Modeling Coordinated Contracts for a Supply Chain Consisting of Normal and Markdown Sale Markets

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Abstract

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The results of a study of the coordination effect in stocking and promotional markdown policies for a supply chain consisting of a retailer and a discount outlet (DCO) are reported here. We assume that the product is sold in two consecutive periods: the Normal Sales Period (NSP) and the subsequent Promotional Markdown Sales Period (PSP). We first study an integrated supply chain in which managers in the two periods design a common system so as to jointly decide the stocking quantities, markdown time schedule, and markdown price to maximize mutual profit. Next, we consider a decentralized supply chain. An uncoordinated contract is designed in which decisions are decentralized to optimize the individual party's objective function. Here, three sources of system inefficiencies cause the decentralized system to earn a lower expected system profit than that in the integrated supply chain. The three sources are as follows: in the decentralized system the retailer tends to (1) stock less, and (2) keep a longer sales period, and the DCO tends to (3) stock fewer leftovers inventories and charge a higher markdown price. Finally, a numerical experiment is provided to compare the coordinated model with the uncoordinated model to explore factors that make coordination an effective approach.

1. Introduction

Consider a supply chain consisting of a retailer and a discount outlet (DCO) selling a "short life cycle good" (e.g., personal computers, consumer electronics, fashion items) to possible consumers. If the product does not sell after the first Normal Sale Period (NSP), the supply chain has an opportunity to try a new price in a secondary Promotional Markdown Sale Period (PSP). When does the supply chain change price and how does the new price relate to the old? How many of these items should the supply chain produce in the first place, and how many should the second agent (discount outlet) stock in the secondary markdown market? Who controls which decisions, and how will agents be compensated? Firms face very similar problems when they market a new product that has a limited product lifetime. Our study tries to provide answers to these questions. In particular, we study two issues of great importance in designing coordinated contracts for a supply chain consisting of normal and secondary markdown sale markets, namely pricing and inventory stocking.

The importance of coordination in the supply chain has recently been discussed in a considerable body of literature. The main argument states that while the importance of achieving integration in the supply chain is generally well recognized, for real-world applications designing a sophisticated integrated system is an arduous task. Few firms are so powerful that they can manage the entire provision of the supply chain so as to drive individual members to a superimposed integrated objective. Rather, a more realistic route is to design a coordination mechanism (including contractual forms, compensation schemes, and side payments) so as to align the self-interests of individuals with supply chain integrated interests. For example, Jeuland and Shugan (1983) suggested using quantity discount as a mechanism for coordinating a bilateral monopoly channel. Monahan (1984) developed a model from a vendor's perspective for establishing an optimal discount schedule, and showed a price discount schedule with a single break point, achieving the desired outcome for the vendor. The purpose of this model is to show a mutually profitable "Joint Order Quantity" that differs from each individual's optimal order quantity, and which can be obtained in a spirit of cooperation. Monahan's work has been advanced by Lee and Rosenblatt (1986), Kohli and Park (1989, 1994), and Weng (1995), among others. Parlar and Weng (1997) described coordination between a firm's manufacturing and supply departments. Weng (1997) assumed demand to be stochastic, and analyzed channel coordination in wholesale and retail prices. The seminar work of Pasterneck (1985) showed that a manufacturer's returns policy not only can induce a larger order by risk sharing but also can coordinate a supply chain so as to eliminate double marginalization phenomenon, and

