{i \neq k} x_{ji}) = 1 \\
 & \sum_s v_s y_{si} - \sum_j u_j x_{ji} \leq 0 \quad \forall i \neq k \\
 & \sum_s v_s y_{sk} - \theta_{kk} \sum_j u_j x_{jk} = 0 \\
 & v_s, u_j \geq 0 \quad \forall s \text{ and } j
 \end{aligned} \tag{3}$$

where DMU  $k$  is the target DMU,  $\sum_s (v_s \sum_{i \neq k} y_{si})$  is the weighted output of composite DMU,  $\sum_j (u_j \sum_{i \neq k} x_{ji})$  is the weighted input of composite DMU, and  $\theta_{kk}$  is the efficiency of DMU  $k$  obtained from the DEA CCR model.

The benevolent formulation uses the same set of constraints except that the efficiency of the composite DMU is maximized. As reported by Angulo-Meza and Lins (2002), these two formulations give very similar results, which is why only one of these formulation is used, generally the aggressive formulation.

Baker and Talluri (1997) provide an effective way of measuring the false positiveness of DMUs by using a false positive index (FPI). The FPI relates to the percentage increment in efficiency that a DMU achieves when moving from peer-appraisal to self-appraisal. A FPI for DMU  $k$  is calculated by the following formulation:

$$FPI_k = (\theta_{kk} - (\sum_i \theta_{ik} / n)) / (\sum_i \theta_{ik} / n) \tag{4}$$

where  $\theta_{kk}$  is the simple efficiency of DMU  $k$  and  $(\sum_i \theta_{ik} / n)$  is the mean score of DMU  $k$  obtained from the CEM. A low FPI for a DMU indicates that it benefited the least when moving from peer-appraisal to self-appraisal.

### Profiling method

Tofalls (1996) presents a profiling method for improving discernment in DEA. The methods retains the original spirit of DEA in trying to extract as much information as possible from the data without applying value judgments in the form of additional constraints. He proposes that inputs which are not substitute for each other be assessed separately and only with respect to outputs which consume them or to which they are otherwise related. The relative efficiency

( $E_{jk}$ ) with which input  $j$  is being used to produce the relevant outputs by DMU  $k$  is evaluated using the following linear program (LP):

$$\begin{aligned}
 \text{Max } \theta_{jk} &= \frac{\sum_{s=1}^p v_{jsk} y_{sk}}{x_{jk}} \\
 \text{subject to} & \\
 \frac{\sum_{s=1}^p v_{jsk} y_{si}}{x_{ji}} &\leq 1, i = 1, K, n \\
 u_{jsk} &\geq \varepsilon, s = 1, K, p
 \end{aligned} \tag{5}$$

where  $\varepsilon$  is a small positive number,  $n$  is the number of DMUs and  $v_{jsk}$  is the weight attached to output  $s$  when evaluating the efficiency of input  $j$  of DMU  $k$ . As with DEA each DMU has its own set of weights. The key difference between this and the DEA formulation is that each linear program only deals with a single input rather than a weighted sum of all inputs. Thus instead of a single efficiency score we now have a score of for each resource input.

### SE-CCR

Andersen and Petersen (1993) present the procedure for ranking efficient units. The procedure is based upon a modified version of DEA. The basic idea of SE-CCR is to compare the DMU under evaluation with a liner combination of all other units in the sample, i.e. the DMU itself is excluded. It is conceivable that an efficient DMU may increase its input vector proportionally while preserving efficiency. The unit obtains in that case an efficiency score above one. The approach provides an efficiency rating of efficient units similar to the rating of inefficient units.

