Box-Jenkins vs. Artificial Neural Networks in Predicting Commodity Prices
Forecasting the Export Prices of Thai Rice

Rujirek Boosarawongse\textsuperscript{1)}, Henry C. Co\textsuperscript{2)}, Ping Feng\textsuperscript{3)}

\textsuperscript{1)} Department of Applied Statistics, Faculty of Science, KMITL, Bangkok, Thailand. (rujirek@mozart.inet.co.th)
\textsuperscript{2)} Technology & Operations Management California Polytechnic and State University 3801 W. Temple Ave., Pomona, CA 91786 (hco@csupomona.edu)
\textsuperscript{3)} Technology and Science Institute of Northern Taiwan, Department of Information Management Taipei, Taiwan (pfeng@tsint.edu.tw)

Abstract

International commodity prices are determined by supply and demand, and to a large extent, governmental interventions through trade barriers and subsidies. Forecasting rice prices has always been a great challenge to researchers because determinants of supply and demand such as agricultural and environmental factors, meteorological factors, biophysical factors, changing demographics, etc. interact in a complex manner. Among statistical techniques used to predict rice prices, researchers have found the Box-Jenkins method to perform well in predicting agricultural farm prices. To study the underlying forces and structure that produced the observed time series data on Thailand’s weekly rice export prices, we first used the Box-Jenkins method to fit the data. Next, we evaluated various aggregate measures of forecast error (the mean absolute deviation, the mean squared error, the mean absolute forecast error, and the root-mean squared error) to assess the performance of the Box-Jenkins models. Then we used the same data to train and cross-validate artificial neural networks. Our findings showed that while both Box-Jenkins and artificial neural networks performed well in forecasting the weekly export prices of Thai rice, the artificial neural networks produce better predictive accuracies in three of the four categories of rice analyzed.

Key words: Forecasting, Neural-Networks, Box-Jenkins, Validation Error

1. Introduction

Rice is the staple food of Asians. Nine of the top 10 rice producers – China, India, Indonesia, Bangladesh, Vietnam, Thailand, Myanmar, the Philippines, and Japan – are in Asia. In these countries, rice cultivation is intertwined with religious observances, festivals, customs, folklore, and traditions.

The world’s top three rice exporters are Thailand, India, and Vietnam, respectively. China, the largest producer and consumer of rice, has recently become a net importer of rice because of changing demographics (industrialization). Thailand is the world’s top rice exporter, with a market share of around 26%. Thailand exports many types of rice. Among these, Jasmine (Hom Mali) or “fragrant rice” is considered Thailand’s specialty and trade mark. Other categories of rice exports include white rice, parboiled rice, and glutinous rice. Table 1 shows the proportion of export quantity of each category in the last decade. Rice export accounts for about 20% of Thailand’s total agricultural export earnings [Vanichanont, 2004].

<table>
<thead>
<tr>
<th>Category</th>
<th>% by Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jasmine</td>
<td>31%</td>
</tr>
<tr>
<td>White</td>
<td>39%</td>
</tr>
<tr>
<td>Parboiled</td>
<td>26%</td>
</tr>
<tr>
<td>Glutinous</td>
<td>4%</td>
</tr>
</tbody>
</table>

Table 1 Proportion of Export by Weight

International commodity prices are determined by supply and demand, and to a large extent, governmental interventions through trade barriers and subsidies. Forecasting rice prices has always been a great challenge to researchers
because determinants of supply and demand such as agricultural and environmental factors, meteorological factors, biophysical factors, changing demographics, etc. interact in a complex manner. Among statistical techniques used to predict rice prices, researchers have found the Box-Jenkins [Box and Jenkins, 1976; Box and Jenkins 1994] method to perform well in predicting agricultural farm prices. Deetee [Deetee, 1991], for instance, applied the decomposition method and the Box-Jenkins model to investigate rice farm-gate prices. The study revealed that the Box-Jenkins model performed better. Kerdsomboon [Kerdsomboon, 1999] applied various statistical forecasting methods to study planting area, rice production, and agricultural farm prices. The author found the regression model to be suitable for forecasting planting area and rice production, while the Box-Jenkins model performed better in predicting agricultural farm prices. Maliwan [Maliwan, 2003] employed the ARIMA model to forecast the domestic prices of jasmine rice. Likewise, Fansiri [Fansiri, 2004] employed the ARIMA model to predict rice export prices. The studies revealed that the autoregressive (AR) model fits sufficiently well. Sangpattaranate [Sangpattaranate, 2005] compared 4 forecasting techniques – Holt-Winters exponential smoothing model, Box-Jenkins model, decomposition method and regression analysis – for forecasting the rice prices in Thailand. The decomposition method was found to be most suitable in forecasting the prices of Jasmine rice and main-crop rice with 5%, 15%, 25% broken, while the Box-Jenkins model was best for main-crop rice with 10% broken.