to generate the greatest supply chain joint profit. Since then, extensions of Pasterneck's basic model have been attempted in many directions. For example, extending the model to consider price-sensitive demands (Emmons and Gilbert [1998], Marvel and Peck [1995], Kandel [1996], and Lau, Lau, and Willett [2000]) and comparing returns policy with other coordinating mechanisms (markdown allowance in Tsay [2001] and two-parts tariff price only contract in Lariviere [1999]) are two important topics among many others. Some of the more recent work has tried to encompass agents' individual agendas and attitudes toward risk into the basic model (Lau and Lau [1999], Tsay [2002], and Webster and Weng [2000]). Others studied the model under a non-Newsboy framework, and showed that a returns policy could stimulate retail competition to benefit the manufacturer (Padmanabhan and Png [1996]). New coordination approaches are also frequently reported. For example, Tsay (1999) studied a Quantity Flexibility Contract in which a manufacturer rebates fully a portion of leftovers (up to the order quantity). Donohue (2000) provided a discussion of a coordinating returns policy under an assumption that the manufacturer can produce a second production lot after a forecast update. Taylor (2001) analyzed a two-period model in which a wholesale price protection policy is employed with mid-of-life (M) and end of life returns (E) to coordinate the supply chain. This two returns opportunities model was also formulated in Lee (2001) for studying a returns policy in a Manufacturer-Retailer-Discount Outlet setting.

This paper is structured as follows. In section 2, a problem description, assumptions, and notations are presented. In section 3.1, an objective function of the retailer and the discount outlet is formulated. Then the optimal policies for the integrated supply chain are developed and analyzed. In section 3.2, we consider a decentralized supply chain. Here, decisions are made for optimizing objectives of the individual systems. The sources of supply chain distortions are verified, and coordinating strategies are suggested. In section 4, numerical examples are provided to explore the coordinated contract. A brief discussion in section 5 completes the paper.

2. The Modeling Issues

Our problem is formulated as a two-period Newsboy model with the objective of maximizing the expected profit. (See Porteus [1990] for a review of the Newsboy problem.) The chronology of events for the model is described as follows:

(1) In the beginning of NSP, the retailer observes the market for a short period of time labeled an

Observation Period (OP) to evaluate market potential. After the observation period a signal about market potential is revealed. The retailer then decides an order size based on a forecasted expected demand conditional on the market signal. Since the forecasted demand is closely related to the time schedule, i.e., the lengths of NSP and PSP, to choose an optimal order size, the retailer needs to simultaneously estimate a time schedule. Notice that the estimated time schedule (the lengths of NSP and PSP) is the retailer's private information, and will only be used by the retailer as an internal aid for the order quantity decision-making. It is not a firm commitment to the discount outlet, and can be changed in the later phase of the selling season. We assume inventory replenishment is allowed only once (at the beginning of the NSP). When the random demand in any point in time exceeds availability, selling opportunities are lost.

(2) At the end of NSP, if stocks are not completely depleted, the retailer initiates the second period (PSP) by offering a leftovers wholesale price, for which the retailer will sell the leftovers to the DCO.

(3) In PSP, the DCO determines a markdown sales quantity (≤leftovers quantity) and a unit markdown sale price (≤normal sale price) for which the DCO will sell the leftovers to the possible consumers.

The following assumptions and notations are used for modeling purposes:

Let τ denote the exogenously determined total life cycle. The lengths of NSP (PSP) are formulated as a fraction $0 \le \alpha \le 1$ of τ, i.e., $\alpha\tau$ ($(1-\alpha)\tau$). Also let γ denote the DCO's markdown price ratio ($\bar{\gamma} = 1-\gamma$). For example, if a product with an initial list price of P =\$10 is later priced at 35% off the original face value, then $P\gamma$ =10(0.65)=\$6.50. Let $\{D_N = k_N\alpha, \sigma_N\}$ and $\{D_P = k_P(1-\gamma)(1-\alpha), \sigma_P\}$ respectively denote mean and standard deviation of the demands during NSP and PSP. We assume that demands in NSP and PSP are Logistically distributed. We choose logistic distribution function because of its flexibility and applicability. It can be used to approximate Poisson and Normal demands (see Bartmann and Beckmann [1992]). For example, if $\sigma_N = \sqrt{k_N\alpha}$ and $\sigma_P = \sqrt{k_P(1-\gamma)(1-\alpha)}$, the logic function can be used to approximate Poisson Process demand.