The input oriented SE-CCR model is as follows:

$$\begin{aligned}
 \text{Min } E_k - \varepsilon(1^T s^- + 1^T s^+) \\
 \text{subject to} & \\
 E_k x_k &= \sum_{\substack{i=1 \\ i \neq k}}^n \lambda_i x_i + s^-, \\
 y_i &= \sum_{\substack{i=1 \\ i \neq k}}^n \lambda_i y_i + s^+, \\
 \lambda_i, s^+, s^- &\geq 0
 \end{aligned} \tag{6}$$

where  $x_k$  is an  $m$ -dimensional input vector and  $y_k$  is an  $p$ -dimensional output vector for the  $k$ th unit,  $E_j$  is a scalar defining the share of the  $k$ th DMUs input vector which is required in order to produce the  $k$ th DMUs output vector within the reference technology,  $Z$  is an intensity vector in which  $\lambda_i$  denoted the intensity of the  $i$ th unit,  $\varepsilon$  is a non-Archimedian infinitesimal, and  $1^T$  is a  $T$ -dimensional vector of 1.

### SE-BCC

Seiford and Zhu (1998) presents SE-BCC)in which increasing, constant and decreasing return to scale are allowed. The SE-BCC model is based on based on a reference technology

constructed from all other DMUs. The super efficiency of DMU  $k$  is evaluated by solving the LP problem below:

$$\begin{aligned}
\rho_k^* &= \text{Min } \rho_k \\
&\text{subject to} \\
&\sum_{\substack{i=1 \\ i \neq k}}^n \lambda_i x_i \leq \rho_k x_k \\
&\sum_{\substack{i=1 \\ i \neq k}}^n \lambda_i y_i \geq y_k \\
&\sum_{\substack{i=1 \\ i \neq k}}^n \lambda_i = 1 \\
&\rho, \lambda_i \geq 0, j \neq k.
\end{aligned} \tag{7}$$

where  $\rho_k^*$  is the optimal value for DMU  $k$  to the input-oriented SE-BCC model,  $x_i$  is a vector of inputs for DMU  $i$ , and  $y_i$  is a vector of outputs for DMU  $i$ .

### MCDEA

Li and Reeves (1999) present the MCDEA which focuses on solving two key problems: lack of discrimination and inappropriate weighting schemes. MCDEA introduces three objective functions into a LP problem. The first objective function seeks minimization of the inefficiency of a target DMU  $k$ , measured by  $d_k$ , such that the weighted sum of outputs is less than or equal to the weighted sum of inputs for each DMU. Thus, we can say that DMU  $k$  is not efficient its efficiency score would be  $\theta_k = 1 - d_k$ . The second objective function aims at the minimization of the maximum deviation, for which the restriction included in the new formulation,  $M - d_i \geq k$  ( $i = 1, K, n$ ), makes  $M$  the maximum deviation. The third objective function seeks maximization of the deviation of all DMUs. All three objective functions are based on the deviation variable. The LP problem is as follows:

$$\begin{aligned}
&\text{Min } d_k \text{ (or Max } \theta_k = \sum_{s=1}^p v_s y_{sk} \text{)} \\
&\text{Min } M \\
&\text{Min } \sum_{i=1}^n d_i \\
&\text{subject to} \\
&\sum_{j=1}^m u_{jh} x_{jk} = 1 \\
&\sum_s v_s y_{si} - \sum_j u_j x_{ji} + d_i = 0, \quad i = 1, K, n, \\
&M - d_i \geq 0, \quad i = 1, K, n, \\
&v_s, u_j \geq 0, \quad \forall s \text{ and } j.
\end{aligned} \tag{8}$$

### SE-SBM

Recently, Tone (2002) proposes a super-efficiency model by using slacks-based measure (SBM). The SBM by Tone (2001) is non-radial and deals with input/output slacks directly. It returns an efficiency between 0 and 1, and gives unity if and only if the DMU concerned is on the frontiers of the production possibility set with no input/output slacks. In this respect, SBM

differs from traditional radial measures of efficiency that do not take account of the existence of slacks.