In this paper, we analyzed Thailand’s weekly rice export prices from January 8, 2003 through September 9, 1996. To study the underlying forces and structure that produced the observed time series data, we first used the Box-Jenkins method to fit the data. Next, we evaluated various aggregate measures of forecast error (the mean absolute deviation, the mean squared error, the mean absolute forecast error, and the root-mean squared error) to assess the performance of the Box-Jenkins models. Then we used the same data to train and cross-validate artificial neural networks (ANN). The ANN have been found to be able to decode nonlinear time series data, and have been applied in stock market [(Chong and Kyong, 1992), (Freisleben, 1992), (Kimoto et al, 1990), (Schoneburg, 1990), (Yao et al, 2000)], bond ratings, (Dutta, 1990), commodity and currency exchange [(Bergerson, 1991), (Grudnitski, 1993), (Hann and Steurer, 1996), (Kuan, 1995), (Refenes, 1993)], and other difficult-to-predict situations.

The paper is organized as follows: Section 2 details the Box-Jenkins models we used to forecast the weekly prices of the 4 categories of rice. The data used in model fitting covered the weeks of January 8, 2003 through May 3, 2006. The data from May 10, 2006 through September 9, 2006 was used for validation. In Section 3, we used the weekly prices through May 3, 2006 to train the artificial neural networks, and the data from May 10, 2006 through September 9, 2006 to cross validate the neural networks. Finally, Section 4 summarizes our findings and provides some concluding remarks.

2. The Box-Jenkins Model

The Box-Jenkins [Box, et al, 1994], [Wei, 1990] method is a statistical technique for time series forecasting. This study used the autoregressive integrated moving average (ARIMA \((p, d, q)\)) model. For any stationary time series, \(Z_t\) , the ARIMA \((p, d, q)\) of \(Z_t\) is \(\phi_p(B)(1-B)^dZ_t = \theta_q(B)a\) , where \(\phi_p(B)\) and \(\theta_q(B)\) are the regular autoregressive and moving average factors, respectively. The Box-Jenkins method consists of the following steps:

1. **Identification** – The Box-Jenkins model assumes that the time series is stationary. This study used logarithmic transformation for non-constant variance, and differencing for non-constant mean. The sample ACF and PACF of the stationary series were then computed and examined to identify the order of \(p\) and \(q\).
2. **Estimation** – The least-squares method for nonlinear parameters was used to determine the parameters of the model.
3. **Diagnostic Checking** – The Ljung-Box Statistic [Ljung and Box, 1978] was used to investigate model adequacy, i.e., to ascertain that the residuals are white noise (zero mean, constant variance, uncorrelated process and normally-distributed).
4. **Forecasting** – The model giving the least mean squared error (MSE) was chosen for forecasting. The forecasting model was then used to compute the fitted values and forecasts.

The data used in model fitting covered the weeks of January 8, 2003 through May 3, 2006. The data from May 10, 2006 through September 9, 2006 was used for validation. The plots in Figure 1 indicate that the time series are non-stationary in the mean and the variance. To have the stationary series, transformation techniques are used. First, the natural logarithm was taken, and then differencing was applied. Figure 2 shows the transformed time series that are stationary. The sample ACF and the sample PACF for the transformed series are plotted in Figure 3 and Figure 4, respectively.

Please insert [Figure 2 The Differenced Series of Natural Logarithms] here.
Please insert [Figure 3 and 4 Sample ACF and PACF for the Transformed Series] here.

