Forecasting Volatility on the Taiwan Futures Market: An Evaluation of Linear and Non-Linear Models

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Abstract

This paper examines the performance of various statistical models to forecast the volatility of daily stock index futures, including TAIEX futures (TX), TSE Electronic Sector Index futures (TE) and TSE Banking Insurance Sector Index Futures (TF) in the Taiwan futures market. The forecasting models chosen for this study are conditionally Gaussian ARCH-class models, including both linear (GARCH and IGARCH) and non-linear (GARCH-M, EGARCH, GJR-GARCH and APARCH) GARCH processes. The models are also augmented by adding lagged volume or open interest to investigate whether there is a relationship between volatility and either volume or open interest. We find that, out of all the linear and non-linear GARCH models examined, the models that perform best in forecasting volatility on the Taiwan futures market are non-linear GARCH models that incorporate market volume information as additional explanatory variables within the volatility forecasting equation.

1. Introduction

In recent years, a great deal of interest in econometrics and empirical finance has recently centered on modeling the temporal variation in financial market volatility. Volatility estimation and forecasting is an important topic for policy makers and financial market participants because volatility is often perceived as a measure of risk, playing a crucial role in many different areas of finance, such as risk management, time series forecasting and the pricing of derivative securities. The volatility of underlying asset prices enters directly into the Black-Scholes formula for deriving the value of traded options.

Volatility estimates implied from option pricing models have frequently been used in conjunction with trading strategies to examine whether arbitrage profits were possible. However, estimating the return volatility of underlying asset is the most difficult and controversial aspect of option valuation. One approach for pricing equity options is to assume that the volatility is constant and use historical prices to calculate the variance of the continuously compounded returns. This forecast is referred to the traditional forecast or estimator. The primary disadvantage with the traditional estimator is that the volatility of returns may change over time, which in turn affects the accuracy of the option valuation.

Chu and Freund [17] compare implied standard deviation (ISD) using option price and GARCH and IGARCH models using underlying asset price to estimate option price and find that the ISD method is best. Similarly, Poon and Granger [50], find that the ISD tends to outperform the volatility forecasts provided by time-series models. However, among time-series models, they find that neither historical volatility nor ARCH models dominate the other in terms of forecasting future volatility, although both of these types of forecasts appear to be better than those provided by stochastic volatility models.

One of the key areas where volatility forecasts are employed is the futures markets. These markets fulfill two social functions. One function is price discovery, which is the revealing of information about future cash market prices. The other is hedging, which is the prime rationale for futures trading. Hedgers are exposed to a preexisting risk of some form that leads them to use futures transactions as a substitute for a cash market transaction. However, arbitrage plays a crucial role for pricing index futures contracts. The spot and futures market prices are linked by arbitrage; i.e., participants liquidate positions in one market and take comparable positions at better prices in another market, or choose to acquire positions in the market with the most favorable prices. Futures index markets typically offer the investor the opportunity to trade at a substantially lower cost and higher liquidity than trading directly on the spot market. Therefore, one would expect futures markets to exhibit more instantaneous reactions to new information than the equity markets. Because volatility measures the magnitude of price movement in a series, it is an appropriate variable for examining the length of time required for the markets to fully incorporate new information. Consequently, volatility can be considered a measure of information flow in derivative instruments (see Ross [54]) and has always been an essential tool for trading strategies. Thus, forecasting volatility has important implications for investors.

Within the futures markets, stock index futures and options on stock index futures are especially important areas of research. These financial instruments have very high trading volumes due to hedging, speculative trading, and arbitrage activities. Insight into the behavior of futures price volatility can have important implications for investors using stock index futures contracts, such as a portfolio manager implementing put-replication portfolio insurance during a period of high market volatility. In calculating the optimal hedge ratio to use in implementing such a strategy, the portfolio manager in this situation would be faced with the issue of whether to use a volatility forecast based on the high level of volatility within the recent past or, alternatively, some other forecast of volatility based on conditioning information appropriate for the insurance horizon.

Related to this issue, Hill et al. [33] show that unexpected changes in volatility are the most important risk factor in determining the cost of portfolio insurance. Similarly, Chu and Bubnys [16] use a likelihood ratio test to compare the variance measure of price volatilities of stock market indices and their corresponding futures contracts during the bull market of the 1980s, and find that spot market volatilities are significantly lower than their respective futures price volatilities. Bera et al. [4] investigate the effectiveness of using conventional OLS estimates of volatility to determine the optimal hedge ratios and find that, compared to ARCH-based hedge ratios, the hedge ratios based on conventional OLS estimates may cause investors to sell short either too many or too few futures contracts.

Most studies of volatility forecasting have noted the empirical observation that volatility in financial time series is highly persistent with clearly demonstrated volatility clustering behavior, and numerous models have been proposed to describe the evolution of volatility over time. On these models, the Autoregressive Conditional Heteroscedasticity (ARCH) and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) processes developed by Engle [21] and Bollerslev [5], respectively, appear to be appropriate models for the daily returns of many financial time series. The GARCH(1,1) and Exponential GARCH (EGARCH) models are two of the most successful such parameterizations for characterizing high-frequency financial market volatility. A common finding in many empirical applications of each of these models concerns the apparent persistence of the estimated conditional variance process (see Bollerslev et al. [7]).

