# Fuzzy VIKOR as an Aid for Multiple Criteria Decision Making

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Abstract

The primary objective of this paper isto propose a fuzzy VIKOR method for solving multiple criteria group decision making problems. Introduction of methodology and resolution referring to multicriteria decision making are first addressed, mainly focusing on the domain of vagueness. It is followed by a brief overview of the fuzzy set theory and description of the proposed fuzzy VIKOR method. Then a hypothetical numeric illustration is conducted to clarify the method's effectiveness and feasibility developed in this study. Finally, conclusion and suggestion for future works are also proposed.

*Keywords*: Fuzzy VIKOR; Group decision making; Decision analysis; Multiple criteria decision making (MCDM)

## 1. Introduction

Making decision and selection is an indispensable part of daily life; of which the chief difficulty is that almost decision issues have multiple, even conflicting criteria. For instance, one may choose a car depending on cost, safety, comfort and gas mileage. The higher gas mileage reduces the comfort intensity due to its smaller passenger room [9]. Applications as well as methodologies toward solving MCDM problems have appeared in professional journals and conferences of diversified disciplines. Karacapilidis [19] presented a computer-supported collaborative argumentation and fuzzy similarity measures in multiple

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criteria decision making. Deng et al. [14] categorized four aspects of evaluation criteria in competitive companies; providing information for selecting adequate improvement actions. Wolters and Mareschal [44] pointed out that sensitivity analysis enables the application of multicriteria decision making methods in dynamic environments. In Chen's study [6], a software company selected three analysis engineer candidates based on five criteria by extended TOPSIS. As indicated by Chen [7], a fuzzy approach was used to select the location of distribution center in accordance with five criteria. In [23,33,35], the analytical hierarchy process (AHP) was elaborated in problems with multiple criteria. Rogers [34] used ELECTRE for weighting multiple environmental criteria. Choo [12] presented an interpretation of criteria weights in multicriteria decision making. Lin and Lin [24] addressed the use of orthogonal array with grey relational analysis to optimize the electrical discharge machining process with multiple performance characteristics. In [3], a case study for interdependence in multiple criteria decision making was taken. In the study of Jae et al. [18], incomplete information was processed in an interactive procedure for multiple criteria group decision making. Many problems related to MCDM are undertaken in a traditional manner such as: dominance, maximin, maximax [25], conjunctive, disjunctive, lexicographic, lexicographic semiorder, SAW, ELECTRE and TOPSIS.

In classical MCDM methods, the crucial goal is to find optimal or best solutions, which have maximum effectiveness with minimum cost. The lower performance rating with respect to specified criteria was frequently ignored among them. To lead a proper agenda for decision makers, a VIKOR method (Serbian: VIseKriterijumsa Optimizacija I Kompromisno Resenje, means: Multicriteria Optimization and Compromise Solution) initiated by Opricovic in 1998 [29], in which the compromise solution should have a maximum 'group utility' ('majority' rule) and minimum individual regret of the 'opponent' is proposed to deal with MCDM problems. In the physical world, crisp data are inadequate to present the real situation since human's intuition, judgment, perception and preference are always vague and difficult to estimate. Dubois and Prade [15] pointed out that statistical decision methods do not measure the imprecision of humanbehavior; rather they are the means of modeling insufficient knowledge about the external environment. Fuzzy set theory approaches toward decision making consider human subjectivity, rather than merely applying objective probabilistic methods. Therefore, when the information in a decision making system is indistinct, uncertain, and vague or represented in linguistic terms, this leads to the study of a new decision analysis field- fuzzy decision analysis. In our study, the applicable VIKOR method is extended to cope with MCDM issues under fuzziness, thereby, we take this method as "fuzzy VIKOR".

#### 2. Basics of fuzzy set theory

Fuzzy set theory, first proposed by Zadeh in 1965 [48], is developed to solve problems in which descriptions are vague, imprecise and uncertain. A larger number of works combine fuzzy set theory with scientific principles and technologies, such as information clustering [27], control engineering [20], data analysis [16,31], pattern recognition [32], neural nets [2], robotics [8], decision and organization sciences [26], artificial intelligence [45], interpolative reasoning [46], preference modeling and multicriteria evaluation [17], production research [21], diagnosis [13], logic programming [42], non-monotonic reasoning [4], expert systems [37] and optimization techniques [28]. Besides, the fuzzy set theory has been widely applied in social sciences [38], management [43], and financial aspects [1].