The input-oriented SE-SBM model is as follows:

$$\begin{aligned} \tau^* &= \text{Min } \tau = \frac{1}{m} \sum_{j=1}^m \frac{\tilde{x}_j}{x_{jk}} \\ \text{subject to} & \\ 1 &= \frac{1}{p} \sum_{s=1}^p \frac{\tilde{y}_s}{y_{sk}}, \\ \tilde{x} &\geq \sum_{i=1, \neq k}^n \lambda_i x_i, \\ \tilde{y} &\geq \sum_{j=1, \neq k}^n \lambda_j y_j, \\ \tilde{x} &\geq t x_k \text{ and } \tilde{y} \leq t y_o, \\ \lambda &\geq 0, \tilde{y} \geq 0, t > 0. \end{aligned} \tag{9}$$

The optimal solution of SuperSBM can be expressed by

$$\begin{aligned} \delta^* &= \tau^*, \quad \lambda^* = \lambda^* / t^*, \\ \bar{x}^* &= \tilde{x}^* t^*, \quad \bar{y}^* = \tilde{y}^* t^*. \end{aligned}$$

where  $(\bar{x}, \bar{y})$  is a DMU belonged to a production possibility set excluding the target DMU  $(x_k, y_k)$ .

## CROSS-EFFICIENCY [RPFI;ING

The idea of CEP is taken from the profiling model by Tofallis (1996) and CEM by Doyle and Green (1994). In CEP, each DMU is evaluated according to the optimal weighting scheme of other DMUs but taking one input at a time and only with its related outputs. To accomplish this, we use the efficiency scores calculated according to the profiling model and the optimal weighting scheme used to obtain such scores. The cross-efficiency with which input  $j$  is being utilized to produce the relevant outputs by DMU  $k$ , using the weighting scheme of other DMUs, is then:

$$\begin{aligned} \text{Min } &\sum_s (v_s \sum_{i \neq k} y_{si}) \\ \text{subject to} & \\ u_j \sum_j x_{ji} &= 1, i \neq k \\ \sum_s v_{js} y_{si} - u_j x_{ji} &\leq 0 \\ \sum_s v_{js} y_{sk} - E_{jk} u_j x_{jk} &= 0 \\ v_s, u_j &\geq 0 \quad \forall s \text{ and } j \end{aligned} \tag{10}$$

where  $E_{jk}$  is the efficiency score for DMU  $k$  using input  $j$  to produce relevant outputs and is obtained from the formulation (5).

Averaging the cross efficiencies of DMU  $k$  using the weighting scheme of other DMUs, we can compute the mean cross efficiencies of DMU  $k$  by the following formulation

$$E_k^{CEP} = \frac{\sum_i \left( \frac{\sum_s v_{si}^* y_{sk}}{u_{ji}^* x_{jk}} \right)}{n}, i = 1 \wedge n \tag{11}$$

where  $E_k^{CEP}$  is the average cross-efficiency profiling score for DMU  $k$ .  $v_{si}^*$  and  $u_{ji}^*$  are the optimal weights of output  $s$  and input  $j$  for DMU  $i$ . These optimal weights are obtained from the formulation (10).

## METHODOLOGY COMPARISON: AN ILLUSTRATION

This section provides a methodology comparison of CEM, CEP, SE-CCR, SE-BCC, SE-SBM and MCDEA through a numerical example of evaluating robot performance to demonstrate the superiority of CEP. The profiling model by Tofallis (1996) is excluded for the analysis, because the model using simple efficiency score may not lead to a clear winner. The robot data set published in Banker and Talluri (1997), as shown in Table 1. Twenty robots are considered as the feasible alternatives. The robot performance measures considered for the analysis are cost (\$10,000), repeatability (mm), load capacity (kg), and velocity (m/s). These measures are not exhaustive by any means, but frequently used in robot's performance evaluation. Cost and repeatability are treated as inputs. Load capacity and velocity are treated as outputs.

It was decided to run a series of seven DEA models, working up from Charnes, Cooper, and Rhodes model (CCR) by Charnes et al. (1978) to CEP. Table 2 shows the results of all seven DEAs undertaken. The results achieved via CCR show standard DEA is not capable of distinguishing between actual and "false" efficiency. Therefore, by fixing weights through traditional DEA, it is hard to identify false positive robots. In order to compare the CEP and other five techniques for improving discrimination power of traditional DEA, the focus of comparison is on which of these proposed methods is capable of identifying false positive robots.