There is a great deal of literature on forecasting volatility, in which many econometric models are compared, yet no single model is shown to be superior. As an example, Akgiray (1989) and Pagan and Schwert (1989) find, using US stock and futures data, that the GARCH(1,1) models outperform most competitors, while Najand (2002) compares linear models with nonlinear models for forecasting the price volatility on S&P 500 Index futures and finds EGARCH to be the best model for forecasting price volatility for stock index futures. However, using data sets from the Japanese and Singaporean markets, respectively, Tse [58] and Tse and Tung [59] find that exponentially weighted moving average models provide more accurate forecasts than GARCH models. Braiisford and Faff [8], on the other hand, find that GARCH models are slightly superior to most simple models for forecasting Australian monthly stock index volatility, and Frances and van Dijk [27] find that the non-linear GARCH models perform no better than the standard GARCH model in forecasting the weekly volatility of various European stock markets. Gokcan [30] finds that, for emerging stock markets, the basic GARCH(1,1) model performs better than EGARCH models, while. Wei [61] presents QGARCH as a better model than either the basic GARCH or the GJR-GARCH model for forecasting the weekly volatility on the China stock market.

Several theories predict a positive relationship between return volatility and trading volume, and, for futures markets, open interest is considered as another important variable. Karpoff [38] argues that the absolute price and volume measures should exhibit positive contemporaneous correlation between each other and reviews eighteen studies examining evidence with regard to this relationship. Tan and Gannon [56] find that, apart from the return-volume relationship, the interrelationships between return, volatility and volume upon information arrival are generally consistent with what theory would anticipate. Chan and Chung [13] argue that mis-pricing produces both subsequent volatility and trading volume, whereas Chen et al. [15] find that the magnitude of mis-pricing is inversely related to volatility in the pricing of US stock index futures. Ferris et al. [26] suggest that the level of open interest is a good proxy for assessing the amount capital flows into and out of the nearest S&P 500 Index futures contracts, and consequently provides information about pricing error shocks. Ragunathan and Peker [51] provide evidence that positive volume shocks have a greater impact on volatility than negative shocks and reach a similar conclusion regarding open interest. Therefore, market depth (reflected in trading volume, open interest, etc.) does appear to have an effect on volatility. Watanabe [60] also presents evidence of a significant positive relationship between volatility and unexpected volume, as well as a significant negative relationship between volatility and expected open interest, but finds that the relationship between price volatility, volume and open interest may vary depending on the market's regulatory structure. Epps [25] proposes a model in which volume tends to be higher when stock prices are rising than falling, although there is no strong reason why the relationship should be contemporaneous rather than lead-lag, since volume may react more quickly to changes in the directing variable than volatility, or vice versa. If market volume, which is used as an exogenous right-hand side variable in the variance equation of the GARCH model, is part of a larger system of equations where volume is itself partly determined by volatility, then failure to appropriately model the system as such will cause a simultaneity bias in the coefficient estimates. One potential solution to this problem is to use lagged measures of volume, which will be predetermined and therefore not subject to the simultaneity problem. Although Lamoureux and Lastrapes [40] find lagged volume to be a poor instrument for forecasting volatility, Najand and Yung [45] find it to be quite acceptable in an analysis of price variability in Chicago Board of Trade futures data. Finally, Brooks [9] presents many models (GARCH, EGARCH, GJR-GARCH, and so on.), which are augmented by the addition of a measure of lagged volume to form more general ex-ante forecasting models, but finds only very modest improvements, if any, in forecasting performance.

In the present study, we focus on Taiwan stock market futures traded on the Taiwan Futures Exchange (TAIFEX). The total trading volume was 7,944,254 and 4,351,390 contracts for the years 2002 and 2001, respectively, which represents a growth rate of 312.31%. This comes on top of a growth rate of 125.84% relative to the 1,926,789 contracts traded in 2000. In terms of trading volume (excluding options), although the TAIFEX is ranked 29th among exchanges worldwide in 2002, taking third place in terms of the growth rate, investors in the emerging Taiwan market suffer from a lack of information as well as reliability. However, Brooks [9] suggests the existence of an informational relationship between volatility and volume or open interest. Open interest represents the number of futures contracts outstanding at any point in time, whereas trading volume captures the number of contracts traded during a specific time period. Open interest supplements the information provided by the trading volume. In this study, all ARCH-class model variance equations are augmented through the addition of lags of market volume or open interest as predictor variables. We then compare, for returns on the TAIEX futures (TX), the TSE Electronic Sector Index futures (TE), and the TSE Banking Insurance Sector Index Futures (TF), respectively, the volatility-forecasting ability of each of the ARCH-class models, both with and without market volume and open interest included in the variance forecasting equestions.

The purpose of this research is to examine the performance of the various models in forecasting daily stock index futures volatility and to see whether volume and open interest data help to improve the accuracy of the forecasting models. The paper is structured as follows. Section 2 describes the data and sample period used. Section 3 provides the methodology and explains the various models and their formulation. The results of In-sample estimation and out-of-sample forecasting, both with and without lagged volume and open interest, are given in Section 4, and the conclusions are drawn in the final section.

2. Data and Descriptive Statistics

The data used in this study include data from the Taiwan Stock Exchange (TSEC)[1] and from the Taiwan Futures Exchange (TAIFEX). Trading on the TAIFEX started in July 21, 1998, with the introduction of TSEC Capitalization Weighted Stock Index (TAIEX) futures (TX), a futures contract on the TAIEX[2] stock index. Subsequently, the TAIFEX issued three additional futures products: the TSE Electronic Sector Index Futures (TE), the TSE Banking Insurance Sector Index Futures (TF) and Mini-TAIEX Futures. TE and TF began to be traded in July 21, 1999. We choose only TX, TE, and TF to forecast return volatility, because TAIEX futures (TX) and Mini-TAIEX Futures have the same underlying assets. Of the five different contract maturities for each kind of futures, we choose the nearest month's contract, up until the beginning of the month of delivery, when the second nearest contract to expiration begins to be used in order to avoids idiosyncrasies that may be specific to the futures markets during the month of delivery..