In the following, some basic definitions and notations of fuzzy set theory will be encompassed, and these will be used throughout the paper unless otherwise stated [5,22]. Let X be the universe of discourse,  $X = \{x_1, x_2, ..., x_n\}$ . A fuzzy set  $\widetilde{A}$  of X is a set of order pairs  $\{(x_1, f_{\widetilde{A}}(x_1)), (x_2, f_{\widetilde{A}}(x_2)), ..., (x_n, f_{\widetilde{A}}(x_n))\}$ , where  $f_{\widetilde{A}}: X \to [0,1]$ , is the membership function of  $\widetilde{A}$ , and  $f_{\widetilde{A}}(x_i)$  stands for the membership degree of  $x_i$  in  $\widetilde{A}$ .

**Definition 2.1.** When X is a continuum rather than a countable or finite set, the fuzzy set  $\tilde{A}$  is denoted as:

$$\widehat{A} = \int_{X} f_{\widetilde{A}}(x) / (x) , \text{ where } x \in X$$
(1)

**Definition 2.2.** When X is a countable or finite set, the fuzzy set  $\tilde{A}$  is represented as:  $\tilde{A} = \sum_{i} f_{\tilde{A}}(x_i)/(x_i)$ ,

where 
$$x_i \in X$$
. (2)

**Definition 2.3.** A fuzzy set  $\tilde{A}$  of the universe of discourse X is convex if and only if for all  $x_1, x_2$  in X,  $f_{\tilde{A}}(Ix_1 + (1-I)x_2) \ge \min[f_{\tilde{A}}(x_1), f_{\tilde{A}}(x_2)]$ , where  $I \in [0,1], x_1, x_2 \in X$ . (3)

**Definition 2.4.** A fuzzy set  $\tilde{A}$  of the universe of discourse X is normal when its membership function  $f_{\tilde{A}}(x)$ 

satisfies: 
$$\max f_{\tilde{A}}(x) = 1$$
. (4)

**Definition 2.5.** A fuzzy number is a fuzzy subset in the universe of discourse X that is not only convex but also normal. (5)

**Definition 26.** The **a** -cut  $\tilde{A}_{a}$  and strong **a** -cut  $\tilde{A}_{a+}$  of the fuzzy set  $\tilde{A}$  in the universe of discourse X is defined by:

$$\widetilde{A}_{\mathbf{a}} = \{x_i \mid f_{\widetilde{A}}(x_i) \ge \mathbf{a}, x_i \in X\}, \text{ where } \mathbf{a} \in [0,1].$$
 (6)

$$A_{a+} = \{x_i \mid f_{\tilde{A}}(x_i) > a, x_i \in X\}$$
, where  $a \in [0,1]$ . (7)

**Definition 2.7.** A fuzzy set  $\widetilde{A}$  of the universe of discourse

X is convex if and only if every  $\tilde{A}_{a}$  is convex, that is  $\tilde{A}_{a}$  is a close interval of  $\Re$ . It can be written as:

$$\tilde{A}_{a} = [P_{1}^{(a)}, P_{2}^{a}], \text{ where } a \in [0,1].$$
 (8)

**Definition 2.8.** A triangular fuzzy number (see Fig. 1.) can be denoted as a triplet  $(a_1, a_2, a_3)$ , the membership function of the fuzzy number  $\tilde{A}$  is taken as:

$$f_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1, \\ (x - a_1) / (a_2 - a_1), & a_1 \le x \le a_2, \\ (a_3 - x) / (a_3 - a_2), & a_2 \le x \le a_3, \\ 0, & x > a_3. \end{cases},$$
(9)



Fig. 1. A triangular fuzzy number A

Let  $\tilde{A}$  and  $\tilde{B}$  be two positive fuzzy numbers parameterized by the triplets  $(a_1, a_2, a_3)$  and  $(b_1, b_2, b_3)$ , the operations (addition, subtraction, multiplication and division) of triangular fuzzy numbers can be performed as [11]:

$$\widetilde{A}(+)\widetilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$
(10)

$$\widetilde{A}(-)\widetilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$$
(11)

$$\widetilde{A}(\times)\widetilde{B} = (a_1b_1, a_2b_2, a_3b_3)$$
(12)

$$\widetilde{A}(\div)\widetilde{B} = (a_1 / b_3, a_2 / b_2, a_3 / b_1)$$
(13)

$$\tilde{A}(x)r = (ra_1, ra_2, ra_3)$$
 (14)