The plots suggest that the ARIMA model is appropriate. Several models were found, and the best fitted model (least MSE) is presented below. At significance level 0.05, the estimates were found significant. The values under each estimator (in parentheses) below are the standard error:

- **Jasmine Rice**
  \[(1 - B)(1 - 0.6737B) \ln Z_t = (1 - 0.3666B)a, \]
  \[\text{ARIMA}(1,1,1)\]
  \[
  \begin{align*}
  (1) & \quad (0.1407) \\
  (2) & \quad (0.1769)
  \end{align*}
  \]

- **White Rice**
  \[(1 - B)(1 - 0.2227B) \ln Z_t = a, \]
  \[\text{ARIMA}(1,1,0)\]
  \[
  \begin{align*}
  (1) & \quad (0.0750)
  \end{align*}
  \]

- **Parboiled Rice**
  \[(1 - B)(1 - 0.2601B) \ln Z_t = a, \]
  \[\text{ARIMA}(1,1,0)\]
  \[
  \begin{align*}
  (1) & \quad (0.0745)
  \end{align*}
  \]

- **Glutinous Rice**
  \[(1 - B)(1 - 0.7116B) \ln Z_t = (1 - 0.5325B)a, \]
  \[\text{ARIMA}(1,1,1)\]
  \[
  \begin{align*}
  (1) & \quad (0.1900) \\
  (2) & \quad (0.2286)
  \end{align*}
  \]

Figure 5 shows the plots of the fitted values versus actual prices. Table 2 compares the aggregate measures of forecast errors. The aggregate measures are the mean-absolute deviation (MAD), the mean-squared error (MSE), the mean-absolute-percentage error (MAPE), and the root-mean-squared error (RMSE).

Please insert [Figure 5: Box-Jenkins Fitting Errors] here.

Table 2  Aggregate Measures of Model-Fitting Errors (Box-Jenkins)

<table>
<thead>
<tr>
<th>Category</th>
<th>MAD</th>
<th>MAPE</th>
<th>MSE</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jasmine</td>
<td>0.1218</td>
<td>1.1034%</td>
<td>0.0377</td>
<td>0.1941</td>
</tr>
<tr>
<td>White</td>
<td>0.0596</td>
<td>0.9753%</td>
<td>0.0068</td>
<td>0.0825</td>
</tr>
<tr>
<td>Parboiled</td>
<td>0.0607</td>
<td>0.9684%</td>
<td>0.0079</td>
<td>0.0887</td>
</tr>
<tr>
<td>Glutinous</td>
<td>0.0763</td>
<td>0.9266%</td>
<td>0.0164</td>
<td>0.1282</td>
</tr>
</tbody>
</table>

As shown in Figure 5, the fitted values closely matched the actual weekly prices in all 4 categories. The MAPE of the 4 rice categories are about 1%.

The data from May 10, 2006 through September 9, 2006 was used for validation. Figure 6 shows plots comparing actual weekly prices with the predicted values for the 4 categories of rice. The validation results for the 4 categories of rice are shown in Table 3. Except for “glutinous rice,” the Box-Jenkins method performed rather well. The MAPE of the three rice categories were less than 5%! The “glutinous rice,” which accounted for only 4% of rice exports, had average forecast errors of about 15%.

Please insert [Figure 6: Box-Jenkins Validation Errors] here.

Table 3  Aggregate Measures of Validation Errors (Box-Jenkins)

<table>
<thead>
<tr>
<th>Category</th>
<th>MAD</th>
<th>MAPE</th>
<th>MSE</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jasmine</td>
<td>0.6156</td>
<td>4.5695%</td>
<td>0.5441</td>
<td>0.7376</td>
</tr>
<tr>
<td>White</td>
<td>0.2181</td>
<td>2.6346%</td>
<td>0.0532</td>
<td>0.2307</td>
</tr>
<tr>
<td>Parboiled</td>
<td>0.3309</td>
<td>3.9291%</td>
<td>0.1580</td>
<td>0.3975</td>
</tr>
<tr>
<td>Glutinous</td>
<td>1.9767</td>
<td>14.7448%</td>
<td>6.4501</td>
<td>2.5397</td>
</tr>
</tbody>
</table>

In the following section, we used the same data to train and cross-validate the artificial neural networks.

