A Final Price Prediction Model for English Auctions—A Neuro-Fuzzy Approach

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Abstract

Online auction, being estimated to account for 25% electronic commerce by 2005, has now been a popular mechanism in setting price for internet users. However, auction price prediction, involving the modeling of uncertainty regarding the bidding process, is a challenging task primarily due to the variety of factors changing in auction settings. Even if all the factors were accounted for, there is still the uncertainty in human behavior when bidding in auctions. In this paper, three models, regression, neural networks and neuro-fuzzy, are constructed to predict the final prices of English auctions, using real-world online auction data collected from Yahoo-Kimo Auction. The empirical results show that neuro fuzzy system can catch the complicated relationship among the variables accurately much better than the others, which is of great help for the buyers to avoid overpricing and for the sellers to facilitate the auction. Besides, the knowledge base obtained from neuro fuzzy provides the elaborative relationship among the variables, which can be further tested for theory building.

Keywords: Online auction; Neuro-fuzzy system

1. Introduction

1.1 Research background and motivation

Since the web-based auction started in 1995, auctions on internet have grown at a tremendous rate. They have been regarded as a popular way of selling used items for scrap value and new items for profit. Every day hundreds of thousands of different online auctions are undergoing with goods ranging from computer, electronics, and toys to real estate, collectibles, and jewelry [17]. Online auctions can not only provide consumers the detailed price bidding process, but also have fueled the fire of dynamic pricing on web and given consumers an alterative to the fixed posted price mechanism [2]. The trading platforms of eBay, Priceline's reverse auction, and price comparison Web sites are all good examples of novel internet pricing models that help create a new pricing paradigm.

Auctions have long been of special interest to economists due to their explicit mechanisms to describe how prices are formed [8]. However, most of the issues are mainly focused on the design mechanism from both the sellers and buyers' point of view. There has been very little research done in the determination of the final prices. Final price prediction for online auctions, which involves the modeling of uncertainty regarding the bidding process, is a challenging task primarily due to the variety of factors changing in auction settings [23]. Even if all the factors were accounted for, there still exists the uncertainty in human behavior when bidding in auctions. The relationship among the final price and the related factors can be more than just the linear one.

From this perspective, as far as the nonlinear problem is concerned, the progress in artificial intelligence technology now provide a possible alternative that deserves further exploration in solving this problem. Among the available methods, expert system, fuzzy logic, and neural network are three of the most commonly used ones in helping managers in making real world decisions. The expert system can embed the past experience into the system; fuzzy logic can describe the problem in a way that is close to the human reasoning process and accommodate the inaccuracy and uncertainty associated with the data; the neural network can learn from historical data. However, the difficulty with the acquisition of the knowledge base for both the expert system and fuzzy logic, and the difficulty with the causal explanation through the construction of appropriate 'real' relations among the variables for the neural network model have constrained the application of these three methods. A method which can combine the advantages of these three methods while avoiding their disadvantages would seem to hold some promise.

Since predicting the final price of the auction can be of great help for bidders to set a reference price to avoid overpricing and for sellers to set the auction rules to facilitate the completion of the auction [6], this paper is trying to apply neuro fuzzy technique to model the price prediction problem. This proposed system can do the fuzzy reasoning and, through learning, can adjust the relative importance of each fuzzy rule. Furthermore, the knowledge base obtained from this technique can be used as an explanation about how the price is affected by the factors, facilitating the understanding about the auctions. Two benchmark models, regression and neural network, are constructed for comparison to show the validation. The rest of this paper is structured as follows. Section 2 briefly provides an overview of online auction literature. Section 3 describes how a neuro fuzzy system is constructed. The research methodology is described in Section 4. Empirical results are shown in Section 5 and finally some conclusions are given in section 6.Since the web-based auction started in 1995, auctions on internet have grown at a tremendous rate. They have been regarded as a popular way of selling used items for scrap value and new items for profit. Every day hundreds of thousands of different online auctions are undergoing with goods ranging from computer, electronics, and toys to real estate, collectibles, and jewelry [17]. Online auctions can not only provide consumers the detailed price bidding process, but also have fueled the fire of dynamic pricing on web and given consumers an alterative to the fixed posted price mechanism [2]. The trading platforms of eBay, Priceline's reverse auction, and price comparison Web sites are all good examples of novel internet pricing models that help create a new pricing paradigm.

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2. Literature Review

2.1 Auction Formats

Basically there are four basic types of auctions when a single item is to be sold, the ascending bid auction (also called the open or English auction), the descending bid auction (also called Dutch auction), the first-price sealed-bid auction, and the second-price sealed-bid auction (also called Vickrey auction) [14, 19]. Among them English auction is the most well-known and most widely used one. As far as it is concerned, the price is successively raised until there is only one bidder remaining. The remaining bidder wins the product at the final price [14]. Examples can be taken for markets of antiques, art, houses, and other collectibles. Lucking-Riley [17] showed that 121 of the 142 internet auction sites surveyed used this format of auction. On the other hand Dutch auction is the converse of the English auction. It starts at a high price and is lowered continuously until the first bidder calls out to accept the price. The bidder wins the object at that called out price. This type of auction has been used particularly in the sale of flowers, fish, and tobacco [14, 19].

For the first-price sealed-bid auction, bidders make independently secret bids without knowing the other's bids and the highest bidder wins the object at the highest price. Examples include the auctioning of mineral rights on government land, procurement auctions, treasury bills, and real estate. The second-price sealed-bid auctions or Vickrey auctions are similar to first-price sealed-bid auctions except the high bidder pays the second highest bid price [14, 19]. Since the English auction is the most commonly used format of auction on the internet, this paper is aiming to explore the relationship among the main attributes and the final price for the English auctions.

2.2 Auction Rules

Basically several parameters needed to be set prior to the beginning of an auction, which can influence the bidding process and consequently the final price, are explained as follows. *Starting bid* restricts participants to bid above a certain level [20]. All bidders have to offer bids that are at least as high as the starting bid. A *reserve price*, also known as the starting bid if being posted, is the lower bound of the price placed by the seller. The seller has the right to withdraw the auctioned item and not make the sale if the reserve price is not met. If the reserve price is kept secret, bidders will be informed when the bid has exceeded the reserve price [20]. The *bid increment* is the minimum amount by which a bid will be raised each time when the current bid is outdone. However, the bidders are not restricted to bid in increments of the bid increment. A *"buy-it-now"* price is placed by the seller and is a form of maximum bid. The existence and amount are publicly known. A "buy-it-now" price establishes a maximum bidding level at which the seller is willing to part with the item immediately which is the opposite of a reserve price. The buyer who bids the "buy-it-now" price immediately wins the auction at a preset price without participating in the bidding process. *Duration,* the length of the time, is also an important factor in determining the number of bidders, thus, potentially affecting the final price [20]. The related papers exploring how the key factors affect the final price are reviewed in the following.

