Optimal Multiple-Periodic Preventive Maintenance Policy for Leased Equipment

Thanawath Niyamosoth[1)], Jarumon Pongpech[2)]

[1)] Thammasat University, Department of Industrial Engineering
(t_niyamosoth@hotmail.com)
[2)] Thammasat University, Department of Industrial Engineering (jjarumon@engr.tu.ac.th)

Abstract

The purpose of this research is to determine the optimal PM actions that minimize the total maintenance costs of the new model of PM policy for leased equipment. The new model proposed in this research is developed by combining the advantages of both sequential and periodic PM policies. The problem is a two-parameter optimization with one and $(k_1 + k_2)$ -dimensional respectively. The optimal solution is obtained by using a two-stage approach where at each stage a one-parameter optimization is solved. The optimal solutions obtained by the model are (i) the optimal constant time intervals to carry out PM, and (ii) the optimal level of PM actions.

1. Introduction

For the lease of equipment, the lessor is responsible to carry out the maintenance. Usually, the contract of lease specifies the penalty for repairs not being finished within a specified time limit. Through PM actions, the cost resulting from failures can be reduced, however, it incurs the cost of PM actions. This implies that optimal PM policy minimizes total maintenance cost through a proper trade-off between the PM cost and the cost due to failures.

Jaturonnatee et al. (2005) proposed the optimal sequential PM policy for leased equipment which the researchers call Policy 1. Under this policy, a unit is preventively maintained at unequal time intervals. Usually, the time intervals become shorter and shorter as time passes, considering that most units need more frequent maintenance with increased ages. Pongpech and Murthy (2005) proposed the optimal periodic PM policy for leased equipment which the researchers call Policy 2. Unlike the sequential PM policy, in the periodic PM policy, a unit is preventively maintained at fixed time intervals of jT , $j = 1,2,...,k$ over the lease period. This implies that periodic PM policy are more convenience in implementation than the sequential PM policy because the time intervals between successive PM actions are constant. However, the expected maintenance cost for Policy 2 is always higher than that for Policy 1. The Policy 3 proposed in this paper develops the new model of PM policy for leased equipment by combining the advantages of both Policy 1 and Policy 2. In Policy 3, we divide the entire lease period into two sub-period called first lease period and second lease period. In each sub-period, we carry out periodic PM at equal time intervals, but the time intervals are different between the first and the second lease period. That is, in the first lease period, the PM actions are carried out at periodic times of $jT, j = 1,2,...,k_1$ and in the second lease period, the PM actions are carried out at periodic times $\frac{j^{'}T}{2}, j^{'} = 1,2,...,k_2$. This implies that in the second lease period, we carry out PM actions more frequent than that in the first lease period, correspond with the increasing rate of failure rate increases with higher rate. Any intervening failures over the lease period are assumed to be rectified through minimal repairs. The parameters of the policy are (i) the constant time intervals to carry out PM actions, and (ii) the level of PM actions.

2. Model formulation

We use the following notation

$F(t)$	failure distribution function
$f(t)$	failure density function associated with $F(t)$
$r(t)$	failure rate (hazard rate) function associated with $F(t)$
$\lambda_0(t)$	failure intensity function with no PM [= $r(t)$]
$\lambda(t)$	failure intensity function with PM actions
$\Lambda_0(t)$	cumulative failure intensity function with no PM $\left[= \int_0^t \lambda_0(x)dx\right]$
$\Lambda(t)$	cumulative failure intensity function with PM actions $\left[= \int_0^t \lambda(x)dx\right]$
$N(t)$	number of failures over $[0,t]$
Y	time to repair
$G(y)$	repair-time distribution function
$g(y)$	repair-time density function [= $dG(y)/dy$]
L	lease period
T	period of time instant to carry out PM
k_1	number of PM actions over the 1st lease period
k_2	number of PM actions over the 2nd lease period
t_j	time instant for j^{th} PM action over the 1st lease period
$t_{j'}$	time instant for j'^{th} PM action over the 2nd lease period
δ_j	reduction in intensity function due to j^{th} PM action over the 1st lease period
$\delta_{j'}$	reduction in intensity function due to j'^{th} PM action over the 2nd lease period
$C_p(\delta_j)$	cost of PM action resulting in a reduction δ_j in intensity function over the 1st lease period
$C_p(\delta_{j'})$	cost of PM action resulting in a reduction $\delta_{j'}$ in intensity function over the 2nd lease period