3. Neural Network Model
Artificial neural networks are nonlinear mapping systems with a structure loosely based on principles observed in biological nervous systems. Artificial neural networks offer many advantages over conventional statistical methods (Shachmurove, 2002). The ANN uses the data to develop an internal representation of the relationship between the variables, and does not make assumptions about the nature of the distribution of the data. Another advantage is that while traditional regression analysis is not adaptive, artificial neural networks readjust their weights as new input data becomes available (Kuo and Reicht, 1994; Pao, 1989; Gilbert, Krishnaswamy, and Pashley, 2000). Applications include pattern classification, clustering and categorization, function approximation, prediction and forecasting, optimization, content-addressable memory, and control of dynamic systems [Jain, Mao, and Mohiuddin, 1996]. The artificial neural networks have also been found to produce better predictive accuracies than conventional statistical methods for a variety of problems in industrial engineering, marketing, banking and finance, insurance, and telecommunications [Smith & Gupta, 2000].

3.1 Modeling the Time Series

The models used in this study are fully-connected, feed-forward multi-layer perception (MLP) neural networks with three layers: an input layer, a hidden layer and an output layer. In a fully-connected MLP neural network, all the input nodes are connected to every hidden node and every hidden node is connected to the output nodes. The connection strengths and the activation function determine when each node is “active.” Activation functions commonly used in MLP neural network include linear, logistic sigmoid, hyperbolic tangent, and Gaussian. Convergence in training depends on the choice of activation function. In MLP neural networks, the logistic sigmoid function is often used, and is used in this paper.

The basic idea is to model the time series (prices of rice export), in terms of explanatory variables, or inputs. The activity of the input nodes represents the raw information that is fed into the network. In this study, the input layer of the ANN has 9 nodes:

1. Rice price for the current week.
2. Absolute difference between the price for current week and that of the previous week.
3. Sign of the difference between current week’s price and that of the previous week, encoded as 0.8 for positive difference, 0.2 for negative and 0 for no change.
4. One month (4 weeks) moving average of rice prices.
5. Absolute first difference of the 4-week (centered) moving average.
6. Encoded sign of the first difference of the 4-week (centered) moving average.
7. One quarter (12 weeks) moving average of rice prices.
8. Absolute first difference of the 12-week (centered) moving average.
9. Encoded sign of the first difference of the 12-week (centered) moving average.

The hidden layer(s) in the network and the nonlinear activation function of nodes allow non-linearity to be introduced into neural networks. The literature claims that a single hidden layer neural network is sufficient for a network to be a universal function approximator [K. Funahashi 1989, K. Hornik, M. Stinchcombe, and H. White 1989, K. Hornik, M. Stinchcombe, and H. White 1990, G. Cybenko 1989]. After experimenting on various topologies, consisting of one to three hidden layers, we found this to be true in this paper.

With sufficient number of nodes in the hidden layer, the MLP can be trained to approximate any nonlinear function to any arbitrary degree of accuracy [White, 1992; Cybenko, 1989]. However, using a large number of hidden nodes can lead to overfitting of the training data. On the other hand, ANN will not train well (large discrepancy between the true target and the expectation of the model output) if the topology is too simple for the data at hand. A complex topology creates large variance while a simple topology creates large bias [German, Bienenstock & Doursat, 1992]. All neural networks used in this study consisted of one hidden layer of 5 nodes. The number of network layers, hidden nodes, and the stopping criteria were determined through trial-and-errors process because no commonly accepted theory exists for predetermining the optimal number of nodes in the hidden layer [Tu, 1996].

The output layer consists of one node: the rice price of the following week

3.2 Training Patterns

Training an ANN involves finding the synaptic weight values such that the output of the model is close to the expected target (i.e., the rice price next week). Most neural networks contain some form of ‘learning rule’ to modify the weights of the connections. The weights are determined to minimize the sum of squared differences between network outputs and its
desired outputs. In this study, we use error back propagation, which is probably the most used algorithm to train MLP [Rumelhart, Hinton and Williams, 1986]. It is basically a gradient descent algorithm of the error computed on a suitable learning set.

The weekly rice prices data provided the training and testing patterns for the neural network. For convenience, we transformed the original data by applying the following linear equation:

\[
\hat{y}_t = \frac{y_t - y_{\text{min}}}{y_{\text{max}} - y_{\text{min}}}
\]

In (5), \(y_t\) is the rice price for week \(t\), and \(y_{\text{min}}\) and \(y_{\text{max}}\) are the minimum and maximum observed values for the entire data set. Note that \(0 \leq \hat{y}_t \leq 1\). Each training pattern consisted of 9 input and 1 output values. The input and output values are numbers between 0 and 1.