2.3 Key Factors Affecting the Final Price

Starting Price

In general, a higher starting price, to some extent indicating a high value of the auctioned item, may consequently lead to a higher closing price. Häubl and Popkowski Leszczyc [7] showed that seller-specified initial starting bid serve as an informative indicator of an item's value and therefore have a positive influence on bidders' valuation of the product. Kauffman and Wood [12] also demonstrated that the online bidder will have higher valuation for an item in an auction if the seller sets a higher starting bid for that auction. However a higher starting price may at the same time reduce the number of bidders, which negatively affect the closing price. Therefore, some sellers set the starting price very low to attract more bidders to enter the auction leading to a higher competitive status that may result in a higher closing price [1, 13].

External Reference Price

Some research pointed out that consumers may obtain a standard price, known as a reference price, for comparison when making buying decisions [21]. They may use this reference price to consider whether to bid or not when faced with new prices. In general, the larger the gap between the bidding price and the advertised reference price (e.g., suggested list price, etc.), the greater the willingness to pay [26]. Hence, the information of the suggested list price will provide the bidders a good reference price and therefore increase the probability of purchase [10]. Lucking-Reiley et al. [18] claimed that market value or "book value" play an important role in the prediction of the final price

Reserve Price

Lucking-Reiley et.al [18] empirically showed that the presence of a reserve price has positive effects on the final price, increasing the final price by about 15% on average significantly. It is explained that the reserve price may acts as a competing bidder, at least until the reserve has been met.

Duration

If the auction duration is too long, the bidders often have to wait for several days until the auction is closed, which imposed a delay cost on bidders. Furthermore, bidders have to monitor the auction item to keep his bid update which incurs the monitoring costs. The longer the duration, the higher the cost is. Consequently consumers who bid in an online auction will prefer the short auction to a longer one in terms of these costs in general. On the other hand, if the auction duration is too short, the auctioned items may not have enough time to attract more bidders' attention to increase the competitive pressure on the auction's closing price. An auctioneer would like to keep the auction open longer to accumulate more bidders to achieve the effect of a longer exposure to the Internet [27]. Lucking-Reiley et.al [18] empirically showed that longer auctions tend to achieve higher prices. Seven-day auction prices are approximately 24% higher and 10-day auctions are 42% higher, on average. It implies that a longer auction should have more bidders resulting in a higher closing price.

Weekend

Some research claimed that "a seller should schedule her auction to close on the weekend" to maximize the final auction price since most bidding activities take place in the final hours or minutes of an auction, and participation rates are higher on weekends as people have more leisure time [18]. Kauffman and Wood [11] also find a *weekend effect* in the auction.

Picture

Unlike the traditional markets, electronic markets seldom let the consumer examine a product before the purchase. Eaton [5] suggested that the uncertainty regarding the condition of a product can be reduced by posting the picture of an item for sale by sellers. It is expected that auction with pictures provided would attract more bidding activities than those without. There are two types of pictures, one is the catalogue and the other is the home made one. The former can give the bidder a better picture about what the product looks like than just providing the product type number, especially when there are a large number of different product types. However it cannot provide the bidders the real information about the product's current condition. The latter (pictures taken by a seller himself) on the other hand, can give such information. The more information about a product the auction provides, probably the more willingness to pay the bidder would show, consequently probably the higher final price the auction is.

Description Length

Resnick et al. [22] claimed that the quality and completeness of the description page may affect the final price. However, it is difficult to judge the quality of a description. The information provided at the product description section of an auction is basically the product's brand name and type which somehow showed the quality, functionality, and even the age of the produc.

Sales Promotion

Consumers' price perceptions have been recognized as important factors affecting purchase decisions and they can be affected by the sales promotion [9]. Sales promotion can trigger unplanned purchases and even encourage consumers to purchase non-promoted merchandise. A promotion usually allows consumers to earn valuable rewards if they buy the products. For example, two-for-one promotions can save a shopper a lot of money and promotional giveaway activities that provides an extra incentive to buy or change the perceived price or value of products [24].

Ratings

Usually the online consumers can not observe or do not know much about the seller, just like the product itself. An untruthful seller could act anonymously by falsifying name, address, phone and email address, resulting in getting hurt felt by the consumers. Thus, information about the seller's credit to provide a product will impact the value of the object that a consumer is interested in. For example, the negative comments or uncertainty to delivery the promise quality would have negative impact on the bidder's utility for an item. On the other hand, the positive comment on the seller's credit to deliver the promised quality would have positive impact on the bidder's value for an item [12].

Online auction sites provide a mechanism for their participants to evaluate each other. These mechanisms are referred to as "reputation mechanisms". Reputation describes how other bidders have judged the seller's previous actions [12]. Lucking-Reiley et al. [18] examined the impact of reputation on final prices in eBay auctions. They found that a one percent increase in a bidder's positive feedback leads to a 0.03% increase in the final bid price while a 1% increase in negative feedback leads to a 0.11% decrease in the final bid price. The effect of negative feedback ratings is significantly large than the positive one.

These related studies are all good exploratory analysis of online auctions, trying to discover the relationships among the key factors and final price. However, so far the predictive model of the final price is still in lack in the literature. Therefore, this paper is trying to construct a predictive model to fill in the deficiency in the literature based on these previous studies. In other words, this paper aims to construct a final price predictive model based on the relationship explored before with the help of neuro fuzzy technique to deal with the complicated mapping among the factors and the final price. The research model is depicted as Figure 1.

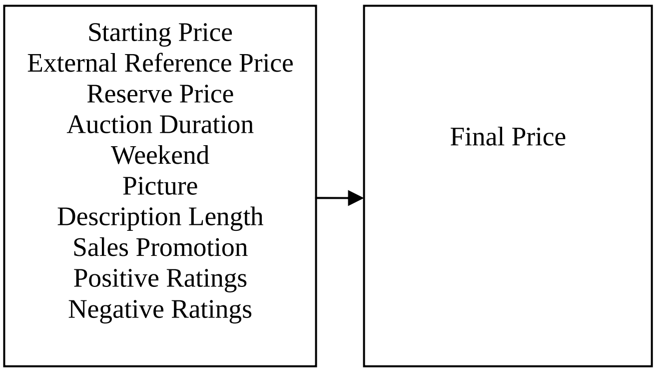

Figure 1. Research Model

3. The Construction of a Neuro Fuzzy System

Since neuro fuzzy is basically a fuzzy logic system combined with the learning ability of neural network, we would briefly introduce the fuzzy logic, neural networks and finally the Neuro Fuzzy system in the following.

