A COMBINED FORECASTING PROCEDURE BASED ON NETWORK SIMULATION AND OPTIMIZATION

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ABSTRACT

Forecasting literature supports the notion that *combined* forecasts are better predictors than *separate* forecasts. This study proposes a procedure using network simulation and optimization to combine outcomes predicted by separate forecasting sources which are assumed to be independent. These sources could include forecasts based on managerial opinion and/or analytical time series forecasting techniques. The outcomes could be treated as point estimates or as random variables, which can be combined as a series of arcs from a stochastic network simulation model. Weights are assigned to each arc and are based on the historical accuracy of the combined forecast from past periods. The weights are generated by an optimization procedure based on preemptive priority goal programming, supplemented by a simple application of perturbation theory. This assures feasible solutions for a set of *best fit* weights from the optimization procedure.

KEYWARDS: MS/OR: forecasting, optimization, simulation

INTRODUCTION

The benefits of combined time series forecasts from multiple independent sources over a single forecast from a single source is strongly supported in the forecasting literature (e.g., Georgeff and Murdick, 1986; Makriddakis et al., 1998; Armstrong, 2001). This is evident in that combined forecasting procedures have the benefit of drawing on several conventional analytical procedures to capture the historical characteristics of demand, as well as the advantage of integrating managerial opinion into the forecast. The significance of the latter follows in that conventional time series is reactive to past events and has no mechanism to capture the impact of highly probable future events (e.g., expected economic downturn, political crisis; newly discovered product safety flaws; new product entries into the market; product entering into the mature/decline stages of its life cycles; improved product entry from a competitor; etc.). Such impending future events are best captured by managerial opinion which draws on the intuition and experience of the managers contributing to the forecast. After examining 30 empirically based comparisons, Armstrong (2001) reported that the *reduction in error* for even (equally) weighted combined forecasts, when compared to conventional non-combined forecasts, *averaged about 12.5% and ranged from 3% to 24%.*

The proposed study uses a series of arcs from a network simulation model to represent the outcomes of the various forecasting sources. Arcs can be summed as random variables using an established simulation language such as *ARENA* (Kelton et al., 2002) or *ProModel* (Harrell et al., 2004). Weights are assigned to each arc in proportion to its expected contribution to an accurate forecast based on the means of historical data. Goal programming (e.g., Schniederjans, 1995) is employed to derive weights that reflect forecast accuracy in each of the past periods, as well as the sum of all periods. To assure feasible solutions, *virtual* forecasting sources were developed to supplement the actual forecasting sources by using a simple framework based on perturbation theory (e.g., Simmonds and Mann). This was required only in situations where the actual demand did not fall within the range of estimated forecasts when fitting weights to historical data.

As reported by Bunn and Wright (1991), research in the area of *combined forecasting* has been sparse, and a review of forecasting literature following their study supports the notion that this continues even to the present. The studies that relate to this study are those that employ *unequal* weights to combined forecasts. Bates and Granger (1969) structure the combined forecast as a network formulation, and utilize linear least squares to optimize the selection of weights with the single criterion of forecasting accuracy. Fralicx and Raju (1982) propose canonical correlations to determine weights based on multiple criteria considerations, and Cosgrove (2003) employs simulation to combine forecasts utilizing weights derived from Bayes' Theorem. The proposed study differs from all the above in its use of goal programming as an optimization tool to find a set of best-fit weights based on historical data.

STRUCTURE OF THE COMBINED FORECAST

Assume N forecasting sources (n=1,2,3,…N) generate N independent forecasts for a given period. If F_n is the n*th* forecast and w_n is the weight corresponding to F_n, the combined forecast of the N forecasting sources is given by

$$Y = \sum_{n=1}^{N} w_n F_n \,, \tag{1a}$$

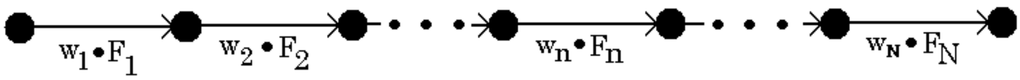
where

$$\sum_{n=1}^{N} w_n = 1 \tag{1b}$$

with $0 \leq w_n \leq 1$ for all n. Note that F_n can be a random variable or a point estimate.

