A Revenue Management Model for Make-To-Order Bidding

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Abstract

This paper extends the single-period price and due date setting model for contingent Make-to-Order (MTO) firm into a finite-period stochastic system. Based on the revenue management framework (viewing every single available timeslot of production capacity as a perishable product), MTO firms can dynamically set price and due date to achieve highest average profitability per timeslot given a certain distribution pattern of future demand within a finite horizon. Heuristically, given a state of production system, admission control policy is then proposed to bid at the shadow price of the required timeslots or higher. An experimental simulation was performed and reported with insightful discussion.

Key-Words: - Revenue Management, Make-to-order bidding, Production planning, Dynamic programming, Price and due date decision.

1 Introduction

In the context of Make-to-order bidding, after quoting the bid, the firm has to wait for customer's decision (to deny or award the job). The contingency of the decision affects the planned capacity allocation. Reserving available timeslots for contingent bids is equivalent to have overbooked capacity in a hotel or airline services. As suggested by [1] and [2], MTO firms can utilize the Markovian's state transition scheme to effectively manage the overbooking scheme that balances the penalty cost of tardiness and opportunity loss of subsequent bids.

Based on the Revenue Management (RM) discipline, we may view every single timeslot of production capacity in an MTO firm as a perishable product, analogous to that of airline seat or hotel night stay in the traditional RM model. A decision of admission control and the setting of price and due date can be dynamically made for every single bid coming in each planning period to achieve the optimal profitability for a given finite period.

Since originated in the airline industry in late 1970s, revenue management (RM) has been widely applied to other industries and has become a generic method that effectively helps a company under constrained capacity to determine selling at what price and to which market segment. However, the application of RM into the MTO environment is still relatively limited and remains a challenging motivation of this study.

According to Weatherford and Bodily [3], RM model is best suited for systems that hold the following characteristics: (i) with perishable products or services; (ii) high fixed cost, generally accompanied by widely fluctuating or stochastic demand; and (iii) possibility of market segmentation. Based on the said requirements, MTO firms can treat its available production timeslots as RM product. Analogously, MTO production timeslots can be viewed as limited inventory of product that is perishable by nature. Some MTO firms, such as in the die and mold and printed-circuit board (PCB) industries, have relatively high investment costs and often need quite a long time to establish new capacity, if required. Lastly, many authors have reported the segmentability of MTO market (Duenyas [4], Carr and Duenyas [11] and Keskinocak et al. [5]).

Extending the single-period contingent multiple product-class model of [1], we propose in this paper a multi-period dynamic programming model that integrates the finite-time RM discipline into the contingent multiclass MTO bidding system. Such the integration leads the system to a class of dynamic price setting under dynamic capacity allocation of RM problem. Also, we evaluate the relative performance of the RM model in a simulated dynamic environment and provide some insightful discussion.

Assumptions made in this work are as follows: (i) the firm makes schedules with no pre-emption on a single workstation or a bottleneck station on which the line capacity is based, (ii) customers must confirm their order within the maximum confirmation lead time (time between the quotation time and the expected critical production start time),

(iii) the production facility requires high investment cost so that costs of machine time and direct labor can be considered as fixed manufacturing costs, (iv) customers, once confirmed, cannot cancel the orders, (v) machines are always in good condition, and raw materials and components are always available when needed, (vi) The work content of any class is in multiple units of production time slots.

The organization of presentation is as follows: Section 2 provides the literature review of the RM and MTO bidding systems. In Section 3, we present the proposed RM bidding model for MTO and the derived decision rule. After then, the design of the experiment and the result of simulation are provided in Section 4. Insightful discussion on comparative study is presented in Section 5. Finally, conclusions are provided in Section 6.

2. Literature Review

The very first publication on RM was in the 1960s when the liberalization of airline industry had just begun. As surveyed by McGill and Van Ryzin [6] and Weatherford and Bodily [3], the researches on RM have evolved and extended widely into many service industries, e.g. demand forecasting, inventory control system (including the reservation system and overbooking control), and dynamic price control.

On the related bid price control problem, Lee and Hersh [7] proposed a dynamic programming approach to resolve the RM's overbooking and optimal allocation problem. Their model relaxed a few strict requirements of the previous works on RM. The first is to relax the strict arrival pattern, i.e. the early arrival for the discounted class and late arrival for the full-price class. The second is to allow the customer to book multiple units of product instead of only one at a time. However, their model assumes a fixed, pre-determined price for each class which is addressed in this paper.

For the manufacturing sector, RM is relatively new. The very first attempt is reported in [8] where a static capacity allocation model is applied into the manufacturing system with multiple classes of customers. The system discriminates the market by pre-identified sets of price and due date. The capacity for each customer class was nest-allocated based on the economic model (EMSRb) modified from the classical EMSR of Belobaba [9]. The pattern of customer arrivals is deterministic with the early coming of low-valued customers. The model could decide to accept/deny an order without determining its optimal price, contingent order was not taken into account.

Prior to [8], Easton and Moodie [2] presented their single-period model for simultaneously deciding optimal price and due date for the MTO bidding system. It is the first paper assuming that the production timeslot is a single type of resources conformed to the basic requirement of RM discipline. The other assumptions are also of RM- oriented, e.g. the high sunk cost of facility investment, the perishability of production capacity, and the overbooking capability due to the existence of contingent bids. Their model assumes a unified market and no consideration of future potential customers.

Subsequently, Watanapa and Techanitisawad [1] extended model in [2] to accommodate multiple classes of customers and improved the search algorithm to efficiently optimize the expected marginal revenue function. They also proposed to explicitly treat different customer classes with different sequencing rules. Lately, Watanapa and Techanitisawad [10] proposed a Genetic Algorithm (GA) in searching for a near-optimal sequence of jobs that interacts with price and due date in maximizing the expected marginal revenue for a single period.