3.1 Fuzzy Logic System

Fuzzy logic utilizes fuzzy sets defined by membership functions in logical expressions to deal with the extent to which the object belongs to the set. The membership function $\mu_A(x)$ with a value varying between 0 and 1 denotes the degree of membership to which object *x* belongs to fuzzy set *A*. The closer this value is to 1, the higher the membership of *x* to set *A*. A fuzzy logic system is constructed by introducing the logical relation of the "IF-THEN" rules to express the relationship among independent and dependent variables. For the clarity of the explanation, we only take two independent variables and one dependent variable for example. Let P_s, P_m, and P_f represent the starting price, external reference price, and final price of an auction.

A fuzzy logic rule is stated as follows:

IF P_s is **low**, and P_m is **medium**, then P_f is **medium**. (1)

Where P_s, P_m, and P_f are called *Linguisic variables* and high, medium, and low are called *linguistic term*s. Basically the construction of a fuzzy logic system consists of three major steps: fuzzification, construction of knowledge base and defuzzificaiton.

3.1.1 Fuzzification

Fuzzification is the process of converting crisp values to fuzzy values (e.g., low, medium, high). For example, we use low, medium, and high to describe the extent of starting price (P_s) and external reference price (P_m). For every linguistic variable, each term is defined by its membership function. Figures 2a and 2b are the membership functions for Ps and Pm respectively. If the figures of one data are $\{P_S, P_m\} = \{6,9\}$, for example, then the corresponding values of each term are described as follows.

P_s : $\mu_{low}(6) = 0.6$, $\mu_{medium}(6) = 0.4$, $\mu_{high}(6) = 0$

P_m : $\mu_{low}(9) = 0.34$, $\mu_{medium}(9) = 0.66$, $\mu_{high}(9) = 0$

In other words, the corresponding values can be written as follows.

P_s : $\{low, medium, high\} = \{0.60, 0.40, 0.00\}$

P_m : $\{low, medium, high\} = \{0.34, 0.66, 0.00\}$

As P_s equal to 6, its membership function values for low, medium, and high are 0.60, 0.40, and 0.00 respectively. Since each linguistic variable after mapping can have difference membership function values for different linguistic term, it breaks traditional binary logic that a case can only belong to or not belong to a category. This process is what we call *fuzzification*. The most commonly used membership function are linear and spline functions, the reader is referred to Zimmermann [31].

3.1.2. The construction of knowledge base

Knowledge base is constructed by a series of "IF-THEN" rules. Each rule has two parts,"IF" and "THEN" parts. "IF" part defines the extent the rule is valid for the current case and "THEN" part defines the response of the system. Take equation (1) for example,

IF P_s is low, and P_m is medium, then P_f is medium, (2)

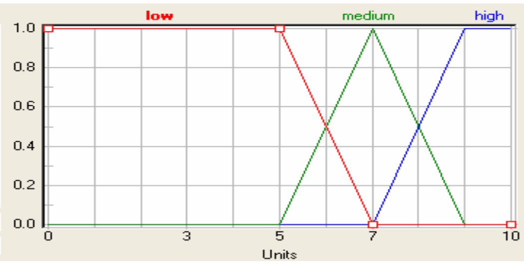

Figure 2a. Membership function for linguistic variable P_s

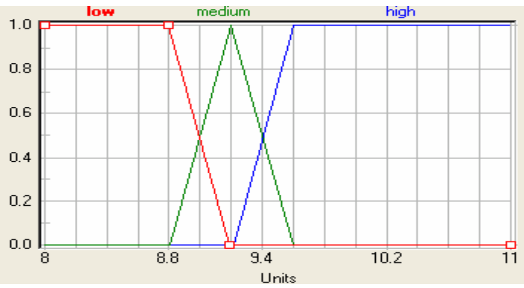

Figure 2b. Membership function for linguistic variable P_m

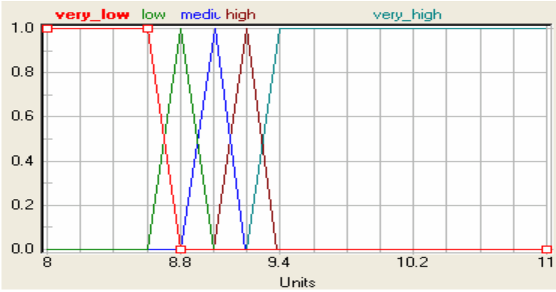

Figure 2c. Membership function for linguistic variable P_f

according to the definition of Zimmerman and Thole [29], the validity degree of the then part depends on the minimum extent of each linguistic term in the if parts. In other words, $\mu_{A\cap B} = \min\{\mu_A; \mu_B\}$. The satisfaction extent of the "IF" part of the above rule is the minimum of the "P_s is low, and P_m is high ", $\min\{0.6, 0.66\} = 0.6$, which is the validity extent of the response. In other words, the response of this system is "the P_f is medium" with validity extent equal to 0.6.

3.1.3 Defuzzification

After fuzzificztion and fuzzy inference, each input value will have a corresponding vlaue for each linguistic term of the output variable. For example the corresponding value of the linguisitc term "the P_f is medium" is 0.6 for equation (2) for the above example. Assume taht the corresponding values for the other linguistic terms are "the P_f is very low" is 0, "the P_f is low" is 0.1, "the P_f is high" 0.35, and "the P_f is very high" is 0. The process to convert fuzzy values to the corresponding crisp value is called defuzzification. Basically it consists of two main steps. In the first step, a representative value is determined for each term in the linguistic variable. In the second step, the best crisp value for the liguistic result is computed. For example, assume the representative values for each linguisitc term of the output variable is $\{8.60, 8.80, 9.00, 9.20, 9.40\}$ as illustrated in Figure 2c, then with the corresponding validity extent of each linguistic term $\{0.00, 0.10, 0.60, 0.30, 0.00\}$, the final output value is equal to

$0.00\times(8.60) + 0.10\times(8.80) + 0.60\times(9.00) + 0.30\times(9.20) + 0.00\times(9.40) =$ 9.04. In other words, the final price for this case is 9.04. This method of defuzzification is called "Center-of-Maximum", which is one of the most commonly used defuzzification method. Please refer to Tong and Bonissone [25] and Zimmermann [30] for the other defuzzification method. And for a detailed discussion of fuzzy logic, the reader is referred to Klir and Yuan [15].