Expression (1a) represents a summation of random variables which can be represented by network arcs which correspond to weighted random service times (i.e., as $w_n \cdot F_n$ for the n*th* arc or forecast as shown in Figure 1). Conversion of any random variable F_n to a random service time at an arc is a simple matter with a network simulation model. For a commercially available simulation language such as *ARENA* or *ProModel*, entities can be released (one at a time with zero queue

Figure 1: Series Network for the Summation of N Random Variables



times) at the source node, and statistics corresponding to the sum of random variables in Expression (1a) can be collected at the sink node. The precision of Y is determined by the number of entities that pass though the network. This can be approximated by the Kolmogorov-Smirnov (K-S) statistic (e.g., collecting the time duration attribute of 18,496 entities through the network will assure a .01 confidence interval in probability with a confidence level of 95%).

GOAL PROGRAMMING FORMULATION FOR GENERATING WEIGHTS

For the purpose of this study, the general preemptive priority goal programming (GP) formulation is modified to specify indices as integers in sets, and to accommodate both single and double subscripted variables and coefficients. These considerations lead to the following:

$$\text{Min } Z = \sum_{k|k\epsilon\mathbf{K}} \sum_{r=1}^{R} P_r\,(d_k^+ + d_k^-) \qquad (2a)$$

subject to

$$\sum_{i|i\epsilon\mathbf{I}} \sum_{j|j\epsilon\mathbf{J}} a_{i,j}\, x_{i,j} - d_k^+ + d_k^- = A_k \qquad (2b)$$

with $d_k^+ \cdot d_k^- = 0$ for all $k|k\epsilon\mathbf{K}$; and with $P_1 >>> P_2 >>> \ldots >>> P_r >>> P_{r+1} >>> \ldots >>> P_R$ and all $x_{i,j}$, d_k^+, and $d_k^- \geq 0$; where r corresponds to R preemptive priority rankings of a given set of goals, and $k\epsilon\mathbf{K}$ are values of k corresponding to goals that are ranked (i.e., values of k in the set **K** which correspond to goal constraints in Expression (2b) that are ranked in the GP *objective function* (2a)). Expression (2a) refers to a set of goal constraints with A_k ($k|k\epsilon\mathbf{K}$) as the k*th* goal. The extent that d_k^+ (defined as a deviational variable overachieving goal A_k) and d_k^-

(defined as a deviational variable underachieving goal A_k) can be driven to zero in (2a) is the extent that the goal A_k can be achieved. The attempt to drive the deviational variables to zero in an ordinal fashion is evident in (2a) since the values of the P_r coefficients drop *dramatically* with any increase in the index r.

The formulation of the GP model as shown in Expressions (2) illustrates that there are no infeasible solutions since the deviational variables can take on values to assure that Expression (2b) is always satisfied. However, the modified simplex algorithm developed by Lee (1972) can accommodate hard constraints in various forms, and with various combinations of indices which could be members of overlapping or disjoint sets. This study will employ such constraints which can be expressed in a general form as follows:

$$\sum_{i|i\in \mathbf{I_2}} b_i\, x_i = B_i \qquad (3a)$$

and

$$\sum_{i|i\in \mathbf{I_3}} \sum_{j|j\in \mathbf{J_2}} e_{i,j}\, x_{i,j} = G_j\ . \qquad (3b)$$

DETERMINING FORECASTING WEIGHTS

This section formulates the preemptive GP model for determining forecasting weights. Goal constraints and hard constraints corresponding to Expressions (2b) and (3) are developed, and later followed by the objective function corresponding to (2a).