**Definition 2.9.** According to [36], the  $\vee$  (max) and  $\wedge$  (min) operations are defined as:

$$\widetilde{A}(\vee)\widetilde{B} = (a_1 \vee b_1, a_2 \vee b_2, a_3 \vee b_3)$$
<sup>(15)</sup>

$$\widetilde{A}(\wedge)\widetilde{B} = (a_1 \wedge b_1, a_2 \wedge b_2, a_3 \wedge b_3)$$
(16)

# 3. The Fuzzy VIKOR method

A systematic approach of a fuzzy VIKOR method for multicriteria group decision making in vague environment is given in this section. As indicated in [29,39,40,41], the basic principle of VIKOR is that each alternative can be evaluated by each criterion function; the compromise ranking can be presented by comparing the degree of closeness to the ideal alternative. Development of the VIKOR methodology started with the  $L_p$  – metric presented in [47],

$$L_{pi} = \left\{ \sum_{j=1}^{n} \left[ w_j (f_i^* - f_{ij}) / (f_i^* - f_i^-) \right]^p \right\}^{1/p}$$

$$1 \le p \le \infty; \quad i = 1, 2, ..., m$$
(17)

where  $f_{ij}$  is the value of j th criterion function for the alternative  $A_i$ .

Formally, a typical fuzzy multicriteria decision making problem can be expressed in the matrix format as:

$$\widetilde{D} = \begin{array}{cccc} C_1 & C_2 & C_n \\ A_1 \begin{bmatrix} \widetilde{x}_{11} & \widetilde{x}_{12} & \cdots & \widetilde{x}_{1n} \\ \widetilde{x}_{21} & \widetilde{x}_{22} & \cdots & \widetilde{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ A_m \begin{bmatrix} \widetilde{x}_{m1} & \widetilde{x}_{m2} & \cdots & \widetilde{x}_{mn} \end{bmatrix}, \quad i = 1, 2, ..., m; \quad j = 1, 2, ..., m$$

$$\widetilde{W} = [\widetilde{w}_1, \widetilde{w}_2, \dots, \widetilde{w}_n], \quad j = 1, 2, \dots, n$$

where  $A_1, A_2, ..., A_m$  are the alternatives to be chosen,  $C_1, C_2, ..., C_n$  are the evaluation criteria,  $\tilde{x}_{ij}$  is the rating of alternative  $A_i$  with respect to  $C_j$ ,  $\tilde{w}_j$  is the importance weight of the *j* th criterion holds,  $\tilde{x}_{ij}$  and  $\tilde{w}_j$  are linguistic variables denoted by triangular fuzzy numbers. Moreover, an algorithm of the proposed fuzzy VIKOR method under fuzziness environment is described in the following.

*Step 1*. Form a group of decision makers, then determine the evaluation criteria and feasible alternatives.

*Step* **2** Identify the appropriate linguistic variables for evaluating the importance weight of criteria, and the rating of alternatives.

Step 3. Pull the decision makers' opinions to get the

aggregated fuzzy importance weight of criteria, and aggregated fuzzy rating of alternatives. If there are kpersons in a decision making committee, the importance weight of criteria and rating of each alternative can be measured by:

$$\widetilde{w}_j = \frac{1}{k} [\widetilde{w}_j^1 \oplus \widetilde{w}_j^2 \oplus ... \oplus \widetilde{w}_j^k]$$
(18)

$$\widetilde{x}_{ij} = \frac{1}{k} [\widetilde{x}_{ij}^1 \oplus \widetilde{x}_{ij}^2 \oplus \dots \oplus \widetilde{x}_{ij}^k]$$
(19)

Step 4. Construct a fuzzy decision matrix, then d etermine the fuzzy best value (FBV,  $\tilde{f}_j^*$ ) and fuzzy worst value

(FWV,  $\tilde{f}_{j}$  ) of all criteria functions.

$$\widetilde{f}_{j}^{*} = \max_{i} \widetilde{x}_{ij} , \quad j \in B$$

$$\widetilde{f}_{j}^{-} = \min_{i} \widetilde{x}_{ij} , \quad j \in C$$
(20)

where B is associated with the benefit criteria, and C is related to the cost criteria.

# Step 5. Compute the index $\widetilde{S}_i$ and $\widetilde{R}_i$ $\widetilde{S}_i = \sum_{i=1}^n \widetilde{w}_j (\widetilde{f}_j^* - \widetilde{x}_{ij}) / (\widetilde{f}_j^* - \widetilde{f}_j^-)$

$$\widetilde{R}_{i} = \max_{j} [\widetilde{w}_{j} (\widetilde{f}_{j}^{*} - \widetilde{x}_{ij}) / (\widetilde{f}_{j}^{*} - \widetilde{f}_{j}^{-})]$$
(22)

where  $\tilde{S}_i$  refers to the separation measure of  $A_i$  from the fuzzy best value, similarly,  $\tilde{R}_i$  is the separation measure of  $A_i$  from the fuzzy worst value, and  $\tilde{w}_j$  is the weight of each criterion.