The Fuzzy system has a characteristic to represent human knowledge or experiences using fuzzy rules; however, the fuzzy systems have some problems. Lack of definite criteria for selection of the shape of membership functions and the relative importance of each rule are two main difficulties associated with this method. Therefore, some self-tuning methods have been proposed to improve this method. Among them the neuro fuzzy technique manipulates the shortcomings and combines the desirable properties of both neural networks and fuzzy systems to form a system that is easy to use, with good performance.

3.2. Neural Network

Neural networks mimic biological information processing mechanisms which are designed to perform a nonlinear mapping form a set of inputs to a set of outputs [3]. The mapping is carried out by the processing elements, called artificial neurons, which are interconnected to form a network divided into layers (usually three): the *input layer* receives inputs from outside, the *output layer* sends outputs to the outside and one or more intermediate layers (hidden layer) connect the input and output layers (see Figure 3). There is a connection strength, *synapses*, or weight associated with each connection. When the weighted sum of the inputs to the neuron exceeds a certain threshold, the neuron is fired and an output signal is produced. The network can identify input patterns once the weights are adjusted or fined tune through some kind of learning process.

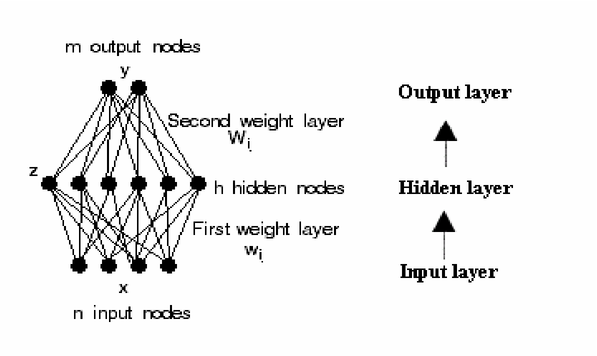

Figure 3. An Example of a simple feed-forward network

The back-propagation learning algorithm is the most widely used method in training the multi-layered feed-forward networks. It iteratively adjusts the network parameters (weights) to minimize the sum of squared approximation errors using a gradient descent technique. The aim of the learning process is to choose values of the weights so as to carry out the desired mapping from input to outputs. The output of the network is compared to a known target in order to define an error and to adjust the existing weights to achieve a better performance. For neural networks the reader can refer to [3] for detail.

Neural networks have been applied to a wide variety of areas, including medical diagnosis, stock market predictions, price forecasts, quality process control, robotics, and water resources. Unfortunately, neural networks are notoriously difficult to interpret – particularly when it comes to assessing the importance of individual weights. Thus, we might see that a neural network model comes up with superior results, but due to the complexity of the model, we might not be able to tell why it makes the recommendations it does. For this reason, neural network models have come to be known as "black box" predictive modeling tools.

<u>The construction of neural networks</u>

In this paper the back-propagation method is used for learning of multi-layered networks with sigmoidal functions. The learning rate of the neural network was set at 0.1, since we found this to be a good number from experimentation. The momentum was set at 0.8. The number of input units was set at 10, corresponding to the number of input attributes. The number of output units was set at 1, corresponding to the numeric output values. The number of hidden units in a hidden layer was set at 13.

In fact, both neural networks and fuzzy logic are powerful design methods which have their strengths and weaknesses. Neural networks can learn from data sets whereas fuzzy logic solutions are easy to verify and optimize. A combination of the explicit knowledge representation of fuzzy logic with the learning ability of neural networks results in Neuro Fuzzy [28].

3.3 Neuro Fuzzy Technique

Basically neuro fuzzy system is a fuzzy logic system with a learning algorithm derived from or inspired by neural network theory to determine its parameters, including the parameters of the membership function and the relative importance of each fuzzy rule [4]. The most common approach to be used to combine these two techniques is so-called Fuzzy Associative Memory (FAM) proposed by Kosko [16] and this paper adopt this approach. A FAM attempts to use neural networks to implement the desired mapping for fuzzy systems by applying fuzzy rules to a set of inputs, combining the consequents of each rule, and producing a value for the output variable. Each rule is associated with a weight factor that represents the importance of the rule in relevance to the other rules in the system. The errors between the results computed by the FAM system and the desired output are used to modify the weights. The training process will stop until the error is less than a certain threshold value.

3.4 The Neuro Fuzzy Research Model

The research model of neuro fuzzy is depicted as Figure 4 and the variables of the model are described in table 1. Table 2 shows all the lingustic Variables, their lingustic terms, and variable type used in this paper.

A fuzzy logic system is constructed by using the complete knowledge base to state the relationship among independent and dependent variables. Then the knowledge base is fine tuned by using the learning ability of neural network based on the training data set. Finally we use the testing data set to validate the obtained model.

