

# Performance Evaluation of an Environmental Conscious Manufacturing System with Stochastic Demand

Kenichi NAKASHIMA

Osaka Institute of Technology, Department of Industrial Management,  
5-16-1 Omiya, Asahi-ku, Osaka, 535-8585 JAPAN

E-mail: [nakasima@dim.oit.ac.jp](mailto:nakasima@dim.oit.ac.jp)

## Abstract

This paper proposes a performance evaluation approach to an environmental conscious manufacturing system such as a remanufacturing system with stochastic variability stemming from customer demand. We here formulate a remanufacturing system with consideration for product life cycle under stochastic demand using a Markov process. The system is composed of the state that denotes the numbers of the products in inventories, the transition probabilities between states and the costs associated with the transitions. In this approach, we consider two types of inventories: one is the actual product inventory in a factory whereas the other is the virtual inventory that is still in use by the customers. This model also includes disposal and return rates. Using the Markov model, we can obtain the steady state distribution of the system and calculate the expected average cost per period. Numerical examples are considered to illustrate the implementation of the methodology.

## 1. Introduction

The continuous growth in consumer waste in recent years has seriously threatened the environment. According to the US Environmental Protection Agency (EPA), in 1990 the amount of waste generated in the USA reached a whopping 196 million tons up from 88 million tons in 1960s [15]. Wann [18] reports that an average American consumes 20 tons of materials every year. Environmentally conscious manufacturing (ECM) is mainly driven by the escalating deterioration of the environment. It has become an obligation to the environment and to the society. Many countries are contemplating regulations that force manufacturers to take back used products from consumers so that the components and materials retrieved from the products may be reused and/or recycled. For example, Germany has passed a regulation that requires companies to remanufacture products until the product is obsolete. Japan has passed similar legislation requiring design and assembly methodologies that facilitate recycling of durable goods [3-7].

Gungor and Gupta [8] reviewed the literature in the area of environmentally conscious manufacturing and product recovery. They summarized wide range of this area including industrial examples, modeling and solutions. Product recovery aims to minimize the amount of waste

sent to landfills by recovering materials and parts from old or outdated products by means of recycling and remanufacturing. It should be considered in designing and managing the manufacturing systems [8]. Here, product recovery includes collection, disassembly, cleaning sorting, repairing bad components, reconditioning, testing, reassembling and testing. Recovered parts/products are used in repair, remanufacturing of other products and components. We focus on a product recovery system in a remanufacturing environment under stochastic variability.

This paper proposes a performance evaluation approach to an environmental conscious manufacturing system such as a remanufacturing system with stochastic variability stemming from customer demand. We here formulate a remanufacturing system with consideration for product life cycle under stochastic demand using a Markov process. Consider a single process that produces a single item product. The finished products are stocked in the factory and are used to fulfill customer demand. Traditional inventory models focus only on the inventory in the factory. In the remanufacturing system, however, we need to account for the outdated products that are collected from the customers as well. That is, a remanufacturing producer has to consider the products in use as the part of the future inventory. Consequently, we treat the products currently in use by the customers as virtual inventory. It is therefore important to manage the virtual inventory until it is collected and used in remanufacturing as well as controlling the inventory on hand. For this virtual inventory, it should be required to consider an operational cost that we need in order to observe and check the quantity of the inventory. We call it the virtual inventory cost and model the system including it.

In section 2, we briefly summarize environmental conscious manufacturing background and the relevant literature. In section 3, we consider a single-item remanufacturing system under stochastic demand. The system is composed of the state that denotes the numbers of the products in inventories, the transition probabilities between states and the costs associated with the transitions. In this approach, we consider two types of inventories: one is the actual product inventory in a factory whereas the other is the virtual inventory that is still in use by the customers and has two classes depending on the status of the inventory. We also consider disposal and return rates in the model. In section 4, we obtain the steady state distribution of the system and calculate the expected average cost per period using

Markov analysis to evaluate the system. Numerical examples are considered to illustrate the implementation of the methodology.

## 2. Environmental Conscious Manufacturing Systems

In order to retrieve components and materials (for reuse, recycling and remanufacturing) from consumer products, the first step is disassembly. *Disassembly* is the process of systematic removal of desirable constituents from the original assembly so that there is no impairment to any useful component [9]. Disassembly can be *partial* (product not fully disassembled) or *complete* (product fully disassembled) and may use a methodology that is *destructive* (focusing on materials recovery) or *non-destructive* (focusing on components recovery). In this paper, we limit ourselves to *complete* and *non-destructive* disassembly in the remanufacturing system.

Our main interest is in the operational aspect of product recovery in the remanufacturing environment. Fleischmann et al. [2] define remanufacturing as a process of bringing the used products back to 'as new' condition by performing the necessary operations such as disassembly, overhaul and replacement. Remanufacturing is also referred as recycling-integrated manufacturing [10]. Similar to the conventional production systems, in remanufacturing systems, there are operational, manufacturing, inventory, distribution and marketing related decisions to be made [12, 16]. In general, the existing methods for conventional production systems cannot be used for the remanufacturing systems. Remanufacturing environments are characterized by their highly flexible structures. Flexibility is required in order to handle the uncertainties which are likely to arise. In addition, we should recognize that the products that are used by customers would be the parts to produce new ones after collecting and recovering them.