$TC_p(\delta_j, \delta_{j'})$	total cost of PM actions
C_f	average cost of CM action to rectify failure
TC_f	total cost of CM actions
τ	repair time limit [parameter of lease contract]
C_t	penalty cost per unit time if repair not completed within τ
ϕ	total cost due to penalty
$J(T, \underline{\delta})$	total expected cost to the lessor

2.1 Lease contract

The equipment is leased for a period L with the penalty associated with failures. The lessor incurs a penalty if the time to repair failures exceeds τ. Let Y denote the time to repair, then there is no penalty if $Y \le \tau$ and a penalty $(Y-\tau)C_t$ if $Y > \tau$.

2.2 Modeling failure and PM actions

We assume that all failures are rectified through minimal repairs. Under minimal repair, the hazard function immediately after repair is the same as that just before failure (Barlow and Hunter, 1960). We further assume that the time needed to rectify failed equipment is small in relation to the mean time between failures and such that it can be ignored. In this case, equipment failures with no PM actions occur according to a Non-Homogeneous Poinsson Process (NHPP) with intensity function $\lambda_0(t) = r(t)$ where $r(t)$ is the hazard function associated with the distribution $F(t)$ (Murthy, 1991).

The lessor carries out periodic PM with a period of T over the first lease period and $\frac{T}{2}$ over the second lease period. The time instants of PM actions are given by $t_j = jT$, $j = 1,2,\ldots,k_1$ for the first lease period and $t_{j'} = L_1 + \frac{j'T}{2}$, $j' = 1,2,\ldots,k_2$ for the second lease period. Each PM action results in a reduction in the intensity function. The reductions resulting from the j^{th} PM for the first lease period and the j'^{th} PM for the second lease period are given by δ_j and $\delta_{j'}$ respectively.

As a result, the failures over the lease period occur according to a NHPP with intensity function given by

$$\lambda(t) = \lambda_0(t) - \sum_{i=0}^{j} \delta_i \quad \text{for } t_j \le t < t_{j+1} \tag{1}$$

where $\delta_0 = 0$

over the first lease period

and

$$\lambda(t) = \lambda_0(t) - \sum_{i'=0}^{j'} \delta_{i'} \quad \text{for } t_{j'} \le t < t_{j'+1} \tag{2}$$

where $\delta_{0'} = \sum_{j=1}^{k_1} \delta_j$

over the second lease period

δ_j and $\delta_{j'}$ are constrained as follows

$$0 \le \delta_j \le \lambda_0(t_j) - \sum_{i=0}^{j-1} \delta_i \text{ for } 1 \le j \le k_1 \tag{3}$$

where $t_0 = 0$ and $\delta_0 = 0$

over the first lease period

and

$$0 \le \delta_{j'} \le \lambda_0(t_{j'}) - \sum_{i'=0}^{j'-1} \delta_{i'} \text{ for } 1 \le j' \le k_2 \tag{4}$$

where $t_{0'} = L_1$ and $\delta_{0'} = \sum_{j=1}^{k_1} \delta_j$

over the second lease period

Fig. 1 shows a plot of intensity function for failure with and without PM actions.