The data used are the daily closing prices, trading volumes, and open interests for the TX, TE, and TF contracts traded on the TAIFEX. We analyze TX data for the period from July 21, 1998, to December 31, 2002, and the data for TE and TF from July 21, 1999, to December 31, 2002. Daily closing prices of the TX used for the in-sample data cover the period from July 21, 1998, to June 28, 2002, yielding a total of 1023 observations. For TE and TF, a total of 754 observations are included within-the sample period.

The daily return series are calculated as the first differences of the logarithms of the daily closing futures prices:

$$r_t = \ln(P_t / P_{t-1}) \tag{1}$$

where P_t is the daily closing price at time t for the relevant TX, TE, or TF futures contract, and r_t is its daily rate of return, assuming continuous compounding. The use of the logarithm of price changes reduces the impact of price-level nonstationarity on the estimated return volatility. Following Chan, Christie, and Schultz [12], Day and Lewis [18], West and Cho [62], and Brooks [9], we measure the daily volatility as simply the square of the day's return. As Jorion [37] notes[3],

$$E_{t-1}[r_t^2] = \sigma_t^2 . \tag{2}$$

Before proceeding to the estimation of the various volatility-forecasting models, we will first examine the distributional properties of the various daily returns series. Descriptive statistics for these return series are reported in Table 1.

Table 1: Descriptive statistics for futures returns

Futures Contract	TX	TE	TF
Beginning Date	98/0722	99/07/22	99/07/22
Ending Date	01/12/31	01/12/31	01/12/31
Sample Size	1023	754	754
Mean	−0.04	−0.05	−0.06
Max	6.77	6.77	7.38
Min	−7.26	−7.26	−7.26
Std. Dev.	2.09	2.69	2.47
Skewness	0.07	0.17	0.22
Kurtosis	4.41	3.53	3.93
Bera-Jarque	85.96***	12.80***	33.58***
Qx(6)	9.95	11.35*	13.29**
Qx(12)	13.91	19.60*	17.32
Qx(18)	33.11**	39.70***	22.89
Qxx(6)	163***	143***	128***
Qxx(12)	233***	219***	177***
Qxx(18)	288***	298***	185***

Note: The symbols ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Over the sample period covered, the mean daily returns of all three futures returns series are all close to zero, though slightly negative (−0.04%, −0.05% and −0.06%). The standard deviations range from 2.09% for the TX futures returns to 2.69% for the TE futures. As is typical with financial time series, all three series exhibit excess kurtosis, and, as a consequence, the Bera-Jarque skewness-kurtosis test of normality results in a rejection of normality at a 1% for all three series. Note that these leptokurtic departures from normality come in spite of the tail truncation effects of the 7% daily price limits that exist on the Taiwan financial markets. As shown by Ammermann and Patterson [2], the effects of these daily price limits not only serve to truncate the tails of the distribution of daily returns, leading to relatively low levels of leptokurtosis as compared to other financial markets throughout the world, but, when Taiwan's markets become especially volatile, the price limits also serve to convert this extant daily return volatility into autocorrelation between daily returns.

The autocorrelations within each of the three futures returns series and the evolution of their levels of volatility over time are explored via the next set of descriptive statistics, the Ljung-Box (Qx) test statistic and the McLeod and Li (Qxx) test statistic. These tests examine the joint significance of the autocorrelations among the first 6, 12, and 18 lags of the return and squared return series, respectively, of the futures contracts. As noted by Taylor [57], among others, these two sets of autocorrelation functions can be used to explore the degree of predictability of various moments within financial data. For the three futures return series, the significant Q-statistics suggest at least a moderate degree of predictability among the returns of each of these series, and, as indicated by the much more highly significant Qxx statistics, a high degree of predictability among the squared returns for each of the three series, reflecting the possible presence of nonlinear serial dependencies, such as autoregressive conditional heteroscedasticity (ARCH) effects, various possible formulations of which will be explored in Section 3.

Before proceeding to the description and estimation of such models for forecasting volatility, the stationarity of the explanatory variables with which we plan to augment such models must first be verified. We used the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests to assess the stationarity of the volume for the nearest month's contract for a given future (VOL), total volume across all the contracts for the given future (TVOL), open interest for the nearest month's contract (OP), and total open interest (TOP), and the null hypothesis of a unit root is rejected for all four variables for each of the three sets of futures contracts. Thus, all of the series are indicated to be stationary, apart from the total volume (TVOL) series for the TE and TF futures contracts, which are found to be trend stationary.

3. Models

3.1. Linear GARCH Models

Our basic model for forecasting the mean of the returns series for each of the sets of futures contracts is the AutoRegressive Moving Average (ARMA) model:

$$r_t = a_0 + \sum_{i=1}^{p} a_i r_{t-i} + \sum_{j=1}^{q} b_j \varepsilon_{t-j} + \varepsilon_t \ , \qquad (3)$$

where r_t is the return on each contract at time t, and the a_0, a_i, and b_j are constant parameters. For each set of futures contracts, we use the likelihood ratio (LR) tests to select the best-fitting autoregressive (AR), moving average (MA) or ARMA model. If $L(\theta_n)$ and $L(\theta_a)$ are the maximum log-likelihood function values under the null and the alternative hypotheses, respectively, then the statistic $-2[L(\theta_n) - L(\theta_a)]$ will be asymptotically χ^2 distributed with the number of degrees of freedom equaling the difference in the number of parameters under the null and the alternative hypotheses. To forecast the volatility of the returns series for each of the three sets of futures contracts, we consider the following linear and non-linear GARCH(p,q) model specifications and, following Akgiray [1], again use the LR test approach to determine the appropriate orders for p and q.