Plambeck [12] proposed a system that can set static price and dynamically quote due date for two classes of time- and price-sensitiveness both sharing the same exponential service time distribution. The model assumes class-dependent arrival rate to be linearly decreasing with prices and due dates. Similar to that of [2], the system gives scheduling priority to time-sensitive orders and enable the system to promise early due date. Cost-sensitive orders will be quoted with lead times that are proportional to the workload.

Assuming that multiple products consume the capacity of the same resource, Maglaras and Meissner [13] modeled the revenue management system for maximizing marginal revenues of those aforementioned products subject to their different rates of resource consumption. This shifted the framework of multiple product pricing decision to the multiple resource capacity controls.

In addition, recently Maglaras [14] showed the applicability of RM model into MTO environment with multiple classes, general demand curves, and facilitated by a single machine with M/M/1 queuing. By analyzing the interaction of dynamic state-dependent pricing and sequencing strategy, the model came up with the maximized total expected revenue. Interestingly, the research proved that (i) dynamic pricing is at least as good as static pricing, (ii) when queues are low the firm may reduce its prices to attract arrivals and reduce idling periods, and (iii) when system is around heavy-traffic regime, the pricing strategy is dependent on the workload, and decoupled from sequencing strategy. The model assumes standard content for each arrival and non-contingent orders.

3. Revenue Management Bidding Model

In this section, we discuss the development of an RM decision model for an incoming bid in an MTO firm with a a finite-horizon RM bidding system and a decision rule. We consider an MTO system with *n* remaining production timeslot and a total contingent work content of φ which is comparable to the booked capacity in the traditional RM context. The optimization is for request *i* requiring *x* timeslots. According to the RM principle, the optimal price $b_{i,n}$ and due date $d_{i,n}$ are dependent on the expected marginal value of those *x* timeslots in the current state of the system (remaining production timeslot *n*, booked capacity of φ, and a set of all pending jobs including contingent orders, Ω), denoted as $\vartheta_x\left(n, \varphi_\Omega\right)$.

Conceptually, the RM marginal value $\vartheta_x\left(n, \varphi_\Omega\right)$ can be interpreted as the shadow price or the incremental profit which could be obtained when freeing x units of timeslots for future demands, instead of being reserved for the current bid request *i*. Computationally, this is the difference between the accumulation of series of expected revenues obtained throughout the remaining time horizon from the current period *n* to the last planning period (period 0) given the reserved capacity φ for all jobs in the queue Ω, denoted as $f^n_{\varphi(\Omega)}$, and the accumulation of series of expected revenues obtained throughout the same remaining service time, but having x pending timeslots more, or φ(Ω)+x, in the queue denoted as $f^n_{\varphi(\Omega)+x}$. This can be depicted as:

$$\vartheta_x(n, \varphi_\Omega) = f^n_{\varphi(\Omega)} - f^n_{\varphi(\Omega)+x} \tag{1}$$

Now, we derive a basic RM principle from the fact that, for the request *i* coming when there is *n* remaining timeslot and booked work content of φ, the bid should be quoted with a set of price and due date that gives an expected contribution margin (Π_i^*) that is higher or equal to the shadow price of the *x* units of timeslot at the given state, or $\vartheta_x\left(n, \varphi_\Omega\right)$, otherwise, it is rational to take no action or enter into period n-1 with the same old level of booked capacity φ:

$$\Pi_i^* + f^{n-1}_{\varphi(\Omega)+x} \geq \vartheta_x(n, \varphi_\Omega) + f^{n-1}_{\varphi(\Omega)+x} \tag{2}$$

The bid policy shown in (2) is analogous to admission control policy in the traditional RM where a decision of accept/reject is made for each request from a prospective customer so that the expected revenue (or contribution margin) is maximized, or:

$$\max\left(\Pi_i^* + f^{n-1}_{\varphi+x-1}, f^{n-1}_{\varphi-1}\right). \tag{3}$$

In our context, the decision is more complicated than that of the traditional RM since each decision period is by itself a product. Rejection to bid and proceeding to the next decision period, hence, equates not only loss of opportunity, but also a waste of the available timeslot. This has to be reflected into the decision model if it is to maximize a total profit for a finite period. Heuristically, we simplify the complexity by introducing an unwilling bid concept and avoiding rejection to bid. The unwilling bid is to bid at unattractive terms (high price and/or long lead time). Even though this leads to low winning probability and little expected profit, opportunity cost is automatically incorporated into the next decision. Reflecting the unwilling bid concept into (2) and (3), we derive an inequality which is a basis for inventing our decision rule as shown in (4). The challenge here is how to derive from (4) a sensible high price to bid unwillingly. The formulation is given when we develop the first decision rule in Section 3.2.

$$\Pi_i^* + f^{n-1}_{\varphi(\Omega)+x} \geq \vartheta_x(n, \varphi_\Omega) + f^{n-1}_{\varphi(\Omega)+x} \geq f^{n-1}_{\varphi(\Omega)} \tag{4}$$

At this stage, based on (3.1-3), we can calculate the accumulated profit by using discrete-time dynamic programming model in a similar approach with those of Gerchak, Parlar and Yee [18] and Lee and Hersh [7]. The model is to recursively estimate the accumulated profit generated in the remaining periods, T^n = {1,2,...n}:

$$f_{\varphi(\Omega)}^{n} = \begin{cases} P_0^n f_{\varphi(\Omega)-1}^{n-1} + \sum_{i=1}^{K} P_i^n \max(\Pi_i^* + f_{\varphi(\Omega)+x-1}^{n-1}, f_{\varphi(\Omega)-1}^{n-1}) \text{ for n>0}, \varphi\text{+x} \le \text{n} \\ 0 \quad \text{otherwise} \end{cases} \quad (5)$$

A prospective bid request *i* coming in period *n* may be from either class 1, 2, up to *K* with the possibility of P_i^n that is assumed to follow a Poisson process and $P_0^n = 1 - \sum_{i=1}^{K} P_i^n$. If dividing the whole planning period into multiple intervals, each with a unique mean of arrival distribution, then we obtain a decision model of MTO system with a lesser restriction on arrival pattern. Moreover, by superpositioning the Poisson process to a set of classes for each interval, we thus have a flexible multi-period multi-classes MTO model. Section 4.1 discusses the development of the request probability in more details.