Step 6. Compute the index 
$$\widetilde{Q}_i$$
  
 $\widetilde{Q}_i = v(\widetilde{S}_i - \widetilde{S}^*)/(\widetilde{S}^- - \widetilde{S}^*) + (1 - v)(\widetilde{R}_i - \widetilde{R}^*)/(\widetilde{R}^- - \widetilde{R}^*)$   
 $\widetilde{S}^* = \min_i \widetilde{S}_i, \widetilde{S}^- = \max_i \widetilde{S}_i,$   
where  $\widetilde{R}^* = \min_i \widetilde{R}_i, \widetilde{R}^- = \max_i \widetilde{R}_i$ 
(23)

The index  $\min_i \widetilde{S}_i$  is with a maximum majority rule, and  $\min_i \widetilde{R}_i$  is with a minimum individual regret of opponent. And v is introduced as the weight in strategy of the maximum group utility, usually v = 0.5.

# Step 7. Defuzzification for triangular fuzzy number $\widetilde{Q}_i$

The process of converting a fuzzy number into a crisp value is called defuzzification. In this paper, Chen's [10] method of maximizing set and minimizing set is applied.

The maximizing set is defined as:  $M = \{(x, f_M(x)) | x \in R\}$  with the membership function:

$$f_M(x) = \begin{cases} (x - x_1)/(x_2 - x_1), x_1 \le x \le x_2, \\ 0, & otherwise \end{cases}$$
(24)

By contrast, the minimizing set is defined as:  $G = \{(x, f_G(x)) \mid x \in R\}$ , with the membership function:

$$f_G(x) = \begin{cases} (x - x_2) / (x_1 - x_2), x_1 \le x \le x_2, \\ 0, & otherwise \end{cases}$$
(25)

Then the right utility  $U_M(F_i)$  and left utility  $U_G(F_i)$  can be denoted as:

$$U_M(F_i) = \sup_{x} (f_{F_i}(x) \wedge f_M(x))$$
(26)

$$U_{G}(F_{i}) = \sup_{x} (f_{F_{i}}(x) \wedge f_{G}(x))$$
(27)

As a result, the crisp value can be obtained by combining the right and left utilities.

$$U_T(F_i) = [U_M(F_i) + 1 - U_G(F_i)]/2$$
(28)

#### Step 8. Rank the alternatives by the crisp value $Q_i$

The index  $Q_i$  implies the separation measure of  $A_i$  from the best alternative. That is, the smaller value indicates the better performance of an alternative.

Step 9. Propose a compromise solution (a) by the index Q, if the condition 'A' is satisfied.

A. Acceptable advantage:  $Q(a') - Q(a') \ge DQ$ 

DQ = 1/(M - 1), M is the number of alternatives  $(DQ = 0.25, \text{ if } M \le 4)$ , and  $a^{"}$  stands for the alternative with second position ranked by index Q. If condition 'A' is not satisfied,  $a^{'}, a^{"}, \dots, a^{(m)}$  are compromise solutions. The best alternative is the one with the minimum of Q.

(21)

# 4. An illustrative example

Suppose a college intends to select the principal. Three candidates ( $A_1$ ,  $A_2$ ,  $A_3$ ) are to be evaluated by three decision-makers ( $D_1$ ,  $D_2$ ,  $D_3$ ) in five benefit criteria: academic accomplishment ( $C_1$ ), research ability ( $C_2$ ), communication skill ( $C_3$ ), maturity ( $C_4$ ) and experience ( $C_5$ ). The proposed fuzzy VIKOR method is employed to solve this multicriteria group decision making problem and the computational procedures are inducted as follows:

*Step 1*. Three decision makers use the linguistic variables, such as very low, low, medium low, medium, medium high, high and very high (the corresponding fuzzy numbers of linguistic terms are shown in Table 1) to assess the importance weight of five criteria, and the results are performed in Table 2.