The training set consisted of weekly prices through May 3, 2006, and the testing set is for the weeks of May 10 through September 6, 2006. The training data was used to update the weights, but not the data for the testing pattern. This way, the testing pattern can be used as an indication of whether or not memorization is taking place. When a neural network memorizes the training data, it produces acceptable results for the training data, but poor results when tested on unseen data.

### 3.3 The Training Sessions

A pattern with a root-mean-square (RMS) training error less than a predetermined error margin is considered learned. In this study, we set the error margin at 0.075. The training session stops when all patterns are correctly classified.

Figure 7 shows the plots of the RMS errors during the training sessions. The ANN uses the training set to learn, thus the RMS error of the training set would be lower than the RMS error of the testing set. In each case, the RMS plot for the training set is below the RMS plot of the testing set.

As the ANN learns, the RMS error plot of all 4 training sets decreased sharply and then tapered off. The training patterns were used to update the weights, until all patterns in the training set have been classified as learned. Except for the case of the “glutinous rice,” the RMS error plot for the testing set also decreased sharply and then tapered off. Figure 8 shows plots comparing actual weekly prices with the fitted values for the 4 training sets. The fitted values closely matched the actual weekly prices.

Not all training sessions were successful. As the ANN learns, the RMS error of the testing set for “glutinous rice” (see Figure 7-d) decreased sharply and then bounced up and continued to rise as the ANN memorized the training data. Table 4 compares the aggregate measures of forecast errors during the training session. The fitted values closely matched the actual weekly prices. The MAPE of all 4 rice categories are below 2%.

### Table 4 Aggregate Measures of Model-Fitting Errors (Neural Networks)

<table>
<thead>
<tr>
<th>Category</th>
<th>MAD</th>
<th>MAPE</th>
<th>MSE</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jasmine</td>
<td>0.1278</td>
<td>1.1473%</td>
<td>0.0273</td>
<td>0.1653</td>
</tr>
<tr>
<td>White</td>
<td>0.0813</td>
<td>1.3217%</td>
<td>0.0106</td>
<td>0.1028</td>
</tr>
<tr>
<td>Parboiled</td>
<td>0.1053</td>
<td>1.7247%</td>
<td>0.0166</td>
<td>0.1290</td>
</tr>
<tr>
<td>Glutinous</td>
<td>0.1243</td>
<td>1.5366%</td>
<td>0.0269</td>
<td>0.1639</td>
</tr>
</tbody>
</table>

### 3.4 Results of Cross Validation

The testing set covers the weeks of May 10 through September 6, 2006. The cross validation results for the various categories of rice are shown in Table 5. Except for “glutinous rice,” the neural networks appear to perform well. The
training session for “Jasmine” rice was the most successful, with an MAPE of about 2%! The other two categories of rice – “White” and Parboiled” – had average forecast errors of 3.2% and 3.8%, respectively.

### Table 5: Aggregate Measures of Validation Errors (Neural Networks)

<table>
<thead>
<tr>
<th>Category</th>
<th>MAD</th>
<th>MAPE</th>
<th>MSE</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jasmine</td>
<td>0.2878</td>
<td>2.1786%</td>
<td>0.1086</td>
<td>0.3296</td>
</tr>
<tr>
<td>White</td>
<td>0.2625</td>
<td>3.1665%</td>
<td>0.0800</td>
<td>0.2829</td>
</tr>
<tr>
<td>Parboiled</td>
<td>0.3232</td>
<td>3.8497%</td>
<td>0.1387</td>
<td>0.3724</td>
</tr>
<tr>
<td>Glutinous</td>
<td>1.4876</td>
<td>11.0815%</td>
<td>3.8002</td>
<td>1.9494</td>
</tr>
</tbody>
</table>

The worst performance, as noted earlier, was the training of the ANN for “glutinous rice.” The MAPE for the category was about 11%. While the training data was able to produce a good fit (Table 4), the ANN for the category gave somewhat disappointing results when tested on unseen data (Table 5). Figure 9 shows plots comparing actual weekly prices with the predicted values for the 4 categories of rice.