The first component, representing the expected demand ($D_P(\alpha,\gamma)$, $D_N(\alpha)$), is influenced by the lapse in the sales period and/or the markdown price. The second component, representing the probabilistic scaling component of the random demand, is independent of the lapse in the sales period and markdown price. This two-component approach was used in various literatures to formulate price-dependent random demands due to its simplicity and flexibility. Leland (1972), for example, has considered two price-dependent random demand models--multiplicative and additive (see also Emmons and Gilbert [1998], Petruzzi and Dada [1999], and Lau and Lau [1988]). The multiplicative model formulates random demand $d = D(P)y$ as the product of an expected demand $D(P)$ (as a function of price P) and a probabilistic scaling component y with $E(y)$=1. The additive model assumes random demand $d = D(P) + y$ with $E(y)$=0. In this work, we will focus on studying the problem assuming that demand is multiplicative; however, we will also briefly discuss the results for the additive demand model in Section 3.1.

Two expected demand functions are considered.

(i) Linear expected demand function: $D_N(\alpha) = k_N\alpha$ and $D_P(\alpha,\gamma) = k_P(1-\gamma)(1-\alpha)$ (Hereafter, we will denote linear model as Model B).

(ii)Square root expected demand functions: $D_P(\alpha,\gamma) = \sqrt{k_P(1-\alpha)(1-\gamma)}$ and $D_N(\alpha) = \sqrt{k_N\alpha}$ (Model A).

Let ($\tilde{Y}, \tilde{X}$) be the probabilistic scaling components of NSP and PSP without the prior knowledge of market signal. We assume that ($\tilde{Y}, \tilde{X}$) can be expressed as a sum of two random components $\tilde{Y} = Z + \varepsilon_y$ and $\tilde{X} = K + \varepsilon_x$ with ε_y (ε_x) and Z (K) being independent, and $E(\tilde{Y}) = 1$ and $E(\tilde{X}) = 1$. Here, Z, denoting "the signal about market potential", can be observed after observation period (OP). This approach is similar to the random demand model formulated in Tsay (1999). We assume that $\tilde{X}$ and $\tilde{Y}$ are correlated. For example, if $K(Z) = a + bZ$ then $Cov(\tilde{Y}, \tilde{X}) = bVar(Z)$, and the random component of demand in the NSP ($\tilde{Y}$) is positively (negatively) correlated with that of the PSP ($\tilde{X}$) if $b > 0 (b < 0)$. Define $Y := \tilde{Y}|Z$ and $X := \tilde{X}|K(Z)$ as random variables given that the market signal Z has been observed. For example, if both ε_y and ε_x are normally distributed with variances σ_Y^2 and

$\sigma_X{}^2$, then Y and X are normally Assuming now $\sigma_X = \sigma_Y = \sigma$, then the demand variance in the NSP and PSP can be expressed as

(i) Square root demand model (Model A): $\sigma_N^2\alpha$ and $\sigma_P^2(1-\alpha)$, where $\sigma_N^2 = k_N\sigma^2$ is a constant, and $\sigma_P^2 = k_P\sigma^2(1-\gamma)$ is a decreasing function of γ. We see that the variance in PSP, particularly σ_P^2, increases as the expected demand increases (due to price (γ) decrease). A justification for this result can be seen in Lau and Lau (1988). They state that for a high demand level (due to low price) beyond the normal operating range, the random demand may have a large variance due to a lack of past experience to draw on. We also see that in the square root demand function when $\sigma_N = \sigma_P$, the total variance is not affected by where the period is divided, and the variance is allocated between the two parts of the period in proportion to their length.

(ii) Linear demand model (Model B): $\sigma_N^2\alpha^2$ and $\sigma_P^2(1-\alpha)^2$, where $\sigma_N^2 = (k_N\sigma)^2$ is a constant, and $\sigma_P^2 = [k_P\sigma(1-\gamma)]^2$ is a decreasing function of γ. We see that in the linear demand model total variance is closely related to the markdown sale time schedule. In particular, the total variance is proportional to the value $\alpha^2 + (1-\alpha)^2$ which is strictly convex between [0,1]; thus, the variance can be reduced to the minimum when the two periods are divided equally. Table 1 lists notations used in the paper.