3.1.1. GARCH Model

Most studies that examine nonlinearity and the time-varying volatility of stock market returns find that GARCH models perform well in explaining and modeling volatility, as well as in capturing the extant nonlinearity within the return series (see, e.g., Bollerslev et al. [7] and Hsieh [34] and [35]). This suggests that the GARCH model would also be useful for predicting volatility. In terms of structure, GARCH models entail the joint estimation of equations for the conditional mean and the conditional variance. Specifically, the linear GARCH model can be formulated as:

$$\begin{aligned} r_t &= \phi_0 + \phi_1 r_{t-1} + \phi_2 r_{t-2} + \varepsilon_t \\ \varepsilon_t \left| \Psi_{t-1} \right. &\sim N(0, h_t) \\ h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1} \end{aligned} \quad , \tag{4}$$

where Ψ_{t-1} denotes all available information at time t–1, α_0, α_1, and β are constant, non-negative parameters, with $\alpha_1 + \beta < 1$. These restrictions on the parameter space prevent negative variances (see Bollerslev [5]). Among all the different linear GARCH models, the GARCH(1,1) depicted above has been found to be the most popular. (see, e.g., Bollerslev et al. [7])

3.1.2. IGARCH Model

In many high-frequency time-series applications, the conditional variance estimated using a GARCH(p,q) process exhibits a strong persistence, that is:

$$\sum_{j=1}^{p} \beta_j + \sum_{i=1}^{q} \alpha_i \approx 1 \tag{5}$$

So long as $\sum_{j=1}^{p} \beta_j + \sum_{i=1}^{q} \alpha_i < 1$, the series ε_t is second-order stationary, and a shock to the conditional variance σ_t^2 has a decaying impact on $\sigma_{t+h}{}^2$ as h increases and is asymptotically negligible. However, if $\sum_{j=1}^{p} \beta_j + \sum_{i=1}^{q} \alpha_i \geq 1$, the effect on $\sigma_{t+h}{}^2$ does not die out asymptotically. This property is called persistence in the literature. When the GARCH parameters sum to one, we are confronted with an Integrated GARCH (IGARCH) process (see Engle and Bollerslev [22]), in which case any shocks to the conditional variance persist indefinitely, meaning that current information remains of importance when forecasting the volatility for all horizons.

The IGARCH model is estimated as a constrained GARCH model, where the GARCH polynomial is constrained to equal one:

$$\sum_{j=1}^{p} \beta_j + \sum_{i=1}^{q} \alpha_i \equiv 1, \tag{6}$$

or:

$$\left[1 - \alpha(L) - \beta(L)\right] \equiv 0, \tag{7}$$

where L is the lag operator. As a reflection of the persistence of volatility shocks to such a process, the IGARCH model is strictly but not weakly stationary.

3.1.3. FIGARCH Model

Occupying a middle ground between the GARCH and IGARCH models is the Fractionally Integrated GARCH (FIGARCH) model of Baillie et al. [3]. Under the FIGARCH(1,d,1) model, the evolution of the variance of the process can be described through the following relationship:

$$\Delta^d \varepsilon_t^2 = \alpha_0 + (\alpha_1 + \beta_1)\Delta^d \varepsilon_{t-1}^2 + \nu_t - \beta_1 \nu_{t-1}, \tag{8}$$

where $\nu_t = \varepsilon_t^2 - h_t$. If $d = 1$, we have an IGARCH process, while $d = 0$ yields the basic GARCH process. Under FIGARCH, $0 < d < 1$. As a consequence, like IGARCH, a

FIGARCH process is strictly but not weakly stationary, but, unlike IGARCH, volatility shocks to a FIGARCH process are not permanent, but instead decay at a hyperbolic rate.

3.2. Non-Linear GARCH Models

In addition to the volatility clustering that is described by the linear GARCH models, a number of researchers have also found asymmetry in financial time series, such that negative return shocks seem to increase volatility more than positive return shocks of the same magnitude (see Bollerslev et al. [7], Engle and Ng [23], and Pagan and Schwert [48]). Despite the success of the linear GARCH models, they cannot capture the asymmetry and skewness of the financial time series. This is the advantage of the non-linear GARCH models, which include the GARCH-in-Mean (GARCH-M) model, the Exponential GARCH (EGARCH) model, the GJR-GARCH model and the Asymmetric Power ARCH (APARCH).model.

3.2.1. GARCH-M Model

One of the earlier extensions of the GARCH model, developed by Engle et al. [24], allows the conditional variance to enter the mean equation, thereby allowing a proxy for risk to directly influence the expected return. The GARCH-in-Mean (GARCH-M) model specification thus includes the conditional variance, from the variance equation, as an added regressor variable within the return equation. A typical specification, the GARCH-M (1,1), can be written as:

$$\begin{aligned} r_t &= \phi_0 + \phi_1 r_{t-1} + \phi_2 r_{t-2} + \delta\, h_t + \varepsilon_t \\ \varepsilon_t \mid \Psi_{t-1} &\sim N(0, h_t) \\ h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1} \end{aligned} \quad , \qquad (9)$$

where Ψ_{t-1} denotes all available information at time t–1 and δ provides a measure of investor risk aversion. The model indicates that a large conditional variance tends to be followed by both another large conditional variance and a higher expected return.

3.2.2. EGARCH Model

An alternative form of non-linear GARCH model is the Exponential GARCH (EGARCH) model, which was developed by Nelson [46]. A description of a typical EGARCH(1,1) model specification is as follows:

$$\begin{aligned} r_t &= \phi_0 + \phi_1 r_{t-1} + \phi_2 r_{t-2} + \varepsilon_t \\ \varepsilon_t \mid \Psi_{t-1} &\sim N(0, h_t) \\ h_t &= e^{\alpha_0 + \alpha_1 \left[\left| \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right| - \sqrt{\frac{2}{\pi}} \right] + \beta \ln(h_{t-1}) + \gamma \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}} \end{aligned} \quad , \qquad (10)$$

where α_0, α_1, β, and γ are constant parameters. The EGARCH specification holds many advantages over the linear GARCH specifications. For example, due to the exponential form of its variance equation (which ensures that h_t is always positive), the EGARCH specification does not require any additional restrictions on the parameter space to ensure non-negativity of the conditional variances, which is in contrast to the linear GARCH model specifications,