To enable the simulation of the MTO system in discrete dynamic programming, we further assume that there is a very little probability (essentially close to zero) of having two or more arrivals in a short interval *t*. Moreover, by the assumption of independent Poisson process, the probability of one arrival in such a short interval *t* is independent of changes in other non-overlapping intervals and is approximately proportional to the length of the interval. For each planning interval, we thus subdivide it into multiple decision periods with the same length which is small enough for each customer class to follow a Poisson process. A request *i* is thus may be from a class in the set {1,2,...,K}.

In addition, due to the contingency of orders confirmation and the variable number of required timeslots, the dynamic programming model (3) needs an extension to treat the realization pattern of order confirmation *r* and the possibility that request *i* is from class *K* as depicted in (6) and (7), respectively.

$$f_{\varphi(\Omega)}^{n} = \sum_{r=1}^{2^{N(r)}} p(r) \cdot f_{\varphi(\Omega_r)}^{n} \quad (6)$$

$$f_{\varphi(\Omega_r)}^{n} = \begin{cases} P_n^0 f_{\varphi(\Omega_r)-1}^{n-1} + \sum_{i=1}^{K} P_n^i \sum_{m=1}^{M_i} \Gamma_{im}^n \cdot \max\left(\Pi_i^* + f_{\varphi(\Omega_r)+x-1}^{n-1}, f_{\varphi(\Omega_r)-1}^{n-1}\right), \text{ for } n > 0, 0 \le \varphi(\Omega_r) + x \le n \\ 0, \text{ otherwise.} \end{cases} \quad (7)$$

For Model (6), (7), and the remainders, we define additional notations as follows:

r	A possible realization for all orders in the queue Ω, with probability $p(r)$,
$N(r)$	Number of contingent orders in the queue Ω,
$\varphi(\Omega_r)$	Total expected capacity required by orders in the queue as of realization r,
P_n^i	Probability that there is a request for class *i* in the decision period *n*,
Γ_{im}^n	Probability that a request for class *i* in the decision period *n* requires *m* timeslots, $m \in \{1, 2, \ldots, M_i\}$.

By (6), the accumulated profits obtained, in *n* decision periods, from the MTO model with contingent orders can be estimated by averaging all expected profits from every possible realization generated from the uncertainty of win or lose for each contingent bid. If the number of contingent bids in Ω is $N(r)$, then the total number of all possible realization is $2^{N(r)}$. For each realization *r*, referring to (7), based on the forward dynamic programming, the accumulated profits can be recursively computed from period *n* to period 1 with the starting state $\varphi(\Omega_r)$ through the varying of all possible classes (*K*) and all possible number of required timeslots (*M*) in each future period. By $0 \le \varphi(\Omega_r) + x \le n,$ the system accumulates profits from those bids which can be completed within *n* periods. This condition forces the system to reject a bid that will overuse the available capacity (e.g., assume that delivering product with outsourcing or extra production time units requires very high costs and thus unworthy).

System (7) needs a unique mechanism for handling awaiting works in the queue. This is due to (i) the contingency of orders in the queue, the booking system in this MTO is comparable to the RM reservation system with cancellation. (ii) The dynamic price setting makes the spread of possible prices of each class overlapped. This bars any pre-determinedly simple rule for bid/no-bid decision as traditionally found in the RM context where nested price structure is preset. (iii) The reduction in available capacity of MTO is similar to that of hotel reservation in a fixed horizon and that is why the indexing of time period is *n*-1 in the RHS but *n* is the LHS of (7). By induction, indexing of the expected number of timeslots being required in the next period is either $\varphi(\Omega_r)$ or $\varphi(\Omega_r)+x$ minuses 1.

3.1. Decision periods and probability of customer arrivals

We aim to optimize a total expected profit obtained throughout a finite planning horizon which is divided into n decision period $\{n,n\text{-}1,....,1\}$. Firstly, the whole planning horizon is divided into several main intervals j in which the arrival pattern of each class can be differentiated from that of the others. For example, the earlier the interval is, the higher probability the price-sensitive customer arrives. On the other hand, the request pattern of time-sensitive customers may start with a low rate of arrival and increase through the remaining intervals (See Fig. 1). By this way, we relax the limitation of the traditional RM model that strictly assumes the sequential arrival pattern of early coming of cost-sensitive customers, followed by time-sensitive customers.

