On the Integrated Supply and Distribution Problem with Heterogeneous Vessels

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Abstract: In many business applications, the supply schedules, which is determined by the manufacturer’s capability, and the distribution schedules, which is determined by the logistics partner’s capability, must be coordinated to form an integrated schedule with respect to a given business objective (e.g., minimizing the total operating cost, maximizing the customer satisfaction, etc.). This leads to the integrated supply-distribution problem. In this paper, we analyze and develop a solution approach for a variation of such integrated supply-distribution problems involving heterogeneous vessels and many customers.

Keywords: Integrated Supply-Distribution scheduling problem, heuristics, heterogeneous vessels

I. Introduction

In today’s industry, manufacturers are outsourcing their productions to low cost countries as a competitive advantage and then ship back the product to serve the domestic markets. This is a significant driver of growth of global containerized trade. According to UNCTAD’s Review of Maritime Transport 2009 reports [29], despite of global economic downturn in 2008, international seaborne trade still grew by 3.6% and the world total of containerized trade has reached to the level of 137 million TEUs (1.3 billion tons), an increase of 5.4% over 2007. With the global economy recovering, ship container volumes from Asia to Europe grew by nearly 10 percent year-on-year in December 2009 [26]. PIERS Global Intelligence Solutions projected that the total U.S. containerized ocean imports will grow 9.1 percent in 2010 [19]. The forecast projects containerized imports on the Trans-Pacific trade lanes will grow 12.1 percent in 2010. According to Xinhua News [28], as the first country emerging from the global economic downturn, China's gross domestic product (GDP) grew 8.7% in 2009 and the projection of growth for 2010 is around 8%. On the other hand, however, managing real-life shipping operations to timely bring back finished goods within the budget and customer expectation has become increasingly challenging. A recent report from GCTIL [14] indicated that on-time container ship arrival rate during the fourth quarter of 2009 was about 50%. Many business executives are concerned that their companies do not have a global procurement, manufacturing, and distribution network to deliver products on time and at the budgeted cost. The survey also indicates that failures in design and execution at the operational level will sidetrack global plans every time. Among the optimization problems to be solved toward a highly integrated and effective supply chain process, one of them is the integrated scheduling of supply and distribution operations.

Our study on the integrated supply (production) and distribution was motivated by the practice of a primary supplier for wood panels used in housing construction. The company outsourced its production to low-cost countries in Asia and has since been utilizing capacitated ocean vessels to ship the products back to North America to serve its regional markets. The products are produced in lots based on the anticipated demand, and transported from the contracted factories to a transshipment location, from where the products are then distributed via barges to the domestic customer ports to meet the demand. Since the actual demand of the customer ports may deviate from the anticipated lot sizes, and since vessels have only a limited capacity and traveling speed, the coordination between the product availability and the distribution operations is critical in order to avoid high operating and shortage cost in the supply chain process.

In this study, we hypothesize our optimization problem based upon the operations of a segment of this supply chain process between a main transshipment port, location 0, and a sequence of customer ports along the coast, \(I = \{1, 2, \ldots, |I|\}\), from where the regional markets will be served. The transshipment port receives a large shipment, \(\Omega_i\), from foreign factories in period \(i\), \(i = 1, 2, \ldots, T\), and dispatches contracted vessels that deliver smaller batches to the customer ports. Each customer port, \(i \in I\), has an anticipated period demand from its regional market, \(d_{ij} \geq 0\), a holding cost \(h_i\), and a penalty cost \(p_i\) for each unit of shortage per time period. The contracted vessels are heterogeneous and capacitated. Let \(V\) be set of vessel types. Each vessel of type \(v, v \in V\), has a maximum loading capacity \(u_v\), an expected traveling time \(t_{ij}^v\) between ports \(i\)
and \( j \), where \( i, j \in I \cup \{0\} \), an available operation time \( \pi_i \) per period, a fixed cost \( c_{ij}^f \) per trip, a variable operating cost \( c_{ij}^v \), an expected berthing time, and cost, at customer port \( i \), denoted by \( l_{ij} > 0 \), and \( c_{ij}^b \), respectively, \( i \in I \). The inventory capacity is not considered here.