Moreover, Eq. (10) is able to incorporate volatility asymmetry through the direct inclusion in the model of the term, ε_{t-1}, normalized by the standard derivation of the data. Consequently, the EGARCH model allows good news (positive return shocks, such that $\varepsilon_{t-1} > 0$) and bad news (negative return shocks, such that $\varepsilon_{t-1} < 0$) to have asymmetrical impacts on volatility, while the linear GARCH model does not (see Engle and Ng [23]). The degree and direction of such asymmetry is a function of the parameter, γ. If $\gamma = 0$, then a positive return shock has the same effect on volatility as a negative return shock of the same magnitude. If $\gamma < 0$, a negative return shock will have a greater impact on future volatility while a positive return shock will actually have an ameliorating effect (which is consistent with the findings of the research on volatility asymmetry), while if $\gamma > 0$, these impacts would be reversed.

3.2.3. GJR-GARCH Model

The GJR-GARCH model was proposed by Glosten et al. [29]. For this model, the conditional mean and conditional variance equation specifications are as follows:

$$\begin{aligned} r_t &= \phi_0 + \phi_1 r_{t-1} + \phi_2 r_{t-2} + \varepsilon_t \\ \varepsilon_t \mid \Psi_{t-1} &\sim N(0, h_t) \\ h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta\, h_{t-1} + \gamma\, S_t^- \varepsilon_{t-1}^2 \end{aligned} , \qquad (11)$$

where α_0, α_1, β, and γ are constant parameters. In Eq. (11) the asymmetry arises from the inclusion of a "negative-return-shock" dummy variable, S_t^-, which takes a value of one whenever $\varepsilon_{t-1} \leq 0$ and zero otherwise. Thus, the GJR-GARCH model specification assumes that negative return shocks will have a greater impact on volatility.

3.2.4. APARCH Model

The Asymmetric Power ARCH, or APARCH, model was introduced by Ding, Granger, and Engle [20]. The general APARCH (p,q) model specification can be expressed as:

$$\begin{aligned} r_t &= \phi_0 + \phi_1 r_{t-1} + \phi_2 r_{t-2} + \varepsilon_t \\ \varepsilon_t \mid \Psi_{t-1} &\sim N(0, \sigma_t) \\ \sigma_t^\delta &= \omega + \sum_{i=1}^{q} \alpha_i \left(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i} \right)^\delta + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^\delta \end{aligned} , \qquad (12)$$

where $\delta > 0$ and $-1 < \gamma_i < 1$ ($i = 1, \ldots, q$).

This model couples the flexibility of a varying volatility exponent, δ, together with the existence of a set of asymmetry coefficients, the set $\gamma_i < 1$ ($i = 1, \ldots, q$), which take the "leverage effect" into account. The flexibility of the APARCH model allows it to comprise seven alternative ARCH model specifications as special cases, including:

(1) The ARCH model of Engle [21], when $\delta = 2$, $\gamma_i = 0$ ($i = 1, \ldots, q$) and $\beta_j = 0$ ($j = 1, \ldots, p$),

(2) The GARCH model of Bollerslev [5], when $\delta = 2$, and $\gamma_i = 0$ ($i = 1, \ldots, q$),

(3) The conditional standard deviation model of Taylor [56] and Schwert [54], when $\delta = 1$ and $\gamma_i = 0$ ($i = 1, \ldots, q$),

(4) The GJR-GARCH model (Glosten, et al. [29]), when $\delta = 2$,

(5) The TARCH model of Zakoian [62], when $\delta = 1$,

(6) The NARCH model of Higgins and Bera [32], when $\gamma_i = 0$ ($i = 1, \ldots, q$) and $\beta_j = 0$ ($j = 1, \ldots, p$), and

(7) The Log-ARCH model of Geweke [28] and Pentula [49], when $\delta \to 0$.

3.3. Forecasting Models with Lagged Volume or Open Interest

In order to improve the prediction ability of the GARCH models beyond what their basic univariate specifications would allow, we add lagged values of market volume and open interest as predictor variables to the volatility equations of both the linear and non-linear GARCH models. Because one week is made up of five trading days, we consider up to five lags of each of these variables. The five lags of market volume or open interest are added to the right-hand side of the variance equations of the ARCH-class models. Thus, the augmented version of the GARCH(1,1) model specification, for example, could be written as:

$$\begin{aligned} & r_t = \phi_0 + \phi_1 r_{t-1} + \phi_2 r_{t-2} + \varepsilon_t \\ & \varepsilon_t \left| \Psi_{t-1} \sim N(0, h_t) \right. \\ & h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1} + \sum_{i=1}^{5} \gamma_i Vol_{t-i} \end{aligned} \quad , \tag{13}$$

where Vol_{t-i} denotes the i[th] lag of the market volume of the nearest month's contract. The other models follow the same method. Then, the final model specification is determined through a sequential process of testing the significance of the γ_is, the coefficients of the lagged volume levels. Specifically, if the coefficient of the Vol_{t-5} (i.e., γ_5) is not significant, then we delete this parameter (Vol_{t-5}) from the model. This process is continued, down from five lags to one, until a lag with a significant coefficient is found. This step helps us both to determine how many lags of market volume are suitable, as well as to understand the relationship between the volatility and market volume. Next, continuing to follow the same procedure, we consider the open interest of the nearest month's contract (OP), the total volume (TVOL), and the total open interest (TOP). This will enable us to better understand the relationship between volatility of the returns on each set of futures contracts and the level of market volume or open interest for each contract.