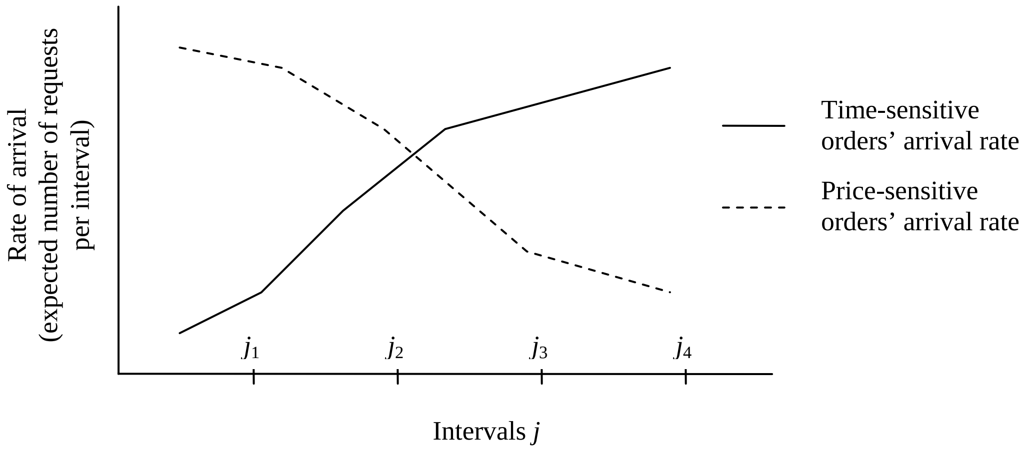

Fig. 1. Comparative rates of customer arrivals

Next, we sub-divide each interval j into a certain number of decision periods t_j with equal lengths so that there is very little chance of having more than one arrival in each decision period. By the Poisson process, if the mean of request arrival in interval j is μ_j, then, the mean of arrival in a decision period t_j is μ_j/t_j, and the function of request probability in the decision period n follows a Poisson distribution with mean μ_j/t_j, or $P_n(x)=\left[\left(\mu_j/t_j\right)^x e^{-\mu_j/t_j}\right]/x!$, where x = 0,1,2…. The value of v_j can be estimated from the inequality of $1-P(0)-P(1)\le\varepsilon$ or:

$$1-e^{-\mu_j/v_j}-(\mu_j/v_j)e^{-\mu_j/v_j}\le\varepsilon \qquad (8)$$

To model the attribute of multiple classes, we apply the superposition of Poisson process by letting μ_j^i be the expected number of requests for class i in main interval j, and ascertain that $\mu_j=\mu_j^1+\mu_j^2+...+\mu_j^i$. We state μ_j as the total expected number of requests for all customer classes in data interval j.

By the formulation, it implies that the pattern of request arrivals for each class in any decision period t_j is independent from the other classes, each follows a Poisson process with different mean μ_j/t_j. Finally, the probability function of having exactly one request arriving in decision period n, or P_n^i, follows a Poisson process with mean μ_j^i/t_j and can be calculated by a simple term: $P_n^i=\left(\mu_j^i/t_j\right)e^{-\mu_j^i/t_j}$.

P_n^i in decision period t is proportional to its length and independent of changes in other non-overlapping periods [19].

For such design of a decision period t, even though it allows us to model the behavior of the system with a Poisson process, there is a problem of practically subdividing each interval into decision periods of which lengths are small enough to satisfy the requirement of having no more than one arrival. This study limits to such cases where a decision period is a unit of production timeslot which does not conflict with the requirements of the superposition of the presumed Poisson processes.

3.2. Bidding decision rules

Based on the concept of marginal value of timeslots as discussed above, we derive a decision rule so that any single request for bidding can be controlled efficiently in terms of total profits acquired in a certain period of time.

Traditionally, the RM model suggests to place a bid or to sell product only when the selling price is greater or equal to the marginal value of the required capacity, otherwise it is better to take no action. Since the assumptions of time-dependent capacity and dynamic nature of pricing and arrival pattern, instead of explicit strategy of bid or no-bid, this model suggests alternative bid prices which are equivalent to "willing to bid" and "unwilling to bid" strategies as justified in the previous section. The willing-to-bid strategy is for the case where a set of optimal bid parameters (obtained from Model A.1 in the appendix A), e.g. price and due date, yields an expected profit Π_i that is of higher value than the marginal value of required timeslots $\vartheta_x\left(n,\varphi_\Omega\right)$ obtained from (1). Here, the decision rule is to bid at the current-state optimal price and due date. The unwilling-to-bid is enforced when the current-state optimal bid parameters provide lower profit than the marginal value. In such a case, instead of taking no action, the bid price will be raised up to a highest possible value which is either the marginal value of required timeslots or the ceiling price of its class, BU. The rule is encoded as:

$$\text{Decision Rule 1: } b^*_{i,n} = \min\left\{B_U, b_{i,n} + \max(\vartheta_x(n,\varphi) - \Pi_i, 0)\right\} \quad (9)$$

$\vartheta_x\left(n,\varphi_\Omega\right)$ can be computed by the dynamic programming as discussed in the previous section while the Π_i is optimal profit computed by the formula given in appendix A.

4. Simulation results and analysis

To gain insights into and to evaluate the proposed RM bidding model, we conducted experiments by simulating simple situations which a bidding firm may encounter in a certain finite period. A conventional next-event time-advance mechanism is adopted for this terminating simulation. Due to the assumption that arrival patterns follow Poisson processes, it is easy to generate discrete random variates based on Monte Carlo sampling technique [16]. The basic parameters were set in the same fashion as in [1]. The particular needs of parameter settings in this study are firstly discussed, while the selected scenarios and conditions of simulation and discussions of the obtained results are subsequently provided.

4.1. Parameters settings

Table 1 summarizes the parameter settings of each customer class. Necessary notations can be found in Appendix B. Due to the computational complexity, we limited our experiments to only four classes with some adjustments on parameters. The standard work content (or the required timeslots) of job *i*, $W_i = m$ was limited to be 1 up to 3 of which, subsequently, is adjusted into estimated work content (or number of timeslots) of job for class i, $W_i = x = \text{int}\left(w_i * \left[\mu_i + Z_\omega \sigma_i\right]\right)$. According to the -value, classes 1 and 2 are cost-sensitive, while classes 3 and 4 are time-sensitive. However, classes 2 and 4 have higher degree of cost-sensitiveness and time-sensitiveness than the others, respectively. Classes 1 and 3 have no competitor in the market, while classes 2 and 4 have 2 competitors, on average. Other parameters of classes 1-4 are referred to classes 2, 2a, 3, and 3a, respectively, in Table 3.2. Note that other parameters of classes 1-4 are referred to as classes 2, 2a, 3, and 3a, respectively, in [1] for backward compatible.