Table 1. Notations

P	the NSP retail price
Q	the retailer's order quantity
C	the retailer's unit order cost
C_r	the wholesale price of leftovers at the end of the NSP
α	the length of the retailer's normal sales period ($\bar{\alpha} = 1-\alpha$)
γ	the DCO's markdown price ratio ($\bar{\gamma} = 1-\gamma$).
q	the DCO's markdown sales quantity
$D_N(\alpha)$	the expected random demand of the retailer in the NSP
$D_P(\alpha,\gamma)$	the expected random demand of the DCO in the PSP
$Y := \tilde{Y}\|Z$ $X := \tilde{X}\|K(Z)$	probabilistic scaling factor after information update (after OP)
$f(y)$, and $g(x)$	the probability densities of Y, X
$F(y)$ and $G(x)$	the cumulative distribution functions ($\bar{F}(y)$ and $\bar{G}(x)$ converse CDF)
$I(Q,\alpha) = \max[Q - yD_N(\alpha),0]$	the retailer's leftovers at the end of the NSP

3. Retailer-DCO Decision-making Systems

In section 3.1, to provide an efficient benchmark, we consider an integrated system in which the retailer and the DCO form a common system, share demand information, and jointly design an integrated ordering, time schedule, and markdown policy (Q, α, q, γ) so as to deliver the greatest possible expected system profits. In section 3.2, a decentralized supply chain is considered. We will focus on verifying possible sources of inefficiencies that cause sub-optimality in the decentralized system.

3.1 Centralized Supply Chain Model

In the centralized system, the supply chain will maximize the joint objective function of the retailer and the DCO. In the centralized supply chain, the market signal Z is known to both parties; thus, decisions regarding $Q(\alpha)$ and α are made based on the forecast about (Y, X). Let Ω_N (Ω_P) denote the retailer's (DCO's) expected profit excluding the wholesale revenue (cost) of leftovers. Denote $\xi_N(Q,\alpha) := Q/D_N(\alpha)$ and $\xi_P(q,\alpha,\gamma) := q/D_P(\alpha,\gamma)$:

$$\begin{cases} \text{Retailer: } \Omega_N(Q,\alpha) := P\left\{\int_0^{\xi_N} yD_N(\alpha)dF + \int_{\xi_N}^{\infty} QdF\right\} - CQ \\ DCO: \ \Omega_P(q,\gamma|I(Q,\alpha),\alpha) := P\gamma\left\{\int_0^{\xi_P} xD_P(\alpha,\gamma)dG + \int_{\xi_P}^{\infty} qdG\right\} \end{cases}.$$

The maximization problem of the expected joint profit, which we denote as Π_{NP}^C is:

$$\max_{Q,\alpha} \Pi_{NP}^C(Q,\alpha) = \Omega_N(Q,\alpha) + E_y\left\{\max_{q,\gamma} \Omega_P(q,\gamma|I(Q,\alpha),\alpha)\right\}. \tag{1}$$

The objective function in (1) reveals that the goal of the integrated supply chain is to choose the centralized optimal policy ($Q^C, \alpha^C, q^C, \gamma^C$). As the first step towards the solution, we solve for the optimal value of q^C for a given $(I(Q,\alpha),\alpha,\gamma)$, and then substitute the result back into the objective function, and solve for the optimal γ^C for a given $(I(Q,\alpha),\alpha,q^C)$. Solving $\partial\Omega_P/\partial q = 0$ leads to $q^C = I$. Clearly, making $q^C = I$ can maximize the supply of leftovers available for sale in the PSP. The only impact of making $q^C < I$ is to reduce the supply of goods available for sale in the PSP, tightening a constraint cannot produce a better solution. Substituting $q^C = I$ into the objective function

in (1), and solving $\partial\Omega_P(q^C = I, \gamma)/\partial\gamma = 0$ give the optimal markdown price ratio $\gamma^C(q^C = I)$, satisfying the following expression. Note that since $q^C = I$, $\xi_P := I(Q,\alpha)/D_P(\gamma,\alpha)$.