#### Table1

Linguistic variables for the importance weight of criteria

Linguistic terms	Corresponding fuzzy numbers
Very Low (VL)	(0.0,0.0,0.1)
Low (L)	(0.0,0.1,0.3)
Medium Low (ML)	(0.1,0.3,0.5)
Medium (M)	(0.3,0.5,0.7)
Medium High (MH)	(0.5,0.7,0.9)
High (H)	(0.7,0.9,1.0)
Very High (VH)	(0.9,1.0,1.0)

Table2
--------

The importance	weight of the criteria	
The importance	weight of the effectu	

	$D_1$	$D_2$	$D_3$
$C_1$	VH	MH	MH
$C_{2}$	М	MH	ML
$C_3$	VH	Н	VH
$C_4$	Н	Н	MH
$C_5$	VH	Н	VH

*Step 2*. Three decision-makers use linguistic variables: very poor, poor, medium poor, fair, medium good, good and very good (the corresponding fuzzy numbers of linguistic terms are shown in Table 3) to evaluate the rating of three

candidates in five criteria, and the fuzzy decision matrix is given in Table 5.

Table 3

]	Linguistic	variables	for the	rating of	f alternative	•
1	Singaistie	, and the second	ioi uie	rating 0	i anconnaci , c	1

Linguistic terms	Corresponding fuzzy numbers
Very Poor (VP)	(0.0,0.0,1.0)
Poor (P)	(0.0,1.0,3.0)
Medium Poor (MP)	(1.0,3.0,5.0)
Fair (F)	(3.0,5.0,7.0)
Medium Good (MG)	(5.0,7.0,9.0)
Good (G)	(7.0,9.0, 10)
Very Good (VG)	(9.0, 10, 10)

**Step3**. According to Eq (18), convert the linguistic variables into triangular fuzzy numbers  $(a_1, a_2, a_3)$  as well aggregate the fuzzy weight of criteria as shown in Table 4. To be more explicit, the weight  $(\tilde{w}_1)$  of  $C_1$  is computed as:

$$\widetilde{w}_1 = \frac{1}{3}[(0.9,1,1) \oplus (0.5,0.7,0.9) \oplus (0.5,0.7,0.9)] = (0.63,0.8,0.93)$$

		Table 4		
Aggregated	d import	tance weight	t of the crit	eria
~	ã	ã	â	~

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$
Weight	(0.63,0.80,0.93)	(0.30, 0.50, 0.70)	(0.83,0.97,1.00)	(0.63, 0.83, 0.97)	(0.83,0.97,1.00)

Step 4. By Eq(19), convert and aggregate the fuzzy rating of three candidates to construct the triangular fuzzy number decision matrix, the results are addressed in Table 6. To make it clearer, the rating ( $\tilde{x}_{11}$ ) of  $A_1$  with respect to  $C_1$  is calculated as:

$$\tilde{x}_{11} = \frac{1}{3}[(7,9,10) \oplus (9,10,10) \oplus (9,10,10)] = (8.33,9.67,10)$$

Step 5. Determine the fuzzy best value  $\tilde{f}_j^*$  and fuzzy worst value  $\tilde{f}_j^-$ . Investigating the aggregated triangular fuzzy number decision matrix with Eqs (15), (16) and (20), the  $\tilde{f}_j^*$ and  $\tilde{f}_j^-$  are listed in Table 7.

Table 5					
The fuzzy rating of three alternatives in five criteria					
		De	ecision-mak	ers	
Criteria	Candidates	$D_1$	$D_2$	$D_3$	
$C_1$	$A_1$	G	VG	VG	
	$A_2$	F	F	MG	
	$A_3$	MG	MP	F	
$C_2$	$A_1$	MP	G	G	
	$A_2$	G	MG	MG	
	$A_3$	VG	F	VG	
$C_3$	$A_1$	MG	F	G	
	$A_2$	G	MG	G	
	$A_3$	G	G	VG	
$C_4$	$A_1$	G	MG	MG	
	$A_2$	VG	G	VG	
	$A_3$	G	VG	MG	
$C_5$	$A_1$	MP	MG	MP	
	$A_2$	VG	MG	G	
	$A_3$	G	VG	MG	

#### Table 6

		C	1		
Aggregated	trianoula	r †1177V	number	decision	matrix
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	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$
$A_1$	(8.33,9.67,10.0)	(5.00,7.00,8.33)	(5.00,7.00,8.67)	(5.67,7.67,9.33)	(2.33,4.33,6.33)
$A_2$	(3.67,5.67,7.67)	(5.67,7.67,9.33)	(6.33,8.33,9.67)	(8.33,9.67,10.0)	(7.00,8.67,9.67)
$A_3$	(3.00,5.00,7.00)	(7.00,8.33,9.00)	(7.67,9.33,10.0)	(7.00,8.67,9.67)	(7.00,8.67,9.67)