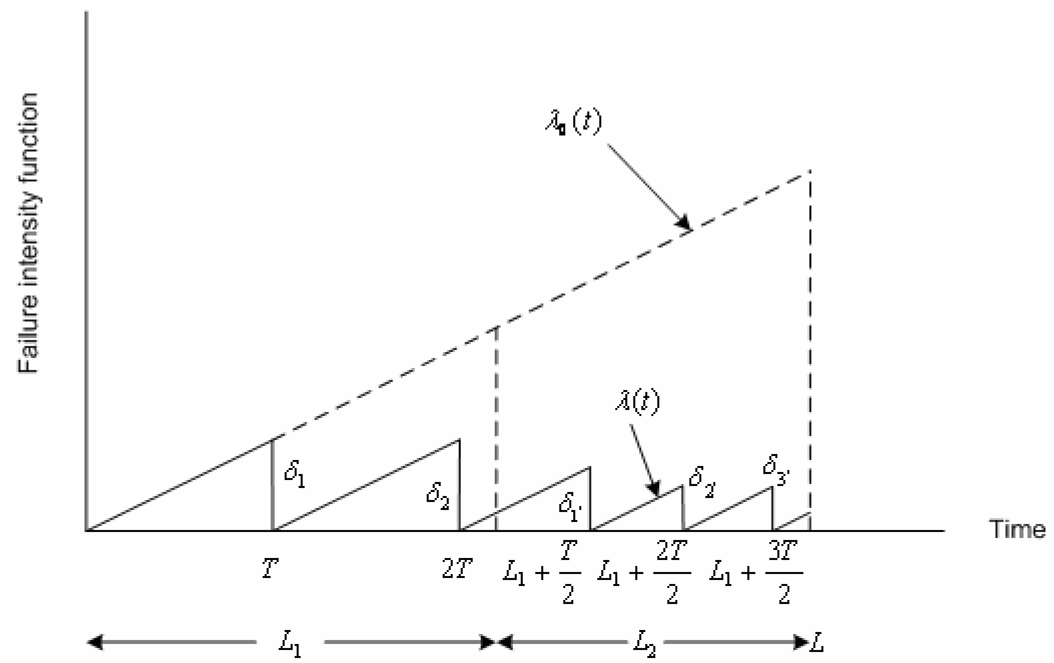

Fig. 1 Plot of failure intensity function

2.3 Cost to the lessor

The cost to the lessor is comprised of the following three costs :

(i) Cost of CM actions

Let $N(L)$ denote the number of failures over the lease period, and let C_f denote the mean cost of repair. Then the cost of repairing failures is given by

$$TC_f = C_f N(L) \quad (5)$$

(ii) Cost of PM actions

The cost of PM action depends on the resulting reduction in the intensity function. We model this through a fixed cost and a variable cost and is given by

$$C_p(\delta_j) = a + b\delta_j, \quad j = 1,2,...,k_1 \quad (6)$$

for the first lease period

and

$$C_p(\delta_{j'}) = a + b\delta_{j'}, \quad j' = 1,2,...,k_2 \quad (7)$$

for the second lease period

with $a > 0$ and $b \geq 0$.

Hence , the total cost of PM actions is given by

$$TC_p(\delta_j, \delta_{j'}) = \sum_{j=1}^{k_1}(a + b\delta_j) + \sum_{j'=1}^{k_2}(a + b\delta_{j'}) \quad (8)$$

where $j = 1,2,...,k_1$ and $j' = 1,2,...,k_2$

(iii) Penalty costs

The penalty cost is a function of the repair time Y (a random variable with a distribution $G(y)$ called the repair time distribution) and τ

Let Y_i denote the time to rectify the i^{th} failure, $1 \leq i \leq N(L)$. Then, the total penalty cost incurred is given by

$$\phi(N(L), Y_i, \tau) = C_t \left\{ \sum_{i=1}^{N(L)} \max[0, Y_i - \tau \right\} \quad (9)$$