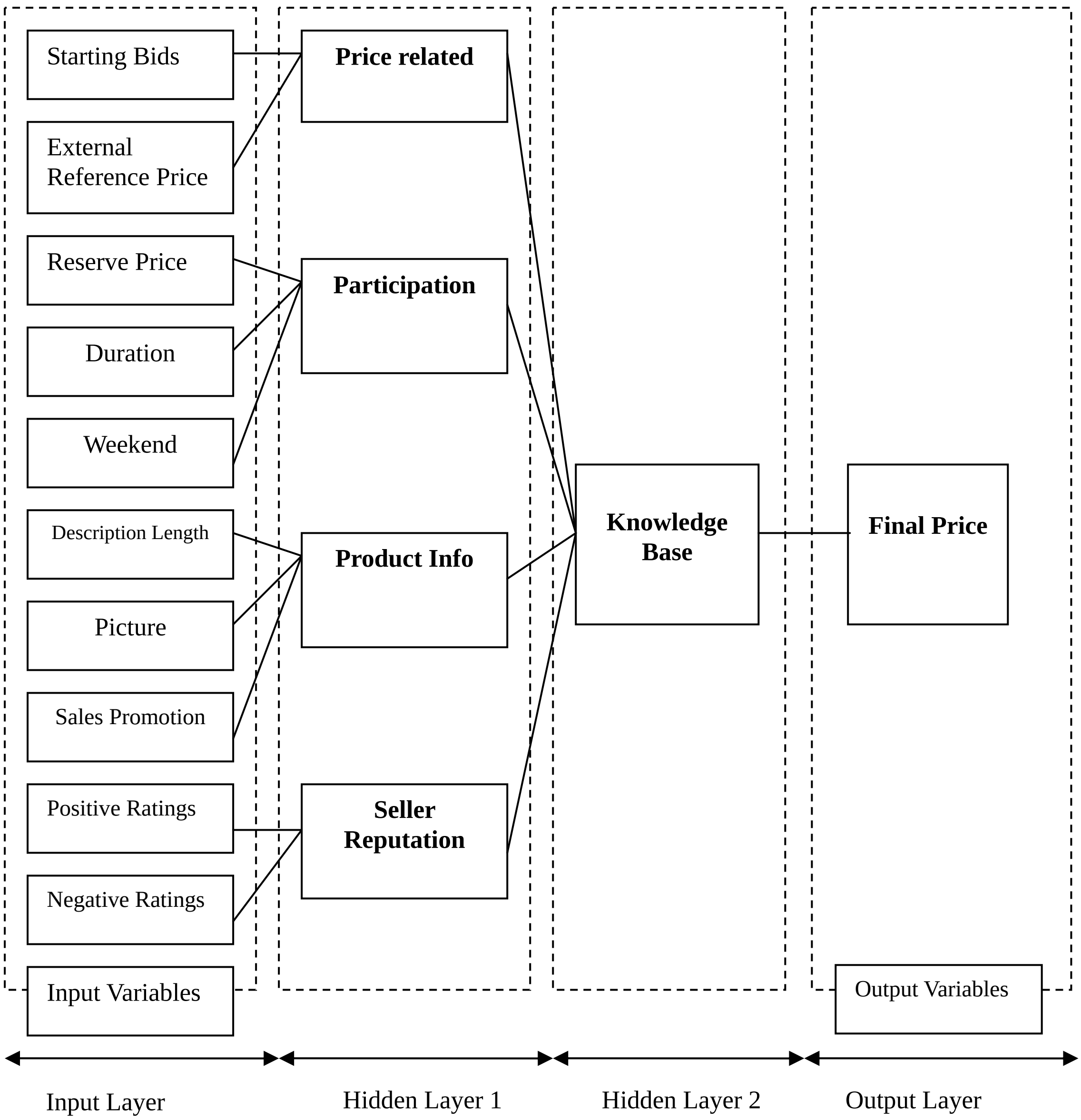

Figure 4. Research model for Neuro Fuzzy

Table 1. Description of variables in the research model

Input Variables	Description
Starting bid	Minimum acceptable starting bid
External Reference Price	Market value for this item
Reserve auction	Dummy variable indicating whether this item used a reserve auction format (*reserve =1*) or did not use a reserve auction format (*reserve = 0*).
Duration	Length of auction—2~10 days.
Weekend	Dummy variable indicating whether the final bid for this item occurred on a weekend (*Weekend* = 1) or on a weekday (*Weekend = 0*).

(continued)

Description Length	Number of bytes contained in the description
Picture	Dummy variable indicating whether this item's description contained a picture (*Picture =1*) or did not contain a picture (*Picture = 0*).
Sales Promotion	Dummy variable indicating whether this auction has promotional programs (Promotion =*1*) or did not have promotional programs (*Promotion = 0*).
Positive Ratings	The number of positive feedbacks left for seller by distinct users
Negative Ratings	The number of negative feedbacks left for seller by distinct users

Table 2. Lingustic variables, variable types and lingustic terms

Lingustic Variables	Type	Lingustic terms
P_s	Input	Low, medium, high
P_r	Input	Yes, No
Duration	Input	Short, mediem, Long
Weekend	Input	Yes, No
P_m	Input	Low, medium, high
Description	Input	Short, mediem, long
Pic	Input	Yes, no
Promotion	Input	New, used
Nnegtive ratings	Input	Low, medium, high
Positive ratings	Input	Low, medium, high
Price_related	Intermediate	Low, medium, high
Participation	Intermediate	Low, medium, high
Product_ Info	Intermediate	Less, medium, more
Seller_ Reputation	Intermediate	Low, medium, high
P_f	Output	Very_low, low medium, high, very high

4. Methodology

4.1. Data set

The data set for this study was collected from the real world online auction website, Yahoo-Kimo Auction, by a spider program written in Java. Digital camera product is chosen as the research target due to its variety of goods and prices. The spider program visited the Yahoo-Kimo Auction home page first, and then obtained the link to Digital camera auction page, collecting the IDs of all Digital camera auctions closed in pervious months. Each auction ID was used to construct a Web URL. After that all the details about each auction can be retrieved. Totally 110 digital camera auctions are collected with due date from December 3, 2004 to January 20, 2005. The data is divided into two sets, training data set and testing data set. The training data set, used for model construction, consisted of 73 examples (66%) and the testing set, used for validation, consisted of 37 examples (34%). To show the robustness of the cross validation, we randomly divide the data into two parts for ten times, and do the model construction and prediction for each division. Mean squared errors (MSE) is used as the performance criterion, which is calculated as $MSE = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 / n$, where y_i and $\hat{y}_i$ represents the actual value and predicted value of observation i respectively , and n is the number of observations.

5. Empirical Results

5.1 Empirical Results

Table 3 shows the parameter estimates, standard errors, and t-statistics for the regression model for the first training data set. Only three parameters are significantly different from zero at 5% significance level. The presence of a reserve price, the information of reference price, and the promotional activities all have positive effects on final price.

Table 3. Parameter estimates, standard errors, and t-statistics

	Coefficient	Standard Error	*t*	*p*
Intercept	-1.025	0.6484	-1.580	0.119
$\ln(P_s)$	-0.0022	0.0058	-0.3793	0.701
P_r	0.14289	0.0549	2.6027	0.012*
Duration	0.01585	0.0098	1.6173	0.112
Weekend	0.00799	0.0430	0.1858	0.853
$\ln(P_m)$	1.04837	0.0673	15.5683	0.000*
ln(Des)	0.0041	0.0180	0.2279	0.820
Picture	-0.0396	0.0882	-0.4490	0.655
ln(Pos)	-0.0127	0.0141	-0.9007	0.371
ln(Neg)	0.0081	0.0486	0.16635	0.868
Promotion	0.1420	0.0421	3.3729	0.001*

DV: $\ln(P_f)$
*$P < 0.05$.