In addition, we make the following assumptions:

The shortage penalty at customer port \( i \) in time period \( t \) is proportional to \( p_i \cdot \max \{0, S_{ij}^t\}^c \) where parameter \( c \geq 1 \) denotes a potential nonlinear relationship between the penalty cost and the shortage quantity;

For some \( t \), we may have \( \Omega_t < \sum_{i \in I} d_{ij}, 1 \leq t \leq T \).

Whenever there is a shortage at the transshipment port, there is a need to balance the limited supplies among the competing customer ports;

Since the vessels will be traveling along the coastline, for any pair of ports \( i \) and \( j \), where \( i \rightarrow j \), \( i \rightarrow j \) is possible but \( j \rightarrow i \) is not considered in any given trip;

Each vessel may take at most one trip/route per period while the trip/route may cover more than one customer port; and each customer port can be visited by at most one vessel in a time period.

The problem is to allocate the supplies \( \Omega_1, \Omega_2, ..., \Omega_T \) to customer ports and to determine the vessel-mix (to be contracted from the vessel owner) for each time period to transport the deliveries so that the sum of the total shortage costs, the vessel (fixed and variable) cost, and the inventory holding cost is minimized subject to the vessel capacity, traveling time, and the product availability. Let \( P \) denote this problem.

If we disregard the product availability and the inventory issue in this supply process, then \( P \) becomes a classical vessel routing problem which has received a significant attention in academic research. Among those, Lane et al. [16] studied the problem of determining the economical ship size and the mix of fleets for a specific route with a known demand over a finite planning horizon. Claessens [8], introduced a shipping model that minimizes the total costs including a penalty cost for cargo not shipped due to capacity constraints. Ranna and Vickson [21] [22] studied the vessel routing problem to maximize a carrier’s profit. Perakis and Jaramillo [20], and Jaramillo and Perakis [15], considered the problem of assigning container vessels to a given set of routes under practical operating costs. Cho and Perakis [2] proposed a model to find the optimal fleet size and the associated liner routes. By generating a priori a number of candidate routes for different vessels, the problem was solved as a linear program. Fagerholt [9] presented an approach to determine the optimal fleet and their weekly liner routes. The problem was formulated as a multi-trip vehicle routing problem and then solved by a set partitioning based approach. Bendall and Stent [1] studied the problem of optimizing the fleet configuration and the associated fleet deployment plan in a container vessel hub and spoke application. A mixed integer program was solved to determine the optimal fleet size and the profitability of a short-haul hub and spoke feeder operation based in Singapore. Sambracos et al. [25] tackled the coastal freight shipping problem via two phases: the strategic planning of fleet size by solving a linear program and the operations scheduling via solving a vehicle-routing problem. Fagerholt [11] presented a decision support system, called TurboRouter, for vessel scheduling. The scheduling problem was solved by the insertion heuristic for an initial feasible solution and then a hybrid local search algorithm to improve the quality of the initial solution. Chen et al. [3] considered a strongly NP-hard container vessel scheduling problem with bi-directional flows. They proved that a special case of this problem is totally unimodular, and then designed a heuristic scheduling algorithm based on this property. Lei et al. [17] proposed two-phase approach to solve the integrated production, inventory, and distribution routing problem. They solve the problem with all constraints but only direct shipments in phase 1, and then solve the associated consolidation problem to handle the potential inefficiency of direct shipment. Several interesting studies on vessel planning and scheduling can also be found in the work by Fagerholt and Lindstad [13], Fagerholt and Christiansen [12], Christiansen and Fagerholt [5] [6], Fagerholt [10], and Lei et al. [18] etc. Four excellent reviews of the results in this area can be found in the work by Ronen [23] [24], Christiansen et al. [7], and Chen [4].

However, if we add the product availability and inventory constraints into consideration, then not many results in the current literature are available.

In this study, we report three fundamental properties of problem \( P \), upon which we propose a greedy heuristic search algorithm. For each time period in the \( T \)-period planning horizon, this heuristic solves a minimal cost flow problem to form an initial vessel schedule and then improves the initial schedule by applying a bin-packing heuristic. After feasible vessels schedule for all the time periods are obtained, a linear programming problem is solved to form an integrated supply-distribution schedule that solves problem \( P \). The remaining part of the study is organized as follows. In Section 2, a mathematical programming model (formulated as a set partitioning problem) that formally defines the problem is presented, and an analysis on the problem properties is conducted. In Section 3, the proposed greedy heuristic search algorithm that solves the integrated supply-distribution problem is introduced, and a numerical example is presented. Finally in Section 4, we conclude the study and discuss its future extensions.
II. An analysis of the integrated supply and distribution problem

To make a formal definition of problem \( P \), let us start with the following parameters and variables.