3. Model analysis

3.1 Expected number of failures

With no PM actions, failures over the lease period occur according to a NHPP with intensity function $\lambda_0(t)$. The expected number of failures over the lease period is given by

$$E[N(L)] = \Lambda_0(L) = \int_0^L \lambda_0(t)dt \quad (10)$$

With PM actions, the expected number of failures over the leased period is also given by a NHPP process with intensity function given by (1) and (2). As a result, we have

$$E[N(L)] = \Lambda(L) = \Lambda_0(L) - \sum_{j=1}^{k_1}\delta_j(L - t_j) - \sum_{j'=1}^{k_2}\delta_{j'}(L - t_{j'}) \quad (11)$$

where $j = 1,2,...,k_1$ and $j' = 1,2,...,k_2$

3.2 Expected cost

(i) Expected CM cost

From (5) and (11), the total expected cost of CM actions is given by

$$E(TC_f) = C_f \Lambda(L) \tag{12}$$

(ii) Expected Penalty cost

From (9) and (11), the total expected penalty cost is given by

$$E[\phi(N(L), Y_i, \tau)] = C_t \Lambda(L) \left\{ \int_{\tau}^{\infty} (y-\tau) g(y) dy \right\} \tag{13}$$

Using integrating by parts on (13) results in

$$E[\phi(N(L), Y_i, \tau)] = C_t \Lambda(L) \left\{ \int_{\tau}^{\infty} (1-G(y)) dy \right\} \tag{14}$$

(iii) Total expected cost to the lessor

Combining the costs given by (8), (12), and (14) yields the total expected cost to the lessor and is given by

$$\begin{aligned} J(T, \underline{\delta}) &= C_f \Lambda(L) + \sum_{j=1}^{k_1} (a + b\delta_j) + \sum_{j^{'}=1}^{k_2} (a + b\delta_{j^{'}}) \\ &+ C_t \Lambda(L) \left\{ \int_{\tau}^{\infty} (1-G(y)) dy \right\} \end{aligned} \tag{15}$$

where $j = 1,2,...,k_1$ and $j^{'} = 1,2,...,k_2$

Define

$$C^{'} = C_f + C_t \int_{\tau}^{\infty} (1-G(y)) dy$$

Then, (15) can be rewritten as

$$\begin{aligned} J(T, \underline{\delta}) &= C^{'} \left[\Lambda_0(L) - \sum_{j=1}^{k_1} \delta_j (L - t_j) - \sum_{j^{'}=1}^{k_2} \delta_{j^{'}} (L - t_{j^{'}}) \right] \\ &+ \sum_{j=1}^{k_1} (a + b\delta_j) + \sum_{j^{'}=1}^{k_2} (a + b\delta_{j^{'}}) \end{aligned} \tag{16}$$

where $j = 1,2,...,k_1$ and $j^{'} = 1,2,...,k_2$

3.3 Optimization

The optimal parameters of PM policy are parameter values that yield a minimum for $J(T, \underline{\delta})$. We obtain the optimal values using a two-stages process. In Stage 1 we apply differential calculus method to obtain $\underline{\delta}^*(T)$. In Stage 2 we obtain T^* by using one-dimensional minimization method with the iterative procedure.

<u>Stage 1</u>

Fix k_1, k_2 and obtain $t_j, t_{j^{'}}$ from $t_j = jT$, $j = 1,2,...,k_1$ and $t_{j^{'}} = L_1 + \frac{j^{'}T}{2}$, $j^{'} = 1,2,...,k_2$.

As a result, $J(T, \underline{\delta})$ is only a function of $\underline{\delta}$, and from (3) and (4), the $\underline{\delta}$ is constrained as follows

$$0 \le \delta_j \le \lambda_0(t_j) - \sum_{i=0}^{j-1} \delta_i \text{ for } 1 \le j \le k_1 \tag{17}$$

where $t_0 = 0$ and $\delta_0 = 0$

over the first lease period

and

$$0 \le \delta_{j'} \le \lambda_0(t_{j'}) - \sum_{i'=0}^{j'-1} \delta_{i'} \quad \text{for } 1 \le j' \le k_2 \tag{18}$$

where $t_{0'} = L_1$ and $\delta_{0'} = \sum_{j=1}^{k_1} \delta_j$

over the second lease period

Determine the extreme point of $J(T,\underline{\delta})$ by determining the first partial derivatives of $J(T,\underline{\delta})$ corresponding to $\underline{\delta}$ as below