3.4. Forecasting Evaluation

3.4.1. Regression Test

In order to assess and compare the out-of-sample forecasting abilities of that various GARCH model specifications that are being fit to the three sets of futures contracts returns, we follow an approach used by Diebold and Mariand [19]. This method uses the following model to compare the various possible pairs of forecasts to each other:

$$d_t = c + \varepsilon_t , \tag{14}$$

where $d_t \equiv e_{it} - e_{jt}$ is the loss differential, $e_{it} = \left|\sigma_{it}^2 - \sigma_t^2\right|$ and $e_{jt} = \left|\sigma_{jt}^2 - \sigma_t^2\right|$. σ_t^2 is defined by Eq. (2). σ_{it}^2 and σ_{jt}^2 denote the variances forecasted by the given model i and the model j, respectively. The null hypothesis is $c = 0$, denoting that the coefficient of the constant is not significant, which in turn would suggest that each of the two sets of forecasts is equally close, on average, to the "true" values for the given daily variances for the futures returns. If we reject the null hypothesis, a finding of $c \neq 0$ with $c < 0$ would denote that model i is preferred to model j, while $c \neq 0$ with $c > 0$ would denote that model j is preferred to model i. In conjunction with this regression approach, we use the Newey and West [47] HAC–consistent standard errors to determine the significance levels of the various parameters of interest. We then use this approach to compare the variance forecasts of each of the various GARCH specifications with those of each of the rest of the various GARCH specifications (i.e., every model is compared with every other model). We then used these results to rank all of the models, for each of the three sets of futures contracts, in terms of their out-of-sample forecasting ability.

4. Empirical Results

4.1. In-Sample Estimation

Before estimating the various GARCH model specifications, we first fit an AutoRegressive (AR) model for the means of each of the three sets of futures returns. We use a log-likelihood-ratio test approach to determine the best order of AR model to fit to the data. For example, the χ_1^2 test statistic for comparing the AR(1) and AR(2) model specifications for the TX futures is -2·[(-2201.48) - (-2198.58)] = 5.8, which is significant at a 5% level (critical χ_1^2 value = 3.84). Similar results were obtained for the other series of futures returns, suggesting that the AR(2) model is a suitable formulation for the mean equations for each of the three series of futures returns.

For the variance equations for the various forms of GARCH models, the specification of p = 1 and q = 1 (implying a GARCH(1,1) specification, for example) appears to show the best fit. Other specifications such as GARCH(p,q) for p = 1,…,5 and q = 1,…,5 did not appear to make any significant improvements in goodness-of-fit with LR tests. Similar results were also found for the categories of linear and non-linear GARCH models that were estimated. Thus, although there is little theoretical justification for this, the GARCH(1,1) model specification appears to work quite well in practice as a general-purpose model for capturing the nonlinearity in financial returns.

Table 2 reports the estimates of the primary parameters of interest for each of the categories of GARCH models that are fitted to each of the three sets of futures returns. The parameters whose values are reported include α_0, α_1, and β_1, for the GARCH(1,1) model, the risk-aversion parameter γ for the GARCH-M model, the volatility asymmetry parameter δ for the GJR-GARCH, EGARCH, and APARCH models, as well as the volatility-scaling parameter γ for the APARCH model.

From the table, it is clear that the α_1 and β_1 parameters in the GARCH(1,1) model are typically significant at the 1% level; hence, the constant variance model can readily be rejected, at least within sample. Moreover, the α_1 and β_1 parameters are positive and sum to less than unity for each of the three sets of contracts, so that an IGARCH specification does not seem to be required. (Related to this issue, it is notable that initial attempts to fit FIGARCH models to these sets of returns were only partially successfully; this may be at least partially a consequence of the conversion, by the price limits imposed on the Taiwan markets, of large magnitude return shocks into autocorrelation instead of into persistent volatility, as was described in the discussion following Table 1.)

Table 2: Estimation results for GARCH, GARCH-M, GJR-GARCH, EGARCH and APARCH models fit to the three series of futures returns

Model Parameters	TX	TE	TF
GARCH(1.1)			
α_0	0.13***	0.31***	0.50***
α_1	0.09***	0.09***	0.12***
β_1	0.88***	0.87***	0.80***
GARCH-M			
δ	0.04	0.06*	0.06*
GJR			
γ	0.11***	0.08***	0.06*
EGARCH			
γ	−0.08***	−0.05***	−0.05**
APARCH			
γ	0.57***	0.24**	0.37***
δ	0.40***	1.89***	0.44*

Note: The symbols ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 2 also reports the estimation results for asymmetry parameters γ incorporated into the GJR-GARCH, EGARCH, and APARCH model specifications. Due to differences in the way volatility asymmetry is incorporated into each of the three model specifications, the γ estimates are positive for the GJR-GARCH and APARCH models and negative for the EGARCH models. Nonetheless, all three model specifications lead to significant γ parameter estimates, suggesting the existence of a significant leverage or volatility asymmetry effect during the sample period for all three sets of returns, such that bad news (negative return shocks) would have a greater impact on future volatility than good news.

In addition to fitting the basic specification forms of the various GARCH-type models described above, we also added lagged values of volume and open interest data to the right-hand sides of the various variance equations[4]. For all three sets of contracts, both lagged total volume and lagged total open interest are found to have a significant relationship with volatility. This result is consistent with Najand and Yung [45]. For TX, only the coefficients out to the second lag of total volume (TVOL) and total open interest (TOP) were significant for all of the GARCH specifications, so only two lags of each of these predictor variables were added to the variance equations for the models fitted to the TX returns. Following the same approach, four lags of total volume and total open interest were found to be suitable for inclusion in the variance equations for the TF returns, while only three lags of total open interest were chosen for inclusion for TE return variance equations.