As can be seen in Table 2, efficient robots 4, 20, 1 and 27 exhibited high false positive indices of 222.58, 194.12, 72.41 and 69.49%, respectively. Therefore, these efficient robots were in fact highly "false positive". Although CEM can identify false positive robots, it can not gain further insights to identify into inefficiencies occurred due to which of input consumed more in order to produce outputs. With our CEP, it not only can identify these four "false positive" robots, but it also shows robot 4 is "false positive" because it consumed more procurement cost to produce output performances, and robots 1, 20 and 27 are "false positive" because these robots used more repeatability to produce output performances.

The results obtained by applying SE-CCR, SE-BCC and SE-SBM show that robots 1, 4, 7, 10, 13, 14, 19 and 20 were identified as efficient robots with different super-efficiency scores over unity. All three methods consistently rated Robot 20 as the best performer which was identified as a "false positive" robot by CEM and CEP. As for MCDEA, it reduced the number of efficient robots from 8 to 2 and identified robots 7 and 27 were efficient. Surprisingly, these four methods produced different results compared to those by CEM and CEP and might not be capable of identifying "false positive" robots.

## DISCUSSIONS

In this section, we would like to discuss the advantages and disadvantages of each method developed for improving discrimination power of traditional DEA method.

CEM can be widely used, particularly when the number of DMUs is relatively small and discrimination among DMU is a major concern. It, however, only provides cross and self-efficiency scores; it does not provide new input and out weights corresponding to those

new efficiency scores; finally it does not explain identify inefficiencies occurred due to which of input consumed more in order to produce outputs. Profiling can provide useful information from the data which in turn will assist the decision maker to make a more informed and defensible decision, but it may not lead to a clear winner due to the usage of CCR efficiency score for each DMU. Value judgments may need for a complete evaluation of all DMUs.

SE-CCR, SE-BCC and SE-SBM do allow for a ranking of the efficient units themselves, but may not be capable of identifying “false positive” DMUs. MCDEA can be used to reduce the number of efficient DMUs identified by traditional DEA method and it thus allows to provide a ranking of the efficient DMUs. It also effectively yield more reasonable input and output weights, but may not be capable of identifying “false positive” DMUs.

As for CEP, it allows to provide a ranking of DMUs based on their cross-efficiency profiling scores with which each input is being utilized by relevant outputs. Thus, it can identify “false positive” DMUs, but it does not yield more reasonable input and output weights. In addition, it is difficult to do computations as the number of inputs is increased. Fortunately, such a difficulty can be solved by using any computer software, e.g. Frontier Analyst.

## CONCLUSIONS

We have presented a CEP method to the super efficiency problem. We argue that the concept of cross efficiency and the profiling method can be further extended, firstly by assessing together only those variables (inputs and outputs) which are relevant, and secondly by assessing separately the cross efficiency with which each input is being utilized. By doing so provides greater discrimination and also shows the weakness and consequences of each DMU. Both of these consequences will aid the decision maker in making a more informed decision which can also be better justified to others.

We also have conducted the methodology comparison of CEM, CEP, SE-CCR, SE-BCC, SE-SBM and MCDEA through a numerical example of evaluating robot performance to demonstrate the superiority of CEP. CEM and CEP can be used to identify “false positive” DMUs, but CEP allows to provide useful information to identify inefficiencies occurred due to consumed more input in order to produce outputs. SE-CCR, SE-BCC and SE-SBM allow to provide a ranking of efficient DMUs, but these methods can not be used to identify “false positive” DMUs. MCDEA allows to provide a ranking of efficient DMUs, reduce the number of efficient DMUs, but it .can not be used to identify “false positive” DMUs.

## ACKNOWLEDGEMENT

The author of this paper would like to give his thank to National Science Foundation of Taiwan for providing financial support to the project of NSC 91-2416-H-123-004.