$$\frac{\partial J(T,\underline{\delta})}{\partial \delta_1} = \frac{\partial J(T,\underline{\delta})}{\partial \delta_2} = \ldots = \frac{\partial J(T,\underline{\delta})}{\partial \delta_j} = \frac{\partial J(T,\underline{\delta})}{\partial \delta_{1'}} = \frac{\partial (T,\underline{\delta})}{\partial \delta_{2'}} = \ldots = \frac{\partial (J,\underline{\delta})}{\partial \delta_{j'}} = 0$$

then we have the constraints of t_j and $t_{j'}$ as follows

$$0 < t_1 < t_2 < \ldots < t_{k_1} < L - \frac{b}{C'} \text{ and } 0 < t_{1'} < t_{2'} < \ldots < t_{k_2} < L - \frac{b}{C'} \tag{19}$$

where $0 < t_j < t_{j'} < L - \dfrac{b}{C'}$

As a result, $J(T,\underline{\delta})$ is a linear function of $\underline{\delta}$ and constrained as indicated in (17)-(19). Therefore, the optimal values are the end points of the constraint intervals. This yields

$$\delta_j^* = \begin{cases} \lambda_0(t_j) - \sum_{i=0}^{j-1} \delta_i & \text{if } t_j < L - b/C' \quad \text{for } 1 \le j \le k_1 \\ 0 & \text{if } t_j \ge L - b/C' \quad \text{for } j > k_1 \end{cases} \tag{20}$$

where $t_0 = 0$ and $\delta_0 = 0$

and

$$\delta_{j'}^* = \begin{cases} \lambda_0(t_{j'}) - \sum_{i'=0}^{j'-1} \delta_{i'} & \text{if } t_{j'} < L - b/C' \quad \text{for } 1 \le j' \le k_2 \\ 0 & \text{if } t_{j'} \ge L - b/C' \quad \text{for } j' > k_2 \end{cases} \tag{21}$$

where $t_{0'} = L_1$ and $\delta_{0'} = \sum_{j=1}^{k_1} \delta_j$

This implies that the optimal PM action at $t_j = jT$, $j = 1,2,\ldots,k_1$ or $t_{j'} = L_1 + \dfrac{j'T}{2}$, $j' = 1,2,\ldots,k_2$ is to reduce failure intensity by the maximum amount when $t_j < L - \dfrac{b}{C'}$ or $t_{j'} < L - \dfrac{b}{C'}$ and not to carry out any PM when

$t_j \geq L - \frac{b}{C^{'}}$ or $t_{j^{'}} \geq L - \frac{b}{C^{'}}$.

Stage 2

In Stage 2 we obtain T^*, the optimal T, by minimizing $J(T, \underline{\delta}^*)$ using $\underline{\delta}^*(T)$ obtained from Stage 1. One can obtain T^* by using one-dimensional minimization method with the iterative procedure according to the algorithm given below.

Step 1: $k_1 = 1$.

Step 2: Evaluate side constraints of T from $\frac{L_1}{k_1 + 1} < T \leq \frac{L_1}{k_1}$.

Step 3: Find T over the interval $\frac{L_1}{k_1 + 1} < T \leq \frac{L_1}{k_1}$ with one dimensional method and step size $\rightarrow$ 0 and then compute k_2 from $k_2 = \frac{2L_2}{T} = \frac{2(L - L_1)}{T}$

Step 4: Compute t_j and $t_{j^{'}}$ from $t_j = jT$, $j = 1,2,\ldots,k_1$ and $t_{j^{'}} = L_1 + \frac{j^{'}T}{2}$, $j^{'} = 1,2,\ldots,k_2$.

Step 5: Evaluate δ_j^* and $\delta_{j^{'}}^*$ by placing t_j and $t_{j^{'}}$ in (20) and (21) respectively.