Table 4 shows the MSE of each method for ten different runs on training and testing data sets. With the analysis of variance (ANOVA) significant at 0.05 level, table 5 shows the pairwise comparisons of Tukey test, indicating that neruo fuzzy performs better than neural network, and neural network performs better than regression significantly on the training data sets. Similarly with ANOVA significant at 0.05 level, table 6 shows the pairwise comparisons of Tukey test, indicating that neuro fuzzy performs the best among these three methods, and there is no significant difference between neural network and regression model. Besides, Table 7 lists the rules with relative importance (Degree of Support, Dos) greater than 0.9. It shows that these rules do make sense. For rule 4, IF Participation is high, Price_related is high, Product_info is medium, and Seller_ratings is high, THEN P_f is very high, the reasoning process follows what we expect. Similarly, for rule 1, IF Participation is low, Price_related is medium, Product_info is less, and Seller_ratings is low, THEN P_f is low, it conforms to our intuition. As for rule 2 and rule 3, they can show the interactions among the variables work.

Table 4. The Comparison for prediction results

	Training			Testing		
	Reg.	NN	NF	Reg.	NN	NF
	MSE			MSE		
1	0.028	0.020	0.015	0.032	0.028	0.009
2	0.032	0.022	0.011	0.028	0.024	0.010
3	0.033	0.023	0.004	0.024	0.025	0.014
4	0.022	0.017	0.003	0.044	0.038	0.012
5	0.030	0.023	0.016	0.029	0.025	0.017
6	0.033	0.024	0.012	0.024	0.021	0.014
7	0.031	0.022	0.004	0.031	0.027	0.005
8	0.027	0.020	0.003	0.039	0.031	0.015
9	0.027	0.020	0.006	0.036	0.030	0.011
10	0.033	0.024	0.016	0.025	0.024	0.017
Ave.	0.030	0.022	0.009	0.031	0.027	0.012

Table 5.Pairwise Comparisons for training data

Method	Mean Difference	Std. Error	*t*	Sig.
Reg-NN	0.00802	0.001791	4.4796	0.0003
Reg- NF	0.02072	0.001791	11.5661	0.0000
NN- NF	0.01269	0.001791	7.0865	0.0000

Table 6.Pairwise Comparisons for testing data

Method	Mean Difference	Std. Error	*t*	Sig.
Reg-NN	0.004110	0.002372	1.7327	0.21171
Reg- NF	0.018965	0.002372	7.9954	0.0000
NN- NF	0.014855	0.002372	6.2626	0.0000

Table 7. Knowledge base with relative importance greater than 0.9

Rule Base						
	IF					THEN
No	Participation	Price_related	Product_info	Seller Reputation	DoS	P_f
1	low	medium	less	low	1.00	low
2	high	medium	more	high	0.92	medium
3	medium	high	more	high	0.91	very high
4	high	high	medium	high	0.91	very high

6. Conclusions and Applications

6.1 Conclusions

This paper is trying to propose a final price prediction model for English auction based on the real world data with the help of neuro fuzzy technique to catch the complicated relationship among the final price and key factors involved in an auction. The empirical results show that neuro fuzzy performs the best no matter in the training data sets or testing data sets. In addition to the better prediction accuracy, neuro fuzzy system also provides the knowledge base obtained from the data set which describes the delicate relationship among the variables.This knowledge can not only provide practitioners some insights in understanding the bidding process but also provide the academia the basis for constructing hypotheses for further investigation. This proposed neuro fuzzy system has shown a promising perspective to explore the online auction.

6.2 Applications

The ability to predict the final price of online auction items can be of great help for both the buyers and sellers, the following applications are recommended for buyers and sellers.

6.2.1 Buyers

Being able to make predictions about the likely closing prices of the various auction items, the buyer can determine which auction is more close to his/her willingness-to-pay to place a bid. In doing so, the bidder can make bidding decision with the aid of the predicting model and diminish the uncertainty with respect to winning the item in the auction.

6.2.2 Sellers

The model of the final-price based on the attributes of the auction can also be used to help sellers to set the auction rules of their items to facilitate the completion of the auction. When the seller enters the information of the item they want to sell, our model would give suggestions for the auction attributes to maximize the final price and enhance the efficiency of the transaction.

6.3 Limitations

The main contribution of this study is expected to construct an effective and accurate final-price model by using the attributes of the auction and external reference price. However, this paper put in the factor, external reference price, in the research model to make final-price prediction so that this model can not be applied to some product categories (such as antiques, art, and collectibles etc.) since this kind of objects involve too much personal preference and not easy to construct a effective reference price.

6.4 Future work

In this paper, we use data from the specific product type of digital camera with the help of neuro fuzzy technique to predict the final price of online auction. One direction that we can conduct is to extend the model to different product types. Another direction is to extend the applicability of our approach and try alternative methods to learn general patterns and make inferences about auctions.

Appendices

Predicted and actual result for testing data

(1)

No	Actual price	Predicted price	Error
3	6500	6289.3	0.0324
4	6000	6218.9	0.0365
5	6000	6577.8	0.0963
6	5650	6335	0.1212
7	7100	6579.1	0.0734
17	4800	5398.1	0.1246
22	6550	6033	0.0789
23	5420	5358.3	0.0114
25	6600	6585	0.0023
27	12000	10713.4	0.1072
28	13500	11101.1	0.1777
37	8500	8013.6	0.0572
39	6500	6324	0.0271
40	9500	9555.8	0.0059
43	8500	8391.7	0.0127
45	8000	8825.1	0.1031
53	7500	7007.9	0.0656
54	8500	7605.3	0.1053
56	6000	6686.5	0.1144
58	6600	7368	0.1164
64	9700	8598.1	0.1136
67	11500	12770.6	0.1105
69	10900	11459.8	0.0514
70	10000	9060.2	0.0940
73	10300	10417.6	0.0114
77	11500	10318	0.1028
79	9000	11098.3	0.2331
82	17530	16532.8	0.0569
86	23600	21063.6	0.1075
93	10900	9969.6	0.0854
97	8800	9186.1	0.0439
98	7700	8628.7	0.1206
103	11000	10013.1	0.0897
107	9100	8883.5	0.0238
108	8100	7205.5	0.1104
109	6200	6881.6	0.1099
110	23500	21250.8	0.0957
Ave.			0.0819

(2)