#### Table 7

	0*	a — .
Fuzzy boot volue	(t) and fuzzy worst value	10(t)
$\Gamma u Z Z V U C S U V a I u C$	i i i i and fuzzy worst van	

	$C_1$	$C_2$	<i>C</i> <sub>3</sub>	$C_4$	$C_5$
${\widetilde{f}}_j^{*}$	(8.33,9.67,10.0)	(7.00,8.33,9.33)	(7.67,9.33,10.0)	(8.33,9.67,10.0)	(7.00,8.67,9.67)
$\tilde{f}_j^{-}$	(3.00,5.00,7.00)	(5.00,7.00,8.33)	(5.00,7.00,8.67)	(5.67,7.67,9.33)	(2.33,4.33,6.33)

**Step 6**. As stated in Eqs (21) and (22), the index  $\tilde{S}_i$  and  $\tilde{R}_i$  are computed respectively in Table 8.

Table 8					
Index $\widetilde{S}_i$ and $\widetilde{R}_i$					
	$\overline{A}_1$	A <sub>2</sub>	A <sub>3</sub>		
$\widetilde{S}_i$	(-0.48,3.27,0.7)	(-0.98,1.35,1.23)	(-0.9,1.22,2.44)		
$\widetilde{R}_i$	(0.80,0.97,11.0)	(0.06,0.69,4.43)	(0.12,0.8,4.90)		

Step 7. By applying Eqs (15), (16) and (23), the index  $S^*$ ,  $S^-$ ,  $R^*$  and  $R^-$  can be seen in Table 9.

Table 9

Index S	$S^*, S^-, R^*$ and $R^-$
$S^*$ : (-0.98,1.22,0.7)	<i>S</i> <sup>-</sup> : (-0.48,3.27,2.44)
$R^*$ : (0.06,0.69,4.43)	$R^-$ : (0.12,0.97,11.00)

Step 8. Calculate the  $\tilde{Q}_i$  for each candidate with Eq (23), and the results are shown in Table 10. To make it more comprehensive,  $\tilde{Q}_1$  can be calculated as:

$$\begin{split} Q_1 &= \\ 0.5 \otimes \frac{(-0.48, 3.27, 0.7) - (-0.98, 1.22, 0.7)}{(-0.48, 3.27, 2.44) - (-0.98, 1.22, 0.7)} + \\ (1 - 0.5) \otimes \frac{(0.8, 0.97, 11) - (0.06, 0.69, 4.43)}{(0.12, 0.97, 11) - (0.06, 0.69, 4.43)} \\ &= (-0.37, 1, -1.98) \end{split}$$

		Table 10	
		Index $\tilde{Q}_i$	
	$A_1$	$A_2$	$A_3$
$\tilde{Q}_i$	(-0.37,1.00,-1.98)	(-0.45,0.03,-1.44)	(-0.43,0.20,-2.01)

Step 9. The triangular fuzzy number  $\tilde{Q}_i$  is defuzzified into a crisp number  $Q_i$  with Eqs (24)-(28), and the values are shown in Table 11.

Table 11 Index  $Q_i$  and rank for candidates

	$A_1$	$A_2$	$A_3$
$Q_i$	0.8434	0.4772	0.3035
Rank	3	2	1

Step 10. As stated in Table 11, the smaller  $Q_i$  implies the better performance of a candidate. Hence,  $A_3$  is given precedence over  $A_2$  and  $A_1$  in that order.

Step 11. Since the  $Q(A_2) - Q(A_3) = 0.1737(<0.25)$  is not satisfied with condition 'A'. It is suggested that the compromise candidate is  $A_3$  owing to its closer degree

toward the best candidate.

## 5. Conclusion and suggestion

The results show that fuzzy VIKOR is very supportive in dealing with situations, which are too complicated to be reasonably stated in conventional quantitative expressions. The chosen candidate could be accepted due to its maximum group utility of the majority as well as the minimum individual regret of opponent. Fuzzy VIKOR method might not only be the compromise foundation stone within mutual communication, negotiation and conflict management, but also a bridge for reaching an agreement among a decision committee. Furthermore, simultaneous consideration of high and low performance rating of feasible alternatives or candidates can help decision makers keep away from improper decisions. Although this method presented in our study is illustrated by a college principal selection problem, it can also be put on many other perspectives of multicriteria group decision making issues.

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