$$D_P(\gamma,\alpha) + \gamma\frac{dD_P(\gamma,\alpha)}{d\gamma} = \frac{-I(Q,\alpha)\overline{G}(\xi_P)}{\int_0^{\xi_P} xdG}. \tag{2}$$

The RHS in (2) is negative, and the absolute value decreases in γ and approaches 0 when γ approaches 1. Since $d^2(\gamma D_P)/d\gamma^2 < 0$, the LHS changes its sign from positive to negative and continuously decreases thereafter. The two properties give a unique optimal markdown price ratio γ^C that is greater than the risk-less price ratio satisfying $D_P + \gamma\, dD_P/d\gamma = 0$. Petruzzi and Dada (1999) state that the optimal price depends on the nature of the uncertainty: additive uncertainty leads to an optimal price that is less than the risk-less price, and multiplicative uncertainty leads to an optimal price that is more than the risk-less price. This property also applies to our study. At the end of this section, we will show that the markdown price of the additive demand model is smaller than that in the deterministic case. Substituting $q^C = I$ into the objective function, the necessary conditions for (Q,α) that maximize (1) satisfy

$$\partial\Pi_{NP}^C/\partial Q = 0 \Rightarrow \overline{F}(\xi_N) - \frac{C}{P} + \int_0^{\xi_N} \gamma\overline{G}(\xi_P)dF = 0 \text{, and} \tag{3}$$

$\partial\Pi_{NP}^C/\partial\alpha = 0 \Rightarrow \quad \partial\Omega_N/\partial\alpha + \partial E_y\{\Omega_P\}/\partial\alpha = 0$, where

$$\frac{\partial\Omega_N}{\partial\alpha} = \frac{dD_N}{d\alpha}\int_0^{\xi_N} ydF \geq 0 \text{ , and } \frac{\partial E_y\{\Omega_P\}}{\partial\alpha} = \frac{\partial D_P}{\partial\alpha}\int_0^{\xi_N}\int_0^{\xi_P} \gamma xdGdF - \frac{dD_N}{d\alpha}\int_0^{\xi_N} \gamma\overline{G}(\xi_P)ydF \leq 0 \tag{4}$$

We see that increasing α by one unit will increase the retailer's objective function by $\partial\Omega_N/\partial\alpha$ but will reduce the DCO's profit by $-\partial E_y\{\Omega_P\}/\partial\alpha$. Clearly, the supply chain has a tradeoff problem at hand. Let $\Omega(1) := \partial\Omega_P/\partial Q = 0$ and $\Omega(2) := \partial\Omega_N/\partial Q = 0$. The optimal order quantity derived from (3) show that $\partial Q^*(\alpha)/\partial\alpha|_{\Omega(1)} < 0$ and $\partial Q^*(\alpha)/\partial\alpha|_{\Omega(2)} > 0$. This tells us that optimization from the view of the DCO requires the retailer to reduce order quantity so as to generate fewer leftovers when the normal sales period is extended ($\partial Q/\partial\alpha|_{\Omega(1)} < 0$). However, the optimization from the retailer's viewpoint

reveals a totally different result ($\partial Q/\partial \alpha|_{\Omega(2)} > 0$). Therefore, a conflict of interest exists between the retailer and the DCO. Proposition 1 shows the properties of the optimal solution.

Proposition 1. (Proof. See Appendix 1.)

1.1 *For models A, B, and C the objective function* Π_{NP}^{C} *is jointly concave with respect to* (Q,γ,α) *if* $g(x)$ *has increasing or constant failure rates (**IFR**).*

1.2 *Comparative statics: Let* $\Pi(1) := \partial \Pi_{NP}/\partial \alpha = 0$ *and* $\Omega(3) := \partial \Omega_P/\partial \gamma = 0$. *The optimum* γ^* *and* α^* *satisfy (2) and (4) show (a)* $\partial \gamma^*(I,\alpha)/\partial I|_{\Omega(3)} < 0$, *(b)* $\partial \gamma^*(I,\alpha)/\partial \alpha|_{\Omega(3)} < 0$, *and (c)* $\partial \alpha^*(Q)/\partial Q|_{\Pi(1)} < 0$.