No	Actual price	Predicted price	Error
8	5500	4576.3	0.1679
10	4171	4523.8	0.0846
24	6800	6170.3	0.0926
32	7700	7663.7	0.0047
33	9000	8440.1	0.0622
39	6500	6204.6	0.0454
42	9300	10174.1	0.0940
44	10000	10968.1	0.0968
46	8100	8820.6	0.0890
48	6900	7542.1	0.0931
50	6500	6757.4	0.0396
56	6000	6292.1	0.0487
61	15500	14337.2	0.0750
73	10300	9740.5	0.0543
74	11100	10174.1	0.0834
75	10300	9878.8	0.0409
78	13200	13569.8	0.0280
79	9000	8394.2	0.0673
80	8900	8746.4	0.0173
90	10000	9740.5	0.0260
91	8500	7900.6	0.0705
93	10900	10007.1	0.0819
94	10300	10099.6	0.0195
95	8800	8907.1	0.0122
98	7700	8773.6	0.1394
99	12000	11930.5	0.0058
100	7900	7363.2	0.0679
101	8000	8244.9	0.0306
102	7600	8127.4	0.0694
103	11000	10161.9	0.0762
104	9800	8865.7	0.0953
105	8200	8260.6	0.0074
106	8100	8015.6	0.0104
107	9100	8753.4	0.0381
108	8100	8037.7	0.0077
109	6200	6729.1	0.0853
110	23500	23509.3	0.0004
Ave.			0.0575

(3)

No	Actual price	Predicted price	Error
1	3400	5842.4	0.7184
9	4500	4740.3	0.0534
11	4950	5440.9	0.0992
12	4600	5166	0.1230
15	6000	5521.5	0.0798
17	4800	5222.6	0.0880
20	6500	6323.6	0.0271
26	6000	5751.1	0.0415
27	12000	12552.8	0.0461
28	13500	13373.1	0.0094
32	7700	7803.7	0.0135
40	9500	10364.1	0.0910
41	9500	9844.8	0.0363
42	9300	9857.1	0.0599
44	10000	10010.1	0.0010
49	6800	7096.5	0.0436
51	8999	8062.7	0.1040
53	7500	7100.7	0.0532
54	8500	8047.4	0.0532
55	9000	8626.1	0.0415
58	6600	6505.2	0.0144
60	11500	13554.2	0.1786
63	15500	15430.5	0.0045
66	10300	10774.1	0.0460
71	10600	11077.8	0.0451
73	10300	11236.2	0.0909
74	11100	11021.5	0.0071
76	9900	10033.2	0.0135
77	11500	11092.8	0.0354
81	15300	15359.7	0.0039
82	17530	16451.1	0.0615
86	23600	20268	0.1412
87	15900	16300.5	0.0252
88	9700	9013.7	0.0708
92	10000	9258.5	0.0742
93	10900	10696.3	0.0187
95	8800	9184.8	0.0437
Ave.			0.0718

(4)

No	Actual price	Predicted price	Error
3	7500	7282.6	0.0290
4	6000	6750.4	0.1251
5	6000	5495.3	0.0841
9	4500	4913.3	0.0918
11	4950	5807.8	0.1733
13	4250	4398.2	0.0349
16	4050	4398.2	0.0860
22	6550	6094.9	0.0695
25	6600	6242.9	0.0541
30	5600	6164.4	0.1008
31	4850	5265.6	0.0857
33	9000	9135.7	0.0151
38	6600	6879.8	0.0424
41	9500	9673.5	0.0183
45	8000	8323.6	0.0405
46	8100	8660.3	0.0692
48	6900	7171	0.0393
53	7500	6857.2	0.0857
56	6000	5708.1	0.0486
59	10699	9881.3	0.0764
75	10300	9491	0.0785
80	8900	8460	0.0494
82	17530	11717	0.3316
83	20000	18570.9	0.0715
86	23600	16329.8	0.3081
89	12500	13706.8	0.0965
92	10000	10622.2	0.0622
98	7700	8412.7	0.0926
102	7600	8268	0.0879
103	11000	10190.4	0.0736
104	9800	10857.9	0.1079
105	8200	7724.5	0.0580
106	8100	8822	0.0891
107	9100	8427	0.0740
108	8100	7773.7	0.0403
109	6200	6743.6	0.0877
110	23500	21437.6	0.0878
Ave.			0.0856

(5)

No	Actual price	Predicted price	Error
1	3400	6309.7	0.8558
8	5500	5655.9	0.0283
9	4500	4863.9	0.0809
11	4950	5471.2	0.1053
12	4600	5002.3	0.0875
17	4800	5214.5	0.0864
22	6550	5956.6	0.0906
23	5420	5947.9	0.0974
26	6000	6046.6	0.0078
27	12000	12153.2	0.0128
28	13500	14746.3	0.0923
31	4850	4992	0.0293
39	6500	6481.1	0.0029
40	9500	9372.2	0.0135
44	10000	10647.2	0.0647
45	8000	8182.1	0.0228
48	6900	6852.4	0.0069
50	6500	7845.1	0.2069
51	8999	9384.8	0.0429
52	7400	7032.2	0.0497
61	15500	14663.3	0.0540
69	10900	11292.6	0.0360
71	10600	11046.3	0.0421
72	7600	7436.5	0.0215
73	10300	10706.4	0.0395
78	13200	13802.4	0.0456
80	8900	9446.5	0.0614
84	18000	16411.7	0.0882
90	10000	10679.2	0.0679
92	10000	9376.4	0.0624
96	6900	6713.7	0.0270
98	7700	7812.3	0.0146
100	7900	8368.7	0.0593
106	8100	8872.4	0.0954
107	9100	9857.1	0.0832
109	6200	5908.8	0.0470
110	23500	22724.5	0.0330
Ave.			0.0774

(6)

No	Actual price	Predicted price	Error
1	3400	5729.9	0.6853
4	7500	5624.6	0.2501
8	5500	5770.4	0.0492
13	4250	4430.6	0.0425
14	5700	6027.9	0.0575
19	5900	6414.1	0.0871
22	6550	7124.9	0.0878
23	5420	5245.4	0.0322
24	6800	6411.8	0.0571
26	6000	6167.2	0.0279
29	4950	5161.1	0.0426
30	5600	5467.4	0.0237
31	4850	5290.7	0.0909
34	6300	6452.4	0.0242
35	5100	5199.1	0.0194
41	9500	9980.6	0.0506
43	8500	8411.9	0.0104
45	8000	8694.1	0.0868
47	8100	8410.6	0.0383
50	6500	7048	0.0843
57	10000	10756.9	0.0757
60	11500	11478.1	0.0019
63	15500	15921.9	0.0272
64	9700	10483.9	0.0808
65	10000	9248.8	0.0751
67	11500	12218.4	0.0625
69	11900	11020.9	0.0739
72	7600	8010.8	0.0541
79	9000	8242	0.0842
80	8900	8898.6	0.0002
85	20000	18294.4	0.0853
87	15900	15225.1	0.0424
89	12500	12028.1	0.0378
92	10000	8891.9	0.1108
94	10300	10804.8	0.0490
100	7900	7958.5	0.0074
101	8000	8030.5	0.0038
Ave.			0.0735