Table 1 Class parameters

Class	Beta-coefficients				Intense of competition	Standard boundaries				Quality Standard			Range of
k	β_0	β_β	β_δ	β_λ	λ	D_L	D_U	B_L	B_U	μ	σ	ω	w_i
1	0.1	0.75	0.5	0	0	W_i	$15W_i$	CW_i	$4CW_i$	1.2	0.2	0.68	1-3
2	0.1	0.8	0.4	0.7	2	W_i	$7W_i$	CW_i	$9CW_i$	1.2	0.2	0.68	1-3
3	0.1	0.5	0.8	0	0	W_i	$14W_i$	CW_i	$4CW_i$	1.1	0.2	0.68	1-3
4	0.1	0.4	1.2	0.7	2	W_i	$8W_i$	CW_i	$8CW_i$	1.1	0.2	0.68	1-3

As shown in Table 2, the initial set of queue and the initial state of the bidding system were randomly generated based on the set parameters. The probability that an arriving request in any decision period n is from class i and requires m time slots, Γ_{im}^{n}, is assumed to be known and static as summarized in Table 3.

Table 2 Status of the orders on-hand at the initial state of simulation

Order number	Customer class	Work contents (Number of time slots)	Due date	Winning probability
1	1	1	3	1
2	3	2	4	1
3	2	2	6	1
4	1	1	7	0.3
5	2	1	9	0.9

Table 3 Probabilities of having work content request of size *m* from each class *k*

Requested number of timeslots ($m = w_i$)	Probability distribution			
	Class 1	Class 2	Class 3	Class 4
1	0.15	0.10	0.15	0.10
2	0.45	0.70	0.45	0.70
3	0.40	0.20	0.40	0.20

4.2. Cases of experiments

To gain insights into the impacts of RM approach on the MTO bidding performance under several circumstances and environments, we set experimental scenarios based on two main factors, namely (i) the varying arrival rate of cost-sensitive orders compared with that of time-sensitive ones and (ii) the distribution pattern of time-sensitive orders arrival. For (i), we define two cases of "sparse" and "normal" markets where the latter case may describe a rather competitive market prevailed by cost-sensitive classes and the former one may represent a more sparse market with no prevailing class. For (ii), two cases are defined: "Early" and "Late", representing the tendency for early and late arrival of time-sensitive orders. By combination, four scenarios can be formed based on these two factors as shown in Table 4 which shows the pre-defined expected numbers of requests for each class, μ_j^i, in each main interval j for different scenarios

Table 4 μ_i^n of Poisson process in different scenarios

Avg. rate of arrival	Sparse & Late arrival				Sparse & Early arrival				Normal & Late arrival				Normal & Early arrival			
	Main data interval (*j*)				Main data interval (*j*)				Main data interval (*j*)				Main data interval (*j*)			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
$\mu_{i=1}$	1.5	1.2	0.8	0.3	0.3	0.8	1.2	1.5	2.8	2.8	2.9	2	2	2.9	2.8	2.8
$\mu_{i=2}$	1	1.4	1.2	0.4	0.4	1.2	1.4	1	2.7	2.5	2.6	2.4	2.4	2.6	2.5	2.7
$\mu_{i=3}$	0.3	0.6	0.8	1	1	0.8	0.6	0.3	0.3	0.6	0.8	1	1	0.8	0.6	0.3
$\mu_{i=4}$	0.4	0.8	1	0.6	0.6	1	0.8	0.4	0.2	0.5	0.6	0.9	0.9	0.6	0.5	0.2

The length of RM planning horizon is assumed to be 31 periods or 16 periods which represent long and short time horizon. The horizon was divided into four main intervals $j = 1,\ldots,4$ whose lengths are of 7, 9, 9, and 6 periods, respectively. Note that a 16 period horizon has 2 intervals of 7 and 9, respectively. According to inequality (4.2) with ε ~ 0.04, this setting was to ensure the requirement of the slight chance of having more than one request arriving in any planning period.

Adding the sequencing rules into consideration, now totally four parameters come into play and give 16 possible cases of experiments. To enable precise analysis, some cases of experiment had been cut off. As seen in Table 5, we put the main focus on the interaction between the pattern and frequency of arrival. Most cases of experiment are performed under the long planning horizon and with FCFS sequencing rule so as to obtain more integrity in the result of the simulation.

In each case, the simulation experiments on the single-period bidding Model (A.1) and on the proposed RM bidding model were performed for 20 replicates and the final experimental results were measured by averaging all runs in terms of total revenues (summation of winning bid prices), total penalty charges for tardiness, total profits (total revenue – total penalty), and total number of winning bids. Their values were accumulated through the whole planning horizon. We also collected other information for analysis purposes, for example, the total number of requests and the total number of requests to bid at adjusted prices.

Table 4 Designed cases for simulation experiments

Cases	Description			
	Planning horizon	Sequencing rule	Rate of arrivals	Arrival pattern of time-sensitive
1	16	Flexible	Normal	Late
2	16	FCFS	Normal	Late
3	16	FCFS	Normal	Early
4	31	FCFS	Normal	Late
5	31	FCFS	Normal	Early
6	31	FCFS	Sparse	Late
7	31	FCFS	Sparse	Early

5. Discussion on simulation results

Next, we discuss the comparative performance of the proposed model against the existing ones according to the key parameters, namely, the length of planning horizon, the pattern of customer arrival, and the interaction between the pattern and the frequency of arrivals.