Step 6: Evaluate $J(T, \underline{\delta}^*)$ from (16)

Step 7: Set new $k_1 \leftarrow k_1 + 1$, and repeat Step 1 onwards until $k_1 = k_{1\max}$ where $k_{1\max} = \frac{C^{'}\Lambda_0(L_1)}{a}$, then go to Step 8.

Step 8: Search for T^* which yields the smallest values for $J(T, \underline{\delta}^*)$. Using this, the optimal PM actions are given by $\underline{\delta}^* = \underline{\delta}^*(T^*)$ and the minimum expected cost to the lessor given by $J(T^*, \underline{\delta}^*(T^*))$.

4. Numerical example

According to Pongpech and Murthy (2005), we assume that the failure distribution for the equipment is given by the two-parameter Weibull distribution. As a result,

$$\lambda_0 = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta - 1} \tag{22}$$

with scale parameter $\alpha > 0$ and shape parameter $\beta > 1$ (implying an increasing failure rate). In the case of the two (or three) parameter Weibull model, the scale parameter α has no influence to the model (Blishchke and Murthy, 2000), therefore, we can assume $\alpha = 1$.

The repair time, Y, is a random variable with distribution function $G(y)$. We assume that $G(y)$ is also a two-parameter Weibull distribution function given by

$$G(y) = 1 - \exp\left[-\left(\frac{y}{\varphi}\right)^m\right], \quad 0 \le y \le \infty \tag{23}$$

with the scale parameter $\varphi < 0$ and the shape parameter $m < 0$ (implying a decreasing repair rate). We consider the following nominal values for the model parameters

$L = 5$ (years), $L_1 = 2$ (years), $C_f = 100\$$, $C_t = 300\$$, $a = 100\$$, $b = 50\$$, $\tau = 2$ (days), $\beta = 3$, $m = 0.5$, $\varphi = 0.5$

In this paper, there are two special cases to be considered.

(i) Special Case 1: [no penalty], and (ii) Special Case 2: [with penalty].

4.1 Special Case 1: [no penalty]

In the case of no penalty, we have $C_t = 0$ that yields $C^{'} = C_f$. As a result, we have $k_{1\max} = 8$, $T^* = 0.6000$ years, $k_1^* = 3$, $k_2^* = 8$ and the minimum expected maintenance cost, $J(T^*, \underline{\delta}^*)$, is \$5,531.20. The optimal parameters for the PM policy are given in Table 1.

Table 1 The optimal parameter for Special Case 1: [no penalty]

The 1^{st} lease period			The 2^{nd} lease period		
j	t_j^*	δ_j^*	$j^{'}$	$t_{j^{'}}^*$	$\delta_{j^{'}}^*$
1	0.6000	1.0800	1	2.3000	6.1500
2	1.2000	3.2400	2	2.6000	4.4100
3	1.8000	5.4000	3	2.9000	4.9500
			4	3.2000	5.4900
			5	3.5000	6.0300
			6	3.8000	6.5700
			7	4.1000	7.1100
			8	4.4000	7.6500

Note that t_j and $t_{j^{'}}$ must be satisfied the constraints

$$0 < t_1 < t_2 < \ldots < t_{k_1} < L - \frac{b}{C^{'}} \text{ and } 0 < t_{1^{'}} < t_{2^{'}} < \ldots < t_{k_2} < L - \frac{b}{C^{'}}$$

where $0 < t_j < t_{j^{'}} < L - \dfrac{b}{C^{'}}$

This implies that the maximum amount of t_j and $t_{j^{'}}$ must be lower than $5 - \dfrac{50}{100} = 4.50$ years, as a result

$t_{8'}^* = 4.40 < 4.50$ years, and not to carry out any further PM actions because the constrains are not satisfied. In this policy the time instant to carry out PM actions, t_j and $t_{j'}$, increase with fixed interval as a result of applying the periodic PM policy, but the increasing for $t_{j'}$ are shorter than that for t_j as a result of applying the sequential PM policy.