□

Proposition 1.1 reveals that the objective function is concave if the density functions have an IFR. While the proposition limits the distribution, IFR class is broad enough to include most of the distribution one would choose to employ. For example, the normal and the exponential are both relatively widely used "long tailed" densities that are quite probable for formulating random demands (see, for example, Parlar and Weng [1997] and Li, Lau, and Lau [1991].) Proposition 1.2 shows that the system gives a greater discount as leftovers increase or the markdown period decreases. Proposition 1.2 also reveals that the supply chain reduces the length of NSP so that the leftovers can be moved to DCO in a more timely fashion as leftovers increase (I increases as Q increases).

Now, let us briefly discuss the result obtained from an additive random demand model. Recall that the necessary condition in (2) is obtained from solving for the optimal value of $q^C = I$ for a given γ first, and then substituting the result back into the objective function, and solving for the optimal γ. This approach was employed in Whitin (1995). Here, we will use a different approach in which we solve for the optimal γ for a given q first, and then solve for the optimal q (see Zabel [1970]). The random demand in the additive model is formulated as $y + D_N(\alpha)$ with $E(Y) = 0$ (we assume $Y \in [B, A]$ is well defined so that $y + D_N(\alpha)$ does not become negative). Similarly, we assume random demand to be $x + D_P(\alpha,\gamma)$, $X \in [D, C]$ with $E(X) = 0$. Define $\xi_N := Q - D_N(\alpha)$, and $\xi_P := q - D_P(\alpha,\gamma)$. The DCO's expected profit is $\Omega_P = P\gamma\left\{\int_D^{\xi_P}(D_P + x)dG + \int_{\xi_P}^{C}(\xi_P + D_P)dG\right\}$. Solving $\partial \Omega_P/\partial \gamma = 0$ reveals γ^C,

satisfying $D_P + \gamma D_P{}' = \int_{\xi_P}^{C} (x - \xi_P) dG$. The right-hand side is positive; thus, γ^C is smaller than the risk-less price ratio satisfying $D_P + \gamma D_P{}' = 0$ (notice that, in the multiplicative model, γ^C is greater than the risk-less price ratio). Substituting the optimal price reveals $\partial \Omega_P / \partial q = \gamma \overline{G}(\xi_P) \geq 0$, and this leads to $q^* \to \infty \Rightarrow q^C = I$. The optimal solution for the order quantity Q^C is identical with that in (3). Solving $\partial \Pi^C / \partial \alpha = 0$ results in α^C, satisfying $D_N{}' \int_B^{\xi_N} dF + \int_B^{\xi_N} \gamma \left\{ D_P{}' \int_D^{\xi_P} dG - D_N{}' \int_{\xi_P}^{C} dG \right\} dF = 0$.

3.2 The Contract Models for the Decentralized Supply Chain

In this section we model a decentralized system as a Stackelberg Game in which the DCO, acting as a follower, chooses a markdown sale quantity $q \leq I$, pays C_r for each leftover item, and designs a markdown price γ. The retailer, on the other hand, acting as a channel leader, designs (Q, α, C_r) to maximize her individual objective function. As in Section 3.1, let Ω_N^{UC} (Ω_P^{UC}) denote the retailer's (DCO's) expected profit excluding the wholesale revenue (cost) of leftovers in the decentralized supply chain. Also denote $\Pi_N^{UC}(Q, \alpha | q, \gamma) := \Omega_N^{UC}(Q, \alpha) + E_Y \{\max_{C_r} C_r \times q\}$ as the retailer's expected profit (including the expected wholesale revenue of leftovers). The retailer's problem is $\max_{Q,\alpha} \Pi_N^{UC}(Q, \alpha | q, \gamma)$, and the DCO's optimization problem is $\max_{q,\gamma} \{\Omega_P^{UC}(\gamma, q | I, \alpha) - C_r \times q\}$. In addition to the notations given in Table 1, Table 2 lists additional notations used in the paper.