5.1 Performance of RM in the short length of planning horizon

Based on the above settings, we summarize the simulation results as shown in Table 6. When the results of cases 1 (with flexible resequencing rule) and 2 (with fixed FCFS rule), both running in 16 periods, are compared, the RM models in both cases outperform the original offline Model [1] in terms of profit improvement, by 3.3% and 6%, respectively. Note for the RM models that there are more than 50% of the requests bid at adjusted higher prices (on average, 7.2 raised bids out of 12.45 requests for case 1, and 6.1 out of 11.9 for case 2). The rates of winning bids for the RM model is about 35% and 38% for case 2 and case 1, respectively, while those for Model (1) are about 41% and 42%, respectively. The lower winning rates of the RM models are as expected due to the higher bid prices. However, this may be interpreted as more effective production for the RM model over Model (1), since less production capacity was used to generate a higher total profit accumulated throughout the horizon.

Table 6 Results of simulation experiments

Scenarios	No. of arrival	Model (1)				RM model				% Price diff.	% Penalty diff.	Avg. profit per order	% Profit improv	% Win bids diff.	No. of price-adj bids	% price-adj bid /total bids
		Sum of Price	Sum of Penalty	Sum of Profit	No. of winning bids	Sum of Price	Sum of Penalty	Sum of Profit	No. of winning bids							
1	12.5	45.1	10.3	34.8	5.2	45.0	9.1	*36.0*	4.8	-0.2	-12.1	2.89	3.3	-7.7	7.2	57.8
2	11.9	37.4	5.7	31.8	4.9	37.5	3.8	*33.7*	4.3	0.1	-32.7	2.83	6.0	-12.4	6.1	51.3
3	10.0	31.8	4.4	27.4	4.0	32.2	3.6	*28.6*	3.7	1.3	-18.2	2.86	4.4	-7.5	4.8	47.5
4	22.4	68.7	6.1	62.6	8.9	70.9	3.9	*67.0*	7.4	3.2	-36.1	2.99	7.0	-16.9	15.4	68.8
5	22.0	65.3	6.2	59.1	9.0	70.7	4.4	*66.4*	7.7	8.3	-29.0	3.02	12.4	-14.4	15.0	68.2
6	13.0	44.8	2.2	42.6	5.6	46.7	1.5	*45.2*	5.2	4.2	-31.8	3.48	6.1	-7.1	7.1	54.6
7	13.1	50.2	2.8	47.5	6.0	49.4	1.9	*47.5*	5.7	-1.7	-32.1	3.63	0.1	-5.0	5.0	38.2

When comparing head-to-head between Cases 1 and 2, a consistent conclusion can be made that the flexible sequencing rule is capable of generating slightly higher profits (2.9 over 2.8 of average profit per orders in cases 1 and 2, respectively). More interestingly, lower profit improvement of the RM model over the single-period Model (1) using the flexible rule (3.3%) when compared with the case using FCFS (6%) implies that there is a smaller gap for the RM system to squeeze profit from the system using the flexible rule. This outcome complements the findings in [1], [10] which noted that EDD in the flexible rule tends to improve the efficiency of capacity utilization by increasing the number of winning bids. The aggressive bidding of EDD rule (more overbooked capacity), thus, erodes the RM bidding strength inevitably.

5.2 Performance of RM under different patterns of time-sensitive customer arrival

Considering the pattern of arrival for time-sensitive orders, the proposed RM model tends to intervene the price setting in the case of late arrival pattern regardless of the type of markets. Simple justification is the higher marginal values of those timeslots, being requested in early interval, caused by the greater booked timeslots for those willing-to-pay orders expected to come in the later interval. This can be noticed in terms of higher numbers of bids with adjusted prices, when comparing cases 2 with 3, 4 with 5, and 6 with 7.

Although we can notice positive correlation between the number of price-adjusted bids and profit improvements in these cases, it seems inconsistent to generally conclude that the RM model helps to achieve greater improvement under a certain arrival pattern. On average, 7.5% and 6.5% improvements for the early and late arrival patterns were achieved, respectively, and it seems that the arrival pattern of time-sensitive orders has no significant statistical effect on the improvement in profits.

5.3 Interactions of arrival rates and arrival patterns

When considering the impact of arrival rate (comparison of cases 4 against 6, and 5 against 7), the RM bidding performs better in the normal markets. Its percentage of improvement on profit over Model (1) evidently shows in Table 6: 7% against 6%, and 12% against 0.1%, for case 4 against 6, and 5 against 7, respectively. On average, profit is boosted by 9.6% in case of normal market, which is larger than the 3.1% of the sparse market environment.

When comparing the results of normal markets in 16 periods (cases 2-3) with those in 31 periods (cases 4-5), we found that the usage of RM model in a longer horizon can improve profit in a larger magnitude than in a shorter horizon. In 31 periods, we achieved around 7% and 12% improvements on profits over Model (1) for late and early arrival patterns for the time-sensitive orders, while the improvements in the 16-period simulations were just 6% and 4.4% on the late and early patterns, respectively. Random effect of simulation and impact of initialized parameters may be of implication to the obtained results.

As seen in case 7, sparse market with pattern of early arrival, there is no much difference between the profits generated from Model (1) and the RM model. The reason is that by knowing that there are not so many potential time-sensitive requests ahead, the RM model can do nothing but suggest bidding at the myopically optimal price of Model (1). Percentage of the number of bids with raised prices is, thus, the least in this case. On the other hand, Model (1) performs better in the sparse market (cases 6 and 7) than in normal market (cases 4 and 5), giving the average profits of 3.6 and 3.3 per order in cases 6 and 7, compared with the corresponding average profits of 2.7 and 2.8 in cases 4 and 5.