**Parameters:**
- \( r \): The index of a vessel route;
- \( k(t) \): The \( k \)-th vessel deployed in period \( t, t=1,2,...,T \);
- \( v(k(t)) \): The vessel-type of the \( k \)-th vessel deployed in period \( t \), \( v(k(t)) \in V \);
- \( R_{k(t)} \): The set of feasible routes (i.e., the total traveling and berthing time does not violate \( \pi_v \)) for the \( k \)-th vessel deployed in period \( t, t=1,2,...,T \);
- \( c^r_{k(t)} \): The operating cost (traveling and berthing cost) of the \( k \)-th vessel in period \( t \) if route \( r \) is taken, \( r \in R_{k(t)} \);
- \( Y_{r,i} \): Binary constant that \( Y_{r,i} = 1 \) if customer port \( i \) is on route \( r \), \( \forall i \in I \); 
- \( K_{\max} \): An upper bound on the maximum number of vessels to be deployed in each time period, where \( K_{\max} \leq |I| \).

**Variables:**
- \( X_{k(t),r,t} \): Binary variables, \( X_{k(t),r,t} = 1 \) if vessel \( k(t) \) takes route \( r \) in period \( t \);
- \( S_{i,t} \): The shortage quantity at port \( i \) in period \( t \);
- \( Q_{k(t),i,t} \): The quantity delivered by vessel \( k(t) \) to port \( i \) in period \( t \);
- \( I_{i,t}, I_{0,t} \): The ending inventory of port \( i \), and the transshipment port, in period \( t \).

[Model P]

\[
\begin{align*}
\text{Minimize:} & \quad \sum_{\forall i} \left[ \sum_{\forall k(t)} (c^r_{k(t)} + c^s_{k(t)}) X_{k(t),r,t} + h_0 \cdot I_{0,t} \right] \\
& \quad + h_i \cdot I_{i,t} + \sum_{\forall i} p_i \cdot (S_{i,t})^x \\
\text{s.t.:} & \\
1. & \quad I_{i,t-1} + \sum_{\forall k(t)} \sum_{\forall r} Q_{k(t),i,t} - d_{i,t} + S_{i,t} = I_{i,t} \\
& \quad \forall i \in I, t = 1,2,...,T
\end{align*}
\]

\[
I_{0,t-1} + \Omega_t - \sum_{\forall k(t)} \sum_{\forall i \in I} Q_{k(t),i,t} = I_{0,t} \\
\forall t = 1,2,...,T
\]  

2. A vessel may be assigned to at most one route per period 

\[
\sum_{\forall r \in R_{k(t)}} X_{k(t),r,t} \leq 1 \\
k(t) = 1,2,...,K_{\max}, \quad t = 1,2,...,T
\]

3. Each port can be visited by at most one vessel in a time period 

\[
\sum_{\forall k(t) \in I} X_{k(t),r,t} \leq u_v \cdot \sum_{\forall r \in R_{k(t)}} X_{k(t),r,t} \\
k(t) = 1,2,...,K_{\max}, \quad t = 1,2,...,T
\]

4. Vessel capacity constraints 

\[
\sum_{\forall r} Q_{k(t),i,t} \leq u_v \cdot \sum_{\forall r \in R_{k(t)}} X_{k(t),r,t} \\
k(t) = 1,2,...,K_{\max}, \quad t = 1,2,...,T
\]

5. Initial inventory 

\[
I_{i,0} = 0 \quad \forall i \in I \cup \{0\}
\]

6. Others 

\[
Q_{k(t),i,t}, I_{0,t}, I_{i,t}, S_{i,t}, u_v, X_{k(t),r,t} \in [0,1] \\
k(t) = 1,2,...,K_{\max}, \forall i \in I, t = 1,2,...,T
\]

Problem \( P \) has several properties. Observations 1, 2, and 3 below reveal three of these properties.

**Observation 1.** Problem \( P \) is NP-hard in strong sense, even with \( |T|=1 \), identical vessels, sufficient supplies, and instantaneous vessel travel times.