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Table 1. Robot attributes

Robot No.	Cost (\$10,000)	Load Capacity (kg)	Velocity (m/s)	Repeability
1	7.20	1.35	60.0	0.150
2	4.80	1.10	6.0	0.050
3	5.00	1.27	45.0	1.270
4	7.20	0.66	1.5	0.025
5	9.60	0.05	50.0	0.250
6	1.07	0.30	1.0	0.100
7	1.76	1.00	5.0	0.100
8	3.20	1.00	15.0	0.100
9	6.72	1.10	10.0	0.200
10	2.40	1.00	6.0	0.050
11	2.88	0.90	30.0	0.500
12	6.90	0.15	13.6	1.000
13	3.20	1.20	10.0	0.050
14	4.00	1.20	30.0	0.050
15	3.68	1.00	47.0	1.000
16	6.88	1.00	80.0	1.000
17	8.00	2.00	15.0	2.000
18	6.30	1.00	10.0	0.200
19	0.94	0.30	10.0	0.050
20	0.16	0.80	1.5	2.000
21	2.81	1.70	27.0	2.000
22	3.80	1.00	0.9	0.050
23	1.25	0.50	2.5	0.100
24	1.37	0.50	2.5	0.100
25	3.63	1.00	10.0	0.200
26	5.30	1.25	70.0	1.270
27	4.00	0.75	205.0	2.030

Table 2. Comparison of efficiency results

Robot No.	CCR	CEM	FPI(%)	SE-CCR	SE-BCC	SE-SBM	MCDEA	CEP	
								$E_1$	$E_2$
1	<b>1.00</b>	<b>0.58</b>	72.41	1.01	6.81	1.00	0.48	0.18	<b>0.49</b>
2	0.90	0.48	87.50	0.93	0.91	0.76	0.42	0.06	0.60
3	0.53	0.30	76.67	0.52	0.67	0.47	0.47	0.20	0.05
4	<b>1.00</b>	<b>0.31</b>	222.58	1.10	2.00	1.05	0.16	<b>0.02</b>	0.67
5	0.59	0.19	210.53	0.59	0.59	0.51	0.11	0.09	0.15
6	0.48	0.28	71.43	0.48	0.86	0.40	0.47	0.06	0.08
7	<b>1.00</b>	0.70	42.86	1.32	1.32	1.17	1.00	0.14	0.27
8	0.78	0.56	39.29	0.78	0.78	0.67	0.62	0.13	0.34
9	0.38	0.27	40.74	0.38	0.38	0.34	0.31	0.05	0.17
10	<b>1.00</b>	0.70	42.86	1.04	1.09	1.03	0.76	0.11	0.56
11	0.67	0.42	59.52	0.67	0.68	0.64	0.67	0.23	0.09
12	0.10	0.06	66.67	0.10	0.14	0.10	0.09	0.04	0.01
13	<b>1.00</b>	0.73	36.99	1.09	1.24	1.07	0.70	0.11	0.71
14	<b>1.00</b>	0.82	21.95	1.77	1.78	1.05	0.66	0.18	0.99
15	0.61	0.36	69.44	0.61	0.62	0.57	0.56	0.27	0.06
16	0.61	0.34	79.41	0.60	0.60	0.59	0.58	0.23	0.08
17	0.41	0.19	115.79	0.40	infeasibl	0.29	0.27	0.07	0.03
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18	0.37	0.26	42.31	0.37	0.37	0.32	0.30	0.05	0.15
19	<b>1.00</b>	0.66	51.52	1.02	1.98	1.01	0.73	0.24	0.28
20	<b>1.00</b>	<b>0.34</b>	194.12	8.26	9.54	4.63	0.00	0.94	<b>0.01</b>
21	0.85	0.34	150.00	0.85	2.26	0.76	0.15	0.26	0.03
22	0.83	0.46	80.43	0.83	0.91	0.77	0.45	0.05	0.49
23	0.69	0.44	56.82	0.69	0.92	0.60	0.69	0.10	0.14
24	0.64	0.41	56.10	0.64	0.85	0.56	0.63	0.09	0.14
25	0.55	0.38	44.74	0.55	0.57	0.48	0.51	0.09	0.20
26	0.58	0.36	61.11	0.58	0.77	0.58	0.58	0.27	0.09
27	<b>1.00</b>	<b>0.59</b>	60.49	3.88	infeasibl	2.73	1.00	0.93	<b>0.08</b>
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