In general, we can summarize that the proposed RM bidding model improves profitability of an MTO firm by encouraging the bidding system to act based on the expected marginal value of timeslots. Profits can be boosted the most in cases of operating in normal market where the RM system has more opportunity to use dynamic pricing strategy to selectively accept or deny (bid with high price) each arriving request.

According to the obtained results of simulation, although the flexible sequencing rule efficiently supports the application of RM to achieve higher profits, the system requires a much longer run time for a lengthened planning horizon. On average, without the memory of state results, the run times for the 16-period FCFS and 16-period flexible rule (EDD + FCFS) cases were 5-10 minutes and 3 hours, respectively. The run time for 31-period FCFS and 31-period flexible rule were 10-15 minutes and more than 24 hours, respectively. With the help of the state memory, the run time for 16-period flexible, and 16- and 31-period FCFS were within 2-5 minutes. Thus, the flexible rule may not be so computationally practical, when operating in the environment that needs an immediate response of bidding action. For a system with longer than 15 planning periods, RM bidding model should be purely equipped with FCFS sequencing rule, for it can better compromise between computational time and quality of solution.

6. Conclusions

Applying the RM approach to the MTO bidding system, we incorporated prospective future demands pattern into the bidding model that dynamically determines the bidding price and the due date for each new order arrival. A dynamic programming algorithm was deployed to estimate the shadow price (also referred as marginal value) of required timeslots for the current state of production system, given the contingent orders in the bidding system of marketing department. The said marginal value can be derived from the difference between the two expected accumulated profits acquired if having and not having booked the production capacity for the current request.

A bidding rule is derived to control each bid so that its contribution is at least the marginal value of the required timeslots. More specifically, to bid at single-period optimal price and due date if this yields higher contribution than the RM marginal value, otherwise to bid with premium price up to the ceiling of its class. The latter strategy is equivalent to bid cancellation by intentionally bidding at a high price which is unattractive to customers.

Based on the conventional discrete next-event time-advance mechanism, the performance of the proposed model is simulated and compared with that of the original model which was proposed by [1] and [2]. The simulation was conducted by varying combinations of order arrival rate, arrival pattern of time-sensitive orders, sequencing rule, and length of planning horizon. The results show that the proposed RM bidding model consistently outperforms in terms of the average profit per order and total profits obtained throughout the planning horizon, although winning less bids. In addition, the result suggests that the RM rule works very well in the environment with (i) higher rate of arrivals and late arrival of time-sensitive classes, (ii) the longer planning horizon, and (iii) equipped with EDD rule.

RM approach is thus a viable decision model for improving the MTO bidding system performance, due to its ability to dynamically select bids and adjust bid prices. However, the improvement is achieved at the expense of computational time, especially where a more complicated sequencing rule, such as EDD, is applied yet selectively.

The next step of this research is to extend the decision rule to cover the case of overbooking and to make further simulation experiment so that more insightful information can be explored. Other research directions may be of, for example, an extension with multiple resource capacities so that different products can be treated differently on certain workcenters, or relaxation of some stochastic terms so that a solution can be searched much more quickly by a dynamic programming tool like a Markovian Decision Process (MDP).

Appendix A
The Single-period Bidding Model

The work of [1] modified the make-to-order (MTO) bidding model of [2] to incorporate the multiple customer segments classified based on parameters of willingness to pay, sensitivity to short delivery time, quality level requirement, and intensity of competition. In the model, they allowed a new prospective order to be placed at any position of the existing job sequence. Even though, enabling a certain level of overbooking (more attractive due date), such a method displaces a certain set of existing jobs and incurs extra cost of tardiness, which is referred as displacement cost in this research. In this section, we briefly summarize the development of this single-period bidding model. The model, referred as Model (A.1), is used in our RM bidding as the base model which is to myopically optimize the bidding terms for an arriving request and also estimate the expected profits obtained from each state of the dynamic programming. Readers are referred to [1] for more details.

To incorporate the multiple customer classes into the bidding model and to allow flexibility in job sequencing of the new and existing orders, Model (A.1) is proposed. For each customer/order arrival specified as job/order i from class k, a set of bid price b and due date d that maximizes the expected marginal revenue Π_i for the MTO firm is determined by:

Model (A)

$$\underset{\forall b,d>0,\ j>1}{Maximize}\ \Pi_i = p_{k(i)}(b,d)\cdot\left[b-\psi_j(i)-\sum_{s\in\Theta}\Delta\psi(s)\right], \tag{1}$$

where:

$$\psi_j(i)=\alpha_1^k E_j(T\,|W_i,d)+\alpha_2^k\cdot \Pr_j(a<W_i,d), \tag{1a}$$

$$\psi_q(s)=\alpha_1^{k(s)}E_q(T\,|W_s,d_s)+\alpha_2^{k(s)}\Pr_q(a<W_s,d_s),\quad \forall s\in\Theta, \tag{1b}$$

$$\Delta\psi(s)=p_{k(s)}(b_s,d_s)\cdot\left[\psi_{new\_q}(s)-\psi_{old\_q}(s)\right],\quad \forall s\in\Theta,\ new\_q\neq j. \tag{1c}$$

Π_i is the multiplication of the winning probability $p_{k(i)}(b,d)$ and the estimated revenue of the bid, $b-\psi_j(i)-\sum_s\Delta\psi(s)$. $\psi_j(i)$ is the estimated tardiness cost of job i given sequencing position j. $\sum_s\Delta\psi(s)$ is the sum of displacement costs incurred to each existing job $s\in\Theta$, the set of all jobs that have been displaced due to the insertion of order i into the existing job sequence at the j^{th} position according to the Earliest Due Date (EDD) rule of sequencing. In case that a *regular or cost-sensitive order i* is being placed at the end of the sequence, Θ is empty.