The GP model consists of period-by-period constraints which attempt to find the *best fit weights* that satisfy the historical demand for *each past period*, and for the sum of all historical demands over *all past periods* (i.e., over the entire forecasting horizon consisting of T periods).

Constraints for Forecasts of Time Period t

For the n*th* forecasting source in time period t, let A_t represent the actual demand in period t, and $F_{n,t}$ the forecast for that period from the n*th* forecasting source. (If the forecast in period t from the n*th* source was a random variable, then $F_{n,t}$ is the mean of the underlying distribution of that random variable.) Consider the following goal and hard constraints for period t:

$$\sum_{n=1}^{N} F_{n,t} \cdot x_{n,t} + A_t \cdot v_t - d_t^{+} + d_t^{-} = A_t \qquad (4a)$$

and

$$\sum_{n=1}^{N} x_{n,t} + v_t = 1 \qquad (4b)$$

for t=1.2,…T, with variable $x_{n,t}$ the weight on $F_{n,t}$, with $x_{n,t}$ such that $0 \leq x_{n,t} \leq 1$, and v_t such that $0 \leq v_t \leq 1$. Expression (4a) requires that the GP model include T goal constraints (t=1, 1, .., T), with each goal constraint attempting to find the best combination of weights that permit the combined forecast in that period to be a perfect match with the actual demand (i.e., seeking a set of weights such that the summed product of weights and forecasts equal the actual demand, according to the structure of Expression (1a)). This perfect scenario for period t assumes that v_t and the two deviational variables in (4a) are all zero. However, in the worse case scenario, A_t could fall outside the range of all the forecasts (i.e., outside the range of $F_{n,t}$ for all n during period t). If so, then v_t must take on the value of 1 to assure that the goal is met, given that (4b) is a hard constraint. Hence, the term $A_t \cdot v_t$ is inserted to assure the goal is met, but it has the unfortunate consequence of the creation of a *virtual* forecasting source (i.e., an unexpected "N+1" forecasting source). Fortunately, the overall value of v_t may be insignificant (i.e., $v_t \rightarrow 0$) as weights from the other time periods are factored into to model. This issue is given further consideration in the development of the next set of goal and hard constraints.

<u>Constraints for Summed Forecasts Over All Time Periods</u>

Expressions (4a) and (4b) limit the impact of weights for separate forecasting periods. The following goal and hard constraints are developed for the sum of all forecasts over all values of T:

$$\sum_{n=1}^{N} \Big[\sum_{t=1}^{T} F_{n,t} \Big] \cdot w_n + \sum_{t=1}^{T} A_t \cdot s - d_{T+1}^{+} + d_{T+1}^{-} = \sum_{t=1}^{T} A_t \qquad (5a)$$

and

$$w_n = \sum_{t=1}^{T} x_{n,t} + p_n - q_n \qquad (5b)$$

with

$$\sum_{n=1}^{N} w_n + s = 1 \qquad (5c)$$

and

$$\sum_{n=1}^{N} (p_n + q_n) - d_k^{+} = 0 \qquad (5d)$$

for k=(T+1)+1, (T+1)+2,…(T+1)+N; and $0 \leq s \leq 1$, $0 \leq p_n \leq 1$, $0 \leq q_n \leq 1$, for all n. Note that to track the deviational variables of the goal constraints, the k index simply picks up after the last goal constraint (i.e., the T*th* goal constraint) from Expression (4a).

Note that the structure of Expression (5a) is similar to that in (4a). Hence, the sum of the product A_t·s in (5a) plays a similar role as the product $A_t \cdot v_t$ in (4a), by assuring the goal is met even if the sum of all past actual demands (i.e., the sum of all the A_t values) falls outside the range of the sum (over t) of the forecast for all of the forecasting sources. While it is expected that the past forecasts should be close enough to prevent s from taking on a nonzero value, it nevertheless cannot be ruled out. This leads to the worst case scenario suggesting that the performance of the forecasting sources was poor, and once again, a *virtual* forecaster is created to meet the goal.