### 4. Summary and Concluding Remarks

Forecasting is predicting the future. To produce a meaningful forecast, the forecast errors must be within acceptable limits. Both Box-Jenkins and artificial neural networks performed well in predicting the weekly export prices of rice.

Figure 6 and Figure 9 compare the actual weekly prices with the predicted values from the Box-Jenkins and the artificial neural networks, respectively. Although the differences are small, it is noteworthy that the predicted values for all 4 categories are consistently below the actual values. This suggests that the underlying forces and structure that produced the observed time series data may have shifted slightly in recent months. Prices of rice are determined by supply and demand. Recently, the USDA reported that world’s rice ending stocks are at 3% below the previous month and projected to be 11% below the previous marketing period. The world’s rice ending stocks are the lowest since 1982/83 [Coats, 2006]. World rice consumption exceeding production, weather conditions reducing production, continued strong global growth, the weakening of the US dollar in the 2006-07 marketing period, changing demographics, have contributed to bullish rice prices in recent months [Bennett, 2006].

Table 6 shows the validation errors of the artificial neural networks as percentages of the validation errors of the Box-Jenkins method. The ANN produced better predictive accuracies, except for “white rice” where the MAPE is 20% above the Box-Jenkins method. For “Jasmine rice,” the MAPE of the ANN is less than half of the Box-Jenkins method. Even with the hard-to-predict “glutinous rice” prices, the MAPE of the ANN is 75% of the Box-Jenkins method.

Figure 10 shows the bar charts comparing the aggregate validation error measures of the Box-Jenkins method and the artificial neural networks. The artificial neural networks produced better predictive accuracies because they are nonlinear mapping systems. The artificial neural networks use the time series data to develop an internal representation of the relationship between the variables, and do not make assumptions about the nature of the distribution of the data. Another advantage is that while traditional regression analysis is not adaptive, artificial neural networks readjust their weights as new input data becomes available. The biggest drawback of the methodology is that artificial neural networks are “black boxes.” It is impossible to figure out how relations in their hidden layers are estimated [Li, 1994, Gilbert, Krishnaswamy, and Pashley, 2000].
Table 7: Validation Errors of the ANN as % of the Validation Errors of Box-Jenkins

<table>
<thead>
<tr>
<th>Category</th>
<th>MAD</th>
<th>MAPE</th>
<th>MSE</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jasmine</td>
<td>47%</td>
<td>48%</td>
<td>20%</td>
<td>45%</td>
</tr>
<tr>
<td>White</td>
<td>120%</td>
<td>120%</td>
<td>150%</td>
<td>123%</td>
</tr>
<tr>
<td>Parboiled</td>
<td>98%</td>
<td>98%</td>
<td>88%</td>
<td>94%</td>
</tr>
<tr>
<td>Glutinous</td>
<td>75%</td>
<td>75%</td>
<td>59%</td>
<td>77%</td>
</tr>
</tbody>
</table>

References


Figure 1: Weekly Prices (January 8, 2003 to May 3, 2006)
Figure 2: The Differenced Series of Natural Logarithms
Autocorrelation Function for DLJas

Autocorrelation Function for DLp

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Autocorrelation Function for DLg

Autocorrelation Function for DLwr

Figure 3: Sample ACF for the Differenced Series of Natural Logarithms
Partial Autocorrelation Function for DL\text{ws}

\begin{figure}[h]
\centering
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{jasmine_pacf.png}
\caption{(a) Jasmine Rice}
\end{subfigure}
\hfill
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{white_pacf.png}
\caption{(b) White Rice}
\end{subfigure}

\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{parboiled_pacf.png}
\caption{(c) Parboiled Rice}
\end{subfigure}
\hfill
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{glutinous_pacf.png}
\caption{(d) Glutinous Rice}
\end{subfigure}

\caption{Sample PACF for the Differenced Series of Natural Logarithms}
\end{figure}
Figure 5: Box-Jenkins Fitting Errors (Through May 3, 2006)
Figure 6: Box-Jenkins Validation Errors (May 10 through September 6, 2006)
Figure 7: Artificial-Neural Networks Training RMSE Plots
Figure 8: Artificial-Neural Networks Fitting Errors (Through May 3, 2006)
Figure 9: Artificial-Neural Networks Validation Errors (May 10 through September 6, 2006)
Figure 10: Validation Errors: Box-Jenkins vs. Artificial Neural Networks