Once we have decided upon the number of lags of the predictor variables with which to augment the various GARCH model specifications, we next test their in-sample performance. We use the values of the log-likelihood function (LnL) and perform Likelihood Ratio (LR) tests to compare the various GARCH model specifications with and without market volume and open interest. For example, for the GARCH(1,1) model fitted to the TX returns, the basic model specification is compared to a specification that includes two lags of total volume. Comparing the two likelihood functions for this situation yields a LR test statistic of 3.92, which is not significant at the 5% level (critical χ_2^2 value = 5.99). Similarly, the augmentative models using two lags of total volume and open interest such as IGARCH, GARCH-M and APARCH models are all found to be significant at the 5% level, which indicates that the addition of two lags of volume and open interest data as predictor variables can improve the performance of these variance prediction models for the TX returns. However, models with three or four lags of these predictor variables do not necessarily produce the same results for the other sets of futures contracts.

For the TE futures, the forecasting models with three lags of total open interest are found to perform better than the models without any predictor variables in the variance equations, as indicated by significant (at a 5% level) LR tests for these augmented models. By contrast, the results for the TF futures returns are mixed. While the models augmented with four lags of total volume as predictor variables are found to provide improved performance, the models augmented with three lags of total open interest, on the other hand, do not appear to offer significant improvement. Overall, the majority of augmentative models (with lagged market volume and open interest) are found to outperform the basic models (without these predictor variables added) in capturing the dynamic behavior of Taiwan futures market returns.

4.2. Forecasting Results

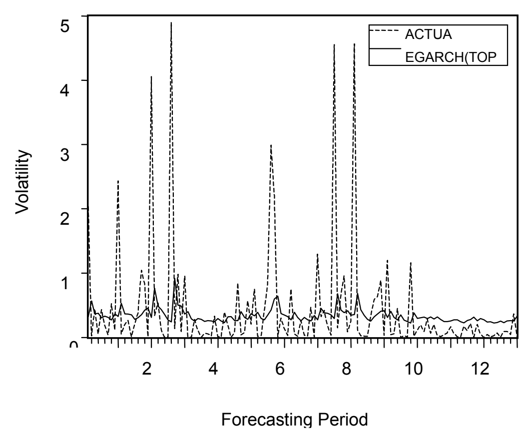
In assessing the out-of-sample forecasting ability of our various candidate models, we calculate the following one-period-ahead forecasting errors for each of the different models:

$$u_{t+1} = \sigma_{t+1}^2 - \sigma_{f,t+1}^2 \qquad (15)$$

where u_{t+1} is the forecasting error of the given forecasting model, $\sigma_{t+1}{}^2$ is defined by Eq. (2), and $\sigma_{f,t+1}{}^2$ is the forecasted variance generated by using the variance equations. In order to find the one-day-ahead forecast of the variance for July 1, 2002, we use the data from July 22, 1998 to June 28, 2002 in TX (from July 22, 1999 to June 28, 2002 in TE and TF) as our initial in-sample modeling period to estimate the parameters of the models. The sample is then rolled forward by removing the first observation of the sample and adding one to the end, and another one-step-ahead forecast of the next day's variance is made. This forecasting procedure is then repeated for each subsequent trading day during the period from July 1, 2002, through December 31, 2002. Computation of forecasts using a rolling window of data should ensure that the forecasts are made using models whose parameters have been estimated using all the relevant information available at the time.

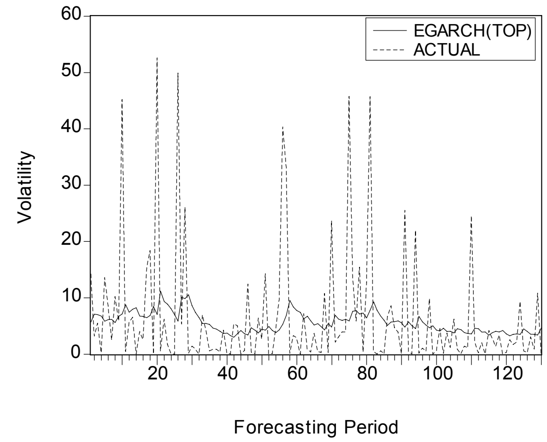
Unfortunately, good performance in parameter estimates and diagnostic statistics does not guarantee good performance in forecasting. Thus the sequences of one-step-ahead forecasts are generated and then evaluated through the regression testing procedure described in Section 3.4.1. Under this test procedure, the variance forecasts of each model are compared, one-by-one, with the forecasts of each of the other models. Using the results of these comparisons, a global ranking of the various models is made.

For the TX futures returns, the model that ranks number one in terms of out-of-sample variance forecast performance is the EGARCH model augmented with two lags of total open interest. Tied for second place in terms forecasting performance are the EGARCH model augmented with two lags of total volume and the augmented GARCH-M model with two lags of total open interest. For the TE futures, the number one model in terms of out-of-sample variance forecast performance is, similarly to the TX futures, the augmented EGARCH model with three lags of total open interest. The second and third place models, on the other hand, are the augmented GJR-GARCH and the augmented simple GARCH models, respectively, each with three lags of total open interest. Finally, for the TF futures, the top three models are all variations of Asymmetric Power ARCH models. The first place model is APARCH augmented with four lags of total volume, the second place model is APARCH augmented with three lags of total open interest, and, finally, the third place model is the basic, unaugmented APARCH model. In general, the non-linear GARCH models augmented with lagged market volume and open interest data as predictor variables within the conditional variance equations appear to be better able to capture and explain the evolution of the daily return variances of the TX, TE, and TF futures.