4.2 Special Case 2: [with penalty]

In the case of with penalty, we have $k_{1\max} = 15$, $T^* = 0.4700$ years, $k_1^* = 4$, $k_2^* = 11$ and the minimum expected maintenance cost, $J(T^*, \underline{\delta}^*)$, is $7,109.69. The optimal parameters for the PM policy are given in Table 2.

Table 2 The optimal parameter for Special Case 2: [with penalty]

The 1^{st} lease period			The 2^{nd} lease period		
j	t_j^*	δ_j^*	j'	$t_{j'}^*$	$\delta_{j'}^*$
1	0.4700	0.6627	1	2.2350	4.3825
2	0.9400	1.9881	2	2.4700	3.3170
3	1.4100	3.3135	3	2.7050	3.6484
4	1.8800	4.6389	4	2.9400	3.9797
			5	3.1750	4.3111
			6	3.4100	4.6424
			7	3.6450	4.9738
			8	3.8800	5.3051
			9	4.1150	5.6365
			10	4.3500	5.9678
			11	4.5850	6.2992

4.3 Comparison between [no penalty] and [with penalty]

Table 3 shows the comparison of the optimal solutions between no penalty case and with penalty case.

Table 3 Comparison between [no penalty] and [with penalty]

Case	k_1^*	k_2^*	k_{total}^*	T^*	J^*
No penalty	3	8	11	0.60	$5,531.20
With penalty	4	11	15	0.47	$7,109.69

As can be seen, the effect of penalty results in dramatic increasing of the total expected maintenance cost. The minimum expected maintenance cost is $5,531.20 in the case of no penalty and being $7,109.69 in the case of with penalty. In addition, the optimal number of PM actions also increase from 11 to 15 corresponding to no penalty case and with penalty case respectively. When $\tau \to \infty$ corresponds to no penalty associated with repair time. In this case $C' = C_f$ so that it reduces to the case of no penalty.

4.4 Comparison between Policies 1, 2, and 3

Table 4 shows the comparison between Policy 1 proposed by Jaturonnatee et al. (2005), Policy 2 proposed by Pongpech and Murthy (2005) and Policy 3 (the new policy proposed in this paper) for $\beta = 3$.

Table 4 Comparison between Policies 1, 2, and 3 [$\beta = 3$]

Case	Policy 1		Policy 2		Policy 3	
	k^*	J^*	k^*	J^*	k^*_{Total}	J^*
No penalty	10	$5,437.03	9	$5,750.00	11	$5,531.20
With penalty	16	$7,009.92	17	$7,312.50	15	$7,109.69

As can be seen, the expected cost for Policy 3 proposed in this paper is always less than that for Policy 2 (periodic PM policy) but higher than that for Policy 1 (sequential PM policy). Therefore, Policy 1 is the lowest cost policy. However, because of unequal time intervals, Policy 1 is more complicated and cause inconvenience in implementation. In addition, the percentage increase in the expected cost is too small (1.73 % in the case of no penalty, and 1.42 % in the case of with penalty). Consequently, Policy 3 is more practical than Policy 1. Beside the lower expected cost than Policy 2, Policy 3 is also more practical than Policy 2 in reliability aspect because in Policy 3 the units are maintained more frequently with increased ages.

5. Conclusion

In this paper we have proposed the new periodic PM policy for leased equipment combining the advantages of both sequential and periodic PM policies into the model. Developing the new policy results in reducing expected cost and being easier in implementation as well as improving reliability.

The authors are currently studying the extended topics as indicated below.

(1) Special Case 3: [penalty 2]

We consider the second type of penalty called Penalty 2. The lessor incurs the Penalty 2 if failures occur during the lease period L. In other words, the penalty 2 occurs immediately when failures occur, without considering the times to restore the equipment back to working state. In Special case 3, we do not include Penalty 1 (the penalty proposed in this paper) in the model.

(2) Special Case 4: [penalty 1 & 2]

In Special Case 4, We consider both Penalty proposed in this paper (called Penalty 1) and Penalty 2 as we have mentioned. This model is the general form of lease contract.

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