Expressions (4a) and (5a) assure that the weights w_n for Expression (1) are cross sectional (across *each* time period t taken separately) and longitudinal (across *all* time periods t taken together as a sum of forecasts by source). Note that Expression (5b) forces adjustments on w_n reflecting the cross sectional weights. These adjustments are either positive or negative depending on values taken by p_n or q_n. Expression (5c) assures that the weights sum to 1, and include s in the event that s takes on a nonzero value.

Goal Constraints to Reduce the Impact of Virtual Forecasting Sources

Virtual forecasting sources were introduced as a perturbation in the GP model to assure feasibility among the hard constraints. Since the existence of virtual sources represent worse case scenarios, it is desirable to remove them entirely from the problem by finding a set of weights (w_n's) where s and all v_t terms are zero. The best chance of achieving this is to include the following goal constraints:

$$s - d_k^+ = 0 \tag{6a}$$

for k=(T+1)+N+1, and

$$\sum_{t=1}^{T} v_t - d_k^+ + d_k^- = 0 \tag{6b}$$

for k=(T+1)+(N+1)+1, (T+1)+(N+1)+2,…(T+1)+(N+1)+N.

Objective Function

The GP objective function is given by

$$\text{Min } Z = P_1 \sum_{t=1}^{T} (d_t^+ + d_t^-) + P_2 \,(d_{T+1}^+ + d_{T+1}^-) + P_3 \sum_{k=T+2}^{T+1+N} (d_k^+)$$

$$+\ P_4\,(d_{T+2+N}^{+})\ +\ P_5 \sum_{k=T+3+N}^{T+2+2N} (d_k^{+} + d_k^{-})\ . \qquad (7)$$

Minimizing the deviational variables associated with P_1 sets the highest priority on the goal specified in Expression (4a). In this manner, the model seeks to find a best fit set of cross sectional weights that match the actual forecasts to actual known demand for each past period (i.e., for t=1, 2,…T). P_2 sets the second priority on achieving the goal specified in (5a) which seeks the best fit for the longitudinal weights, subject to the hard constraint in (5c). These are the weights required in Expression (1a) for making the combined forecast. P_3 refers to Expression (5d), which sets the third priority on minimizing the impact of the variables that adjust the longitudinal weight of each forecasting source with its corresponding cross sectional weights over T time periods (with t=1,2,…T), subject to (5b). P_4 and P_5 focus refer to the goal constraints of (6a) and (6b). Meeting these goals eliminates the need of the virtual forecasting sources, which were introduced to assure that a feasible set of both longitudinal and cross sectional weights would be found.

Final Determination of the Forecasting Weights

If it is assumed that the virtual forecasting source of goal constraint (5a) does not appear in the final solution (i.e., s=0), then the weights for the combined forecast for Expression (1a) are simply the values of w_n. However, if s > 0 (or more likely s=1 given the current form of the model), then a practical means for the elimination of s is to follow the usual practice from perturbation theory. This would require reruns of the GP model with some adjustments in constraints, while making small incremental reductions of s (e.g., $\Delta s \approx .01$) to observe changing patterns in the w_n values as $s-\Delta s \to 0$.

CONCLUSION

The generality of the preemptive GP model permits it to be subject to a number of modifications and variations. For example, the formulation in this study gave a higher priority to fitting period-by-period cross sectional goals over the longitudinal goal. These priorities could be reversed. Also, conditions shown as hard constraints that include virtual, cross sectional, and longitudinal weights (all of which help avoid multiple solutions in the current GP model) could be modified and supplemented with additional constraints to proportionally adjust the weights. Overall, the flexibility of the GP formulation allows for testing and experimentation to tailor the final form of the model as a best approximation with historical data.

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