Table 2. Notations

$\tilde{g}(x), \tilde{G}(x)$	probability density and cumulative distribution of $\tilde{X}$
$\chi := (x \text{ or } \tilde{x})$	
$\varphi := (g \text{ or } \tilde{g})$, $\Gamma := (G \text{ or } \tilde{G})$, and $\rho(\_) := \varphi(\_)/\overline{\Gamma}(\_)$	
$\xi_L(Q, \overline{q}, \alpha) = (Q - \overline{q})/D_N$	(see equation (6) for the definition of $\overline{q}$)
$\xi_{P1} = \overline{q}/D_P(\gamma_1, \alpha)$, and $\xi_{P2} = I/D_P(\gamma_2, \alpha)$	

Two scenarios regarding information sharing are assumed. (1) Scenario 1: The retailer does not share the market signal Z with the DCO, and the DCO has no way to know market signal Z; thus, the DCO's decisions are made based on $\tilde{g}(\tilde{x}), \tilde{X}$, and (2) Scenario 2: the retailer truthfully shares the market signal Z with the DCO, or the DCO has a way to obtain information regarding Z; hence, the DCO's decisions

are made based on $g(x), X$. The retailer's optimal wholesale price satisfies:

$$C_r^{UC} = \begin{cases} C_{r1} \quad satisfies \quad \bar{q} + C_r \, \partial\bar{q}/\partial C_r = 0 & if \quad \bar{q}(C_{r1}) \le I \\ C_{r2} = P\gamma\bar{\Gamma}(\xi_{P2}) & if \quad \bar{q}(C_{r1}) > I \end{cases}. \tag{5}$$

The (q, γ) maximizing the DCO's objective function are given by the following expression:

$$q^{UC} = \begin{cases} \bar{q}(C_{r1}) & if \quad \bar{q}(C_{r1}) = D_P(\gamma_1, \alpha)\Gamma^{-1}(1 - C_{r1}/P\gamma_1) \le I \\ I & if \quad \bar{q}(C_{r1}) > I \end{cases}, \text{ and}$$

$$\gamma^{UC} = \gamma_A \quad A\text{=1,2 satisfies} \quad D_P(\gamma, \alpha) + \gamma \frac{dD_P}{d\gamma} = \frac{-q^{UC}\bar{\Gamma}(\xi_{PA})}{\int_0^{\xi_{PA}} \chi d\Gamma}. \tag{6}$$

Upon substituting $C_{r2} = P\gamma_2\bar{\Gamma}(\xi_{P2})$, the optimal order quantity Q^{UC} satisfies:

$$\bar{F}(\xi_N) - C/P + \int_{\xi_L}^{\xi_N} \gamma_2 \bar{\Gamma}(\xi_{P2})(1 - \rho(\xi_{P2})\xi_{P2}) dF = 0, \tag{7}$$

and α^{UC} satisfies:

$$\int_0^{\xi_N} y \frac{dD_N}{d\alpha} dF + \int_{\xi_L}^{\xi_N} \gamma_2 \left\{ \varphi(\xi_{P2})(\xi_{P2})^2 \frac{\partial D_P}{\partial \alpha} - y\bar{\Gamma}(\xi_{P2})(1 - \rho(\xi_{P2})\xi_{P2}) \frac{dD_N}{d\alpha} \right\} dF + \int_0^{\xi_L} \xi_{P1}\gamma_1\bar{\Gamma}(\xi_{P1}) \frac{\partial D_P}{\partial \alpha} dF. \tag{8}$$

Proposition 2. (See Appendix 2 for the Proof of Proposition 2.)

2.1 *For both models A and B,* $\Pi_N^{UC}(Q, \alpha, C_r^{UC}(Q, \alpha))$ *is concave in* (Q, α) *if* $\varphi(x)$ *has an IFR.* $\Pi_P^{UC}(q, \gamma)$ *is concave in* (q, γ).