The adjusted work content or processing time W_i in Equations (1*a*) and (1*b*) is used primarily to characterize the distinct requirements of different customer classes. Given that the promised due date is d, Equation (1*a*) computes for job i, $\psi_j(i)$ as the sum of $\alpha_1^k E_j(T\,|W_i,d)$, tardiness charge for the total expected tardy periods, and other penalties $\alpha_2^k \Pr_j(a<W_i,d)$, including the expected cost of image loss and/or expediting cost. Define the Markovian state (a, t), where (a, t=d) is the state of being in time period d and having exactly a cumulative idle timeslots, $\Pr_j(a<W_i,d)$ is the cumulative distribution function for the probabilities of being in state (a, t=d), for a = 0, 1,…,W_i-1, or equivalently, $\sum_{a=0}^{W_i-1}\Pr_j(a,d)$. Given that job i is exactly T periods late,

$$E_j(T\,|W_i,\ d)=\sum_{T=1}^{\infty}T\left[\sum_{a=0}^{W_i-1}\Pr_j(a,d+T-1)-\sum_{a=0}^{W_i-1}\Pr_j(a,d+T)\right].$$

Equation (1*b*) calculates the tardiness cost for each job $s\in\Theta$ with the quoted due date d_s and the sequencing position q^{th}. $E_q(\cdot)$ and $p_q(\cdot)$ can be calculated in the same way as in Equation (1*a*). Equation (1*c*) computes the expected cost of displacing job s from the old_q^{th} to the new_q^{th} position, caused by the arrival of new job i.

The search for a winning probability is obtained from the response function of winning probability to the change in each key parameter b or d. Based on the S-shape curve of the surface response of the winning probability function to the changes in price and due date, given a fixed number of competitors joining the bid for job i, the winning probability for a change of n units of price or due date can be simply calculated as (Proof in [20]):

$$p_k(b+n,d)=1/\left[1+\left(\frac{1-p_k(b,d)}{p_k(b,d)}\right)\exp\left(\frac{-n[1+\eta_b^k\delta_B]\beta_b^k}{C_k w_i}\right)\right], \text{ when } d \text{ remains the same.} \tag{2a}$$

$$p_k(b, d+n) = 1 \Big/ \left[1 + \left(\frac{1 - p_k(b,d)}{p_k(b,d)} \right) \exp\left(\frac{-n[1+\eta_d^k \delta_D]\beta_d^k}{w_i} \right) \right], \text{ when } b \text{ remains the same.} \quad (2b)$$

To optimize the objective function (1), they proposed a modified pattern search algorithm of Adby and Dempster [17] in searching for an optimal price and due date using dynamic values of discrete step sizes that lead to faster convergence. Regarding the sequencing rule, two types of job orders, namely cost-sensitive (regular) orders and time-sensitive (urgent) orders, are treated differently according to the policy of flexible sequencing rule. Specifically, the First-Come, First-Serve rule (FCFS) is recommended for a regular or cost-sensitive order, and the EDD rule for a time-sensitive (or urgent) order. Such a flexible sequencing rule interplays with contingent available capacity and renders improved performance of the bidding system. The simulation study conducted under the stochastic demands and dynamic environment of finite periods did positively confirm in their paper that the multi-class Model (A.1) can significantly enhance the marginal revenue for the MTO bidding system.

Appendix B.

Indices:

i	= An arriving job for class 1, …, K
k	= Customer class, k = 1, ..., K
s	= A job that is affected or displaced by the arriving order

Decision variables:

b	= Bid price
d	= Promised due date
j	= Sequencing position for a job

Result variable:

Π_i	= Expected marginal revenue obtained from job i

Intermediate variables:

T	= No. of tardy periods = planned delivery date minus promised due date
w_i	= Preliminary estimated work content for job i
W_i , x	= The adjusted work content estimated for job i
a	= Available machine hours in the number of periods
Ω	= Set of all jobs currently in the system, including contingent orders
Θ	= Set of all displaced jobs s as a consequence of putting the arriving order i into position j
$p_k(b,d)$	= Winning probability for a job in class k, quoting price b and due date d
$E\left(T \mid W_i, d\right)$	= Expected tardiness for job i class k with content W_i^k and due date d
$\Pr\left(a < W_i, d\right)$	= Probability of being tardy for job i class k, with work content W_i^k and promised due date d
$\psi_j(i)$	= Estimated tardiness costs of job i, being sequenced at j^{th} position,
$\Delta\psi(s)$	= Displacement cost incurred if displacing job s from the current post

Parameters:

λ_k	= Expected number of bidders for customer class k
μ_k, σ_k	= Expected work content and its standard deviation for customer class k
ω	= Confident interval of adjusted work content
α_1^k	= Direct tardiness penalty per period for class k
α_2^k	= Embedded cost, e.g. cost of special transportation or expediting the job, or image losses
C_k	= Direct cost estimated per one unit of work content for job in class k
β_n^k	= Coefficient representing the weight of its corresponding term in predicting the winning probability, where n = 0, b, d, and λ
B_L, D_L	= Lower bounds of the acceptable price and due date, respectively
B_U, D_U	= Upper bounds of the acceptable price and due date, respectively
δ_B, δ_D	= Triggers, activating new values of coefficients when price and due date are beyond the upper bounds, respectively
η_b^k, η_d^k	= Multiplying factors, activated by the triggers, to operate with the betas resulting in higher values of coefficients for price and due date.

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