In practice, sometimes the orders from individual customer ports are non-splittable in the shipment (i.e., the quantity in one order must arrive via the same vessel-trip) and non-substitutable (i.e., the orders are customized subject to manufacturing specifics, and \( d_{i,t} \) denotes the total order quantity that is expected to arrive on or before time period \( t \) at port \( i \)). Observation 2 below reveals a strongly polynomial time solvable case encountered under such conditions.

**Observation 2.** If \( |I|=1 \), \( p_i = \infty \), \( \Omega_1 \geq \sum_{t=1,2,...,T} d_{i,t} \), \( \Omega_t = 0 \) for all \( t > 1 \), \( h_0 = 0 \) and the orders are non-
Substitutable and non-splitable, then problem \( P \), even with heterogeneous vessels, is strongly polynomial time solvable.

This result is generalized from Lei et al. (2006) where the case with identical vessels was considered.

**Observation 3.** If \( T=1 \), \( \Omega_i \geq \sum_{v \in \sigma_i} d_i + h_0 = 0 \), \( p_i \cdot d_i > c_v^L + 2 \cdot c_v^P \cdot \tau_{i}^K + c_v^K \), and each vessel may serve at most two customer ports per period, then \( P \) is solvable in strongly polynomial time.

This result is generalized from Lei et al. (2007) where the vessel traveling factor was excluded.

Note that the optimality of the minimal cost flow in Observation 3 is no longer guaranteed when the fleet size becomes a bottleneck. That is, if the number of vessels being deployed by the minimal cost flow solution exceeds the fleet size, then, the problem of reallocating the ports to fit the fleet size becomes NP-hard. On the other hand, if each vessel can visit/service more than two ports, then the optimality of the minimal cost flow in Observation 3 is not guaranteed either, and the vessel utilization needs to be improved. In this case, the available heuristics, such as the first-fit decreasing (FFD) heuristic, for the bin-packing problem can be applied. Among FFD related studies, Yue [29] provided a simple proof that FFD runs in \( O(n^2) \) time and for every instance \( L \) of bin-packing provides the solution performance of \( FFD(L) \leq (11/9)OPT(L) + 1 \). However, due to the heterogeneity of the vessels (i.e., heterogeneous bins), such a heuristic must be extended before it can be applied to our case. Table 1 outlines an extended FFD heuristic.

| Table 1 The Extended FFD Heuristic |

**The Extended FFD (EFFD) Heuristic**

**Step 1.** Solve the single period minimal cost flow problem (Observation 3). Let \( K \) be the number of vessels being deployed in the minimal cost flow solution.

**Step 2.** For the \( k \)-th vessel, \( k=1,2,\ldots,K \), with its given route \( r_k \), compute the saving factor by using

\[
\Lambda_k = \sum_{v \in \sigma_k} p_i \cdot d_i - c_v^f
\]

and form a non-increasing sequence \( \sigma = \langle 1, \ldots, K \rangle \mid \Lambda_{j} \geq \Lambda_{j+1}, 1 \leq j < K \rangle. \) Let \( k=K.\)

**Step 3.** (Given \( \sigma \) and index \( k \)) Remove vessel \( k \) from \( \sigma \) and apply the classical FFD heuristic to assign the customer ports \( i, i \in r_k \), to the remaining vessels (bins) in \( \sigma \). If success, then permanently delete vessel \( k \) from \( \sigma \). Let \( k \leftarrow k - 1 \) and repeat Step 3 until none of the vessels in the current sequence \( \sigma \) can be deleted.

Applying the minimal cost flow algorithm and then improving the vessel utilization by EFFD determine an independent vessel schedule for a single time period assuming that we have sufficient supplies at the transshipment port and that we do not have to be concerned with the inventory. To build a complete solution to the integrated supply-distribution problem, \( P \), however, it is necessary for us to combine the \( T \) vessel schedules together by considering the supply capacity \( (\Omega_1, \Omega_2, \ldots, \Omega_T) \) and the inventories carried at each customer port \( i, i \in I \). In next section, we shall address this issue.