2.2 *Comparative statics: Let* $\Pi(1) := \partial\Pi_N^{UC}/\partial\alpha = 0$ *and* $\Omega(4) := \partial\Omega_r^{UC}/\partial\gamma = 0$. *The optimum* γ^* *and* α^* *satisfy (6) and (8) show (a)* $\partial\gamma^*(I)/\partial I|_{\Omega(4)} < 0$, *(b)* $\partial\gamma^*(\alpha)/\partial\alpha|_{\Omega(4)} < 0$, *and (c)* $\partial\alpha^*(Q)/\partial Q|_{\Pi(1)} < 0$.

2.3 *System Inefficiency (Double Marginalization): Under scenario 2 (* $\varphi(x) = g(x)$ *), if* $g(x)$ *has an increasing failure rate (IFR)*

(a) Given $(\alpha^C, q^C) = (\alpha^{UC}, q^{UC}), Q^{UC} \le Q^C$.

(b) Given $(Q^C, q^C) = (Q^{UC}, q^{UC}), \quad \alpha^{UC} \ge \alpha^C$.

(c) *Given* $(\alpha^C, Q^C) = (\alpha^{UC}, Q^{UC}), \quad q^C = I \ge q^{UC} \Rightarrow \gamma^{UC} \ge \gamma^C$. □

Proposition 2.3 reveals three sources of system distortions (Double Marginalization). In general, these distortions stem from making decisions based on "local costs or revenues" rather than on "system costs or revenues".

4. Discussion and Conclusion

In this paper, we analyzed a model to study the effects of retailer-DCO coordination in supply chain stocking and promotional markdown operations. Our purpose was to develop an understanding of how, when, and why coordination helps to improve expected profits.

We first developed an integrated system as a benchmark case in which the retailer-DCO alliance jointly decides the stocking quantity, a plan for markdown time schedule, and markdown price to maximize mutual profit. Next, we considered a decentralized system in which the DCO, acting as a follower, individually optimizes her objective function by choosing a markdown sale quantity and a markdown price. The retailer, on the other hand, acting as a channel leader, designs order quantity, markdown time schedule, and wholesale price of leftovers to maximize her individual objective function. Three sources of system inefficiencies cause the decentralized system to generate a lower expected profit than that for the integrated system. These are as follows: in the decentralized system the retailer tends to (1) stock less, and (2) keep a longer sales period, and the DCO tends to (3) stock lesser inventories and charge a higher markdown price. A two-part tariff coordination contract (see Appendix 3) is then designed to modify the terms of trade so as to eliminate these inefficiencies and to improve system performance; thus, an arbitrary split program of net profit increases can be structured to induce both the retailer and the DCO to indifference.

A numerical experiment is provided to compare centralized model and decentralized model. Our study indicates that the coordination approach outperforms the uncoordinated approach on every occasion, but the benefits are most significant when the market signal Z and price P are relatively high, or the production cost C is relatively small. We also see that the coordination model generates higher profits when normal sale demand rates k_N are relatively high or relatively low (demands are either extremely sensitive or extremely insensitive to the length of NSP).

We have observed several distinctive decision-making patterns that might contribute to the sub-

optimality of the uncoordinated model. First, in the uncoordinated model, the two parties frequently show disconnected decision-making patterns. For example, when P increases, the retailer is overly aggressive while the DCO is overly pessimistic in their stocking policies. As a result the supply chain generates a large quantity of wasted leftovers. Second, we have observed that in the decentralized model, the retailer tends to hold the inventories for an excessive period of time. In the coordination model, the retailer reduces her sales period so that the leftovers can be moved to the DCO earlier to take advantage of a more time-elastic market. Finally, in the uncoordinated model the DCO stocks less inventory and charges higher prices for the leftover items, thereby resulting in a lower system profit.

Our focus thus far does not allow us to study the possibility of a situation involving multiple markdown periods. Generally, in a real-world application a markdown operation may consist of more than one discount period. The multiple markdown periods problem has been studied by Khouja (1995) on a single company level, but in this model demand in the markdown period is treated independently of markdown rate and markdown timing. Future work on the two-party progressive promotional markdown model could certainly shed further light on the topic. Finally, the problem of jointly deciding normal sales price and markdown discount rates also should receive future research effort.

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