As for the periodic review models, Cohen *et al.* [1] developed the product recovery model in which the collected products are used directly. Inderfurth [11] discussed effect of non-zero leadtimes for orders and recovery in the different model. As for continuous review models, Muckstadt and Isaac [13] dealt with a model for a remanufacturing system with non-zero leadtimes and a control policy with the traditional  $(Q, r)$  rule. Van der Laan and Salomon [17] suggested push and pull strategies for the remanufacturing system. They, however, consider that demand and procurement are independent in the inventory systems. All the above studies, however, considered the demand and procurement are independent in the systems. Nakashima *et al.* [14] dealt with a product recovery system with a single class of product life cycle. They proposed a new analytical approach to evaluate the system using Markov chain and gave numerical examples for various conditions.

## 3. Model Description

We formulate a generalized remanufacturing system with stochastic variability such as demand using a discrete time Markov model. Let us consider a single process that produces single item product. The finished products are stocked in the factory and faced according to the customer demand. Traditional inventory management focuses on only the inventory in the factory. In the remanufacturing system, however, we should focus on the outdated products that are collected from the customers. That is, a remanufacturing producer has to consider the products in use as the part of the future inventory. We here take the products used by customers as the virtual inventory. It is important managing the virtual inventory until products are collected and used in the remanufacturing as well as controlling the inventory on hand. For this virtual inventory, it should be required to consider an operational cost that we need in order to observe and check the quantity of the inventory. We call it the virtual inventory cost and model the system including it.

### 3.1 A Remanufacturing System with Stochastic Demand

Figure 1 shows the remanufacturing system with stochastic demand. Remanufacturing preserves the product's or the part's identity, and performs the required disassembly and refurbishing operations to bring the product to a desired level of quality with remanufacturing cost. On the other hand, we define normal manufacturing as producing the products using new resources. The number of products by normal manufacturing at period  $t$  is defined by  $P(t)$ . Products are produced by normal manufacturing and/or remanufacturing with the parts taken back from the customers. All production begins at the start of the period and all the products are completed by the end of the period. All the products bought by customers are new. We assume that the number of finished products is given by  $I(t)$ . There are two types of conditions on the products bought by customers; one is the product of which remaining life time is one that is denoted by  $J_1(t)$ , and the other is that of which remaining life time is two which is denoted by  $J_2(t)$ . If the backlog occurs,  $I(t)$  is negative value. Demand in successive periods,  $D(t)$  is independent random variables with known identical distributions and densities. When sold, products are recovered from  $J_1(t)$  at the recovery rate,  $\alpha_1$  and from  $J_2(t)$  at the recovery rate,  $\alpha_2$ . The products in  $J_1(t)$  are discarded at the discarded rate,  $\beta$  with out-of-date cost. It is supposed that  $\alpha_1 + \alpha_2 + \beta \leq 1$ .

### 3.2 Formulation

The state of the remanufacturing system is denoted by

$$s(t) = (I(t), J_1(t), J_2(t)). \quad (1)$$

The transition of the each inventory is given by

$$I(t+1) = I(t) + P(t) + \alpha_1 J_1(t) + \alpha_2 J_2(t) - D(t) \quad (2)$$

and

$$J_2(t+1) = \min(\lceil I(t) \rceil + P(t) + \alpha_1 J_1(t) + \alpha_2 J_2(t), \lceil I(t) \rceil + D(t)) \quad (3)$$

$$J_1(t+1) = (1 - \alpha_2) J_2(t) - (\alpha_1 + \alpha_2) J_1(t) \quad (4)$$

where means  $\lceil I(t) \rceil = \max(0, I(t))$ . It is assumed that

$$P(t) = \max\{0, \text{Imax} - I(t) - \alpha_1 J_1(t) - \alpha_2 J_2(t)\}. \quad (5)$$

The transition probability is defined as

$$P_{s(t)s(t+1)} = \begin{cases} \Pr\{D(t) = d\} & \text{if } S(t+1) = (I(t) + P(t) + \alpha_1 J_1(t) + \alpha_2 J_2(t) - d, \\ & \min(\lceil I(t) \rceil + P(t) + \alpha_1 J_1(t) + \alpha_2 J_2(t), \lceil I(t) \rceil + d), \\ & (1 - \alpha_2) J_2(t) - (\alpha_1 + \alpha_2) J_1(t) \\ & \text{otherwise.} \\ 0 & \end{cases} \quad (6)$$

If the system is in  $s(t)$ , the expected average cost per period,  $R(t)$  is given by

$$R(t) = cP(t) + \alpha_1 \alpha_1 J_1(t) + \alpha_2 \alpha_2 J_2(t) + h[I(t)]^+ + h_1 J_1(t) + h_2 J_2(t) + b[I(t)]^- + \alpha J_1(t) \quad (7)$$

where the parameters are as follows:

- $c$  : the normal manufacturing cost of a new product
- $\alpha_1$ : the remanufacturing cost of a product using the part of which remaining life time is one
- $\alpha_2$ : the remanufacturing cost of a product using the part of which remaining life time is two
- $h$  : the holding cost per unit product
- $h_1$  : the virtual inventory cost per unit product of which remaining life time is one
- $h_2$  : the virtual inventory cost per unit product of which remaining life time is two
- $b$  : the backlog cost per unit
- $\alpha$  : the out-of-date cost per unit

In this model, we consider the virtual inventory costs that we need in order to observe and check the quantity of those as the operational costs. The stationary distribution of the remanufacturing system can be calculated by solving a liner equation of the steady state distribution using Markov analysis. In this system, we obtain the total expected average cost per period and evaluate the

performance of the system under the stochastic demand.

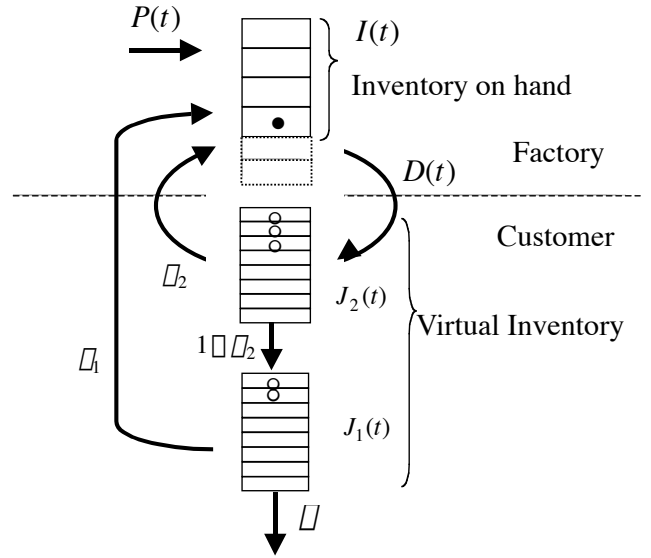


Figure 1: A Remanufacturing System with Stochastic Demand

## 4. Numerical Experiments

In this section, we show the numerical example of the suggested approach to evaluating the performance of the remanufacturing system with stochastic demand described in section 3. It is assumed that the maximum number of products inventory,  $\text{Imax}$  is 5 and that of the maximum backlog demand,  $\text{Imin}$  is set as -5. Moreover, the maximum numbers of virtual inventories,  $J_1(t)$  and  $J_2(t)$  are given by 4 and 4, respectively. We also assume that  $\alpha_1=0.1$  and  $\alpha_2=0.1$ .

The cost parameters in the previous section are set as below:

$$c=1, \alpha_1=3, \alpha_2=2, h=1, h_1=2, h_2=2, b=10 \text{ and } \alpha=10.$$

The distribution of the demand is given by

$$\Pr\{D_n = D - \frac{1}{2}Q + j\} = \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2} \binom{Q}{j} \frac{1}{2^Q}, (0 \leq j \leq Q) \quad (8)$$

where  $D=3$  and  $Q$  is an even number and the variance( $\sigma^2$ ) is  $Q/4$ . We can obtain the expected average cost per period under the steady state of the system.

Figure 2 illustrates the effect of the product recovery rate derived from  $J_1(t)$ ,  $\alpha_1$  on the expected average cost per period when the variance is 3. We can find the expected average cost tends to decrease when the recovery rate,  $\alpha_1$  increases. This means that promoting the remanufacturing activity focusing on taking back the end-of-life products from the customers seems to be good performed in this system.

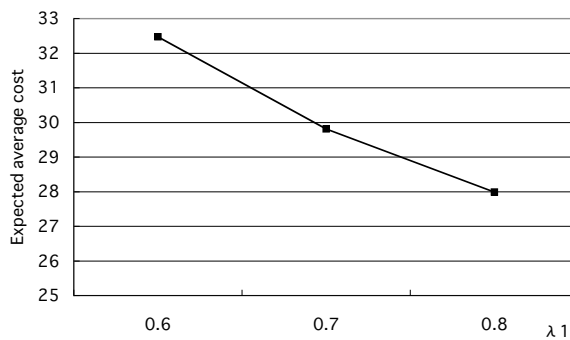


Figure 2: The effect of the recovered rate

## 5. Conclusions

This paper proposed the performance evaluation approach to the environmental conscious manufacturing system such as the remanufacturing system with stochastic variability stemming from customer demand. The system was formulated into the Markov chain model that was composed of the state that denoted the numbers of the products in inventories, the transition probabilities between states and the costs associated with the transitions. Using the Markov analysis, we obtained the steady state distribution of the system and calculated the expected average cost per period. Numerical examples are considered to illustrate the implementation of the methodology.

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