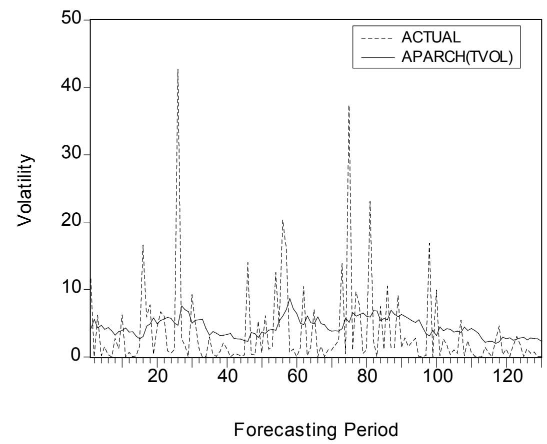
Note: Actual denotes the volatility defined by Eq. (2).

Figure 1: Esimation of TX volatility via EGARCH model augmented with two lags of total open interest (7/1/2002 – 12/31/2002)



Note: Actual denotes the volatility defined by Eq. (2).

Figure 2: Esimation of TE volatility via EGARCH model augmented with three lags of total open interest (7/1/2002 – 12/31/2002)



Note: Actual denotes the volatility defined by Eq. (2).

Figure 3: Esimation of TF volatility via APARCH model augmented with four lags of total volume (7/1/2002 – 12/31/2002)

The variance forecasting performance of each of the three first-place models, for the TX, TE, and TF futures, are displayed in Figures 1, 2, and 3, respectively. Each figure compares the actual daily futures return variances to the sequence of in-sample and one-step-ahead out-of-sample variance forecasts provided by the model found to best forecast the variances for the given set of futures contracts. These figures demonstrate how well the first-place candidate models perform at capturing the trends in the daily futures volatility of these three sets of futures contracts on the Taiwan market.

5. Conclusions

This paper examines the temporal behavior of the volatility of daily returns on stock index futures in the Taiwan futures markets, using various specifications of GARCH-type models, with and without market volume and open interest data as added predictor variables. The three sets of futures contracts examined are the TAIEX futures (TX), the TSE electronic sector Index futures (TE), and the TSE banking insurance sector index futures (TF). The candidate models chosen for forecasting the variances of the given sets of futures returns are conditionally Gaussian GARCH-type models, including both linear (GARCH and IGARCH) and non-linear (GARCH-M, EGARCH, GJR-GARCH and APARCH) GARCH model specifications.

Other potential models for variance prediction that were not chosen for examination in this study include stochastic volatility models and conditionally leptokurtic GARCH models. While the GARCH models that were examined in this study were all conditionally Gaussian, or Normal GARCH models, such models generally cannot account for the degree of leptokurtosis typically found in financial time series. This observation led to the development of the conditionally leptokurtic GARCH models, such as, most notably, the Student's t GARCH (GARCH-t) model of Bollerslev [5] (see Bollerslev et al. [7] and Mills [43] for additional examples of conditionally leptokurtic GARCH models), which has been found to provide a better fit than the Normal GARCH model for most other financial time series, However, as a consequence of the tail truncation that results from the daily price limits imposed on Taiwan's financial markets, the Normal GARCH models actually appear to provide a better fit than the conditionally leptokurtic models.

With regard to the other category of time series volatility model, the stochastic volatility models, Poon and Granger [50] find that they do not perform as well as GARCH models. However, a recent variation of stochastic volatility model, the Multifractal Model of Asset Returns (MMAR) (see Mandelbrot et al. [42], Calvet & Fisher [10][11], and Lux [41]) shows promise in providing better longer-term volatility forecasts than the GARCH models and would provide a promising area for future research into volatility on the Taiwan markets.

For the conditionally Gaussian GARCH-type models that we did fit to the Taiwan stock index futures returns, we also examine the relationship between volatility of the futures returns and market trading volume and open interest by directly adding the lagged volume and open interest data to the right-hand side of the variance prediction equations. We find significant relationships between the futures' daily volatilities and both the lagged total market volume and the total open interest, with the exceptions of the lagged volume and open interest of the nearest month's contract. This result may be attributed to the data selection method and the fact that the second nearest month's futures contract is not very actively traded in the Taiwan futures market. Regarding the in-sample estimation performance of the various models examined, the forecasting models with lagged total volume and open interest appear to outperform the other model specifications for all three sets of futures contracts, as determined by the LR test.

With regard to the out-of-sample forecasting performance of the various candidate linear and non-linear GARCH models, the regression test yields the conclusion that lagged values of either market volume or open interest data play a critical role in improving the out-of-sample forecasting performance of volatility models for all three sets, TX, TE, and TF, of the futures returns. Moreover, the non-linear GARCH models whose variance equations are augmented with lagged market volume and open interest values for all contracts perform better at out-of-sample forecasting than the equivalent linear GARCH models.

In summary, when the futures return series exhibit significant volatility asymmetries, non-linear GARCH models can better explain the volatility of the time series. In addition, the inclusion of lagged total volume and open interest can further improve their forecasting ability for the volatility of the Taiwan futures markets.

Endnotes

[1] The TSEC maintains a total of 27 stock price indexes, to allow investors to grab both overall market movement and different industrial sectors' performances conveniently. The TSEC Capitalization Weighted Stock Index (TAIEX) is the most widely quoted of all TSEC indexes. The base year value as of 1966 was set at 100.

[2] TAIEX covers all of the listed stocks excluding preferred stocks, full-delivery stocks and newly listed stocks, which are listed for less than one calendar month. Up to December 2001, 557 issues were selected as component stocks from the 584 listed companies on the Exchange.

[3] See Jorion (1995). We know:

$$\sigma^2 = E_{t-1}[r_t - E_{t-1}(r_t)]^2 = E_{t-1}[r_t^{\,2}] - [E_{t-1}(r)_t]^2,$$

and $E_{t-1}[r_t] \approx 0$, so $\sigma^2 = E_{t-1}[r^2{}_t]$.

[4] Following Akigray (1991), we also use LR test to find the fitted p and q of the variance equations when adding the lagged market volume and open interest to the right-hand side of the variance equations of the forecasting models

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