(7)

No	Actual price	Predicted price	Error
3	7500	6484.7	0.1354
5	6000	5906.2	0.0156
11	4950	5610	0.1333
14	5700	6463.3	0.1339
15	6000	6182.3	0.0304
18	6900	5976	0.1339
20	6500	5848.8	0.1002
21	3800	4278.7	0.1260
28	13500	10936.9	0.1899
29	4950	5985.2	0.2091
31	4850	4838.7	0.0023
32	7700	7354.4	0.0449
33	9000	8896.4	0.0115
35	5100	5661.3	0.1101
38	6600	6429.8	0.0258
46	8100	7764.4	0.0414
47	8100	8154.7	0.0068
48	6900	6340.1	0.0811
51	8999	8149.8	0.0944
54	8500	9121.1	0.0731
55	9000	8397.2	0.0670
56	6000	5281.4	0.1198
57	10000	9653.3	0.0347
59	10699	11657.4	0.0896
63	15500	15410.4	0.0058
71	10600	9692.4	0.0856
74	11100	11755.8	0.0591
76	9900	9284.9	0.0621
79	9000	9682.3	0.0758
82	17530	17339.6	0.0109
86	23600	21788.8	0.0767
88	9700	10735.4	0.1067
91	8500	8130.3	0.0435
93	10900	10196	0.0646
96	6900	7548.1	0.0939
98	7700	9416.8	0.2230
104	9800	10277.9	0.0488
Ave.			0.0802

(8)

No	Actual price	Predicted price	Error
2	6500	11102.8	0.7081
3	7500	7062.8	0.0583
12	4600	4880.2	0.0609
15	6000	5552.2	0.0746
16	4050	4468.2	0.1033
18	6900	7883.7	0.1426
21	3800	4203.4	0.1062
22	6550	6474.7	0.0115
29	4950	5573.1	0.1259
36	6000	6375.1	0.0625
38	6600	7072.7	0.0716
39	6500	7316.2	0.1256
40	9500	9767.8	0.0282
42	9300	8680.6	0.0666
43	8500	9752.2	0.1473
44	10000	8608	0.1392
46	8100	9771.7	0.2064
56	6000	6782.8	0.1305
57	10000	9221.6	0.0778
60	11500	9998.6	0.1306
63	15500	15141.6	0.0231
65	10000	8982.2	0.1018
68	11000	9920.4	0.0981
71	10600	9860.6	0.0698
74	11100	9837.9	0.1137
77	11500	10131.5	0.1190
80	8900	9803.6	0.1015
83	20000	16388.7	0.1806
85	20000	18685.4	0.0657
87	15900	15063	0.0526
88	9700	8649.9	0.1083
90	10000	10754.2	0.0754
91	8500	8314.4	0.0218
92	10000	9920.9	0.0079
94	10300	9990.1	0.0301
97	8800	8726.3	0.0084
98	7700	7544.7	0.0202
			0.1020

(9)

No	Actual price	Predicted price	Error
1	3400	6294.3	0.8513
3	7500	7150.2	0.0466
4	6000	5786.3	0.0356
8	5500	5913.8	0.0752
10	4171	4375.8	0.0491
11	4950	5417	0.0943
14	5700	5970.9	0.0475
16	4050	4412.7	0.0896
20	6500	6779.8	0.0430
23	5420	5491.7	0.0132
37	8500	8000.8	0.0587
40	9500	9618.6	0.0125
42	9300	8906.6	0.0423
46	8100	8105.9	0.0007
56	6000	6064.5	0.0108
62	16500	17047.3	0.0332
64	9700	10540.2	0.0866
65	10000	10833	0.0833
66	10300	10381.7	0.0079
68	11000	10801.1	0.0181
71	10600	9860.1	0.0698
77	11500	11136.7	0.0316
78	13200	12126.5	0.0813
79	9000	8615.8	0.0427
82	17530	16820.4	0.0405
92	10000	10815.7	0.0816
94	10300	9840.9	0.0446
95	8800	8277.9	0.0593
98	7700	8849.8	0.1493
100	7900	8243.3	0.0435
103	11000	10914	0.0078
105	8200	8983.5	0.0955
106	8100	8673.3	0.0708
107	9100	8835.7	0.0290
108	8100	8533.5	0.0535
109	6200	6722.4	0.0843
110	23500	22445.6	0.0449
Ave.			0.0738

(10)

No	Actual price	Predicted price	Error
2	6500	7549.6	0.1615
3	7500	8154.3	0.0872
6	5650	5747.7	0.0173
8	5500	5899.1	0.0726
9	4500	5093.7	0.1319
10	4171	4713.1	0.1300
16	4050	4879.3	0.2048
24	6800	7554.9	0.1110
29	4950	6166.3	0.2457
33	9000	8230.9	0.0855
34	6300	7180	0.1397
39	6500	5990	0.0785
41	9500	9047.1	0.0477
42	9300	9785.9	0.0522
48	6900	6043	0.1242
50	6500	5976.9	0.0805
52	7400	5675.1	0.2331
61	15500	12113.2	0.2185
62	16500	17006.5	0.0307
66	10300	13412.6	0.3022
67	11500	12809.6	0.1139
68	11000	10379.1	0.0564
69	10900	11680.2	0.0716
76	9900	10268.1	0.0372
77	11500	10555.5	0.0821
79	9000	9201.8	0.0224
83	20000	17033.7	0.1483
85	20000	17981.5	0.1009
88	9700	12237.4	0.2616
89	12500	12682.1	0.0146
93	10900	10406.1	0.0453
96	6900	7108.9	0.0303
97	8800	7568.1	0.1400
98	7700	6051.1	0.2141
104	9800	9649.9	0.0153
105	8200	7351.8	0.1034
110	23500	22011.1	0.0634
Ave.			0.1101

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