**III. A greedy solution for Problem P**

The greedy solution that we propose for solving problem \( P \) calls the minimal cost flow algorithm and the EFFD heuristic to form the vessel schedule for each time period \( t=1,2,\ldots,T \). Let \( Z_{k(t),i,t} \in \{0,1\}, \forall k(t), \forall i \in I \), be binary variables that \( Z_{k(t),i,t}=1 \) if the \( k \)-th vessel in period \( t, k(t) \), visits customer port \( i \) in period \( t \). Then, the values of \( Z_{k(t),i,t} \), \( \forall k(t), \forall i \in I, \forall t \), become known as EFFD terminates. Since the search process for the vessel assignment does not consider the inventory issue, one remaining problem is how to connect the \( T \) independent vessel schedules \( \{Z_{k(t),i,t}\} \), one for each time period \( t=1,2,\ldots,T \), through inventory planning. To do so, let \( K(t) \), and \( I(t) \), be the set of vessels being dispatched, and the set of customer ports being visited, in period \( t \), \( t=1,2,\ldots,T \), respectively, where \( i \in I(t) \) and \( k(t) \in K(t) \) if and only if \( Z_{k(t),i,t}=1 \). Let \( S_{i,t} \) be the shortage quantity at port \( i \) in time period \( t \), \( Q_{k(t),i,t} \) be the quantity delivered by vessel \( k(t) \) to port \( i \) in period \( t \), where \( k(t) \in K(t) \), and \( I_{i,t} \) be the ending inventory of port \( i \) in period \( t \).

Then, the respective inventory planning problem can be defined as follows.

\[
P_L: \text{Min} \sum_{t} \left[ \sum_{v \in I(t)} h_i \cdot I_{i,t} + \sum_{v \in \sigma \setminus i} p_i \cdot S_{i,t} \right]
\]

s.t.

\[
I_{i,t+1} = \sum_{v \in \sigma \setminus i} Q_{k(t),i,t} - d_i + S_{i,t} = I_{i,t}
\]

\[
I_{i,0} = 0, \forall i \in I(t), \forall t \in 1,2,\ldots,T
\]
I_{0,t-1} + \Omega_t - \sum_{k(t) \in K(t)} \sum_{i \in I(t)} Q_{k(t),i,t} = I_{0,t}, \quad I_{0,0} = 0 \quad \forall t \in 1,2,...,T \tag{12}

\sum_{i \in I(t)} Q_{k(t),i,t} \leq u_{k(t)}, \quad \forall k(t) \in K(t), \quad \forall i \in I(t), \forall t \in 1,2,...,T \tag{13}

Q_{k(t),i,t}, I_{i,t}, S_{i,t}, I_{0,t} \geq 0, \quad \forall i \in I(t), \forall k(t) \in K(t), \forall t = 1,2,...,T \tag{14}

Since P_L is a linear programming problem, it can be solved quickly via CPLEX. The resulting solution together with the vessel schedules discussed in Section 2 defines a feasible heuristic solution to problem P. This is outlined in Table 2.

Table 2: A greedy search algorithm for solving P

| Step 1. (Initialization) Let \( I_{0,0} = 0 \), \( i \in I \cup \{0\} \);  
Step 2. For each time period \( t \), \( t = 1,2,...,T \), apply the Minimal Cost Flow Algorithm and the EPPF heuristic to obtain the vessel schedule: \( \{Z_{k(t),i,t} \mid k(t) \in K(t), \forall i \in I(t) \} \);  
Step 3. Based upon the given \( K(t), I(t), \) for \( t = 1,2,...,T \), solve the respective linear programming problem \( P_L \) to obtain \( Q_{k(t),i,t}, I_{i,t}, S_{i,t} \), and \( I_{0,t} \). Compute the total operating cost (1). |

To demonstrate the use of this greedy search algorithm, we solve an integrated supply-distribution problem by considering a distribution network with the New York port (0-Supply) as the transshipment port, and five customer ports: Philadelphia (P-1), Norfolk (P-2), Wilmington (P-3), Charleston (P-4), and Savannah (P-5). The distance between ports (Nautical Miles) are from the Office of Coast Survey, National Ocean Service (NOS) and the traveling time between ports (hours) are calculated based on the average vessel speed 15 knots/hour. The other related data are given in Table 3 and Table 4. The total operation cost obtained from our proposed algorithm is $1,650,200. We have also solved the original problem P by using CPLEX Optimizer 9.0 on a Dell LATITUDE D600 (Pentium(R) M, 1.4GHz, 1.00GB of RAM). The optimal schedule was obtained after 6899.16 seconds with a total cost of $1,596,200. The gap between the optimal solution and that obtained by the proposed greedy search algorithm for this case is 3.38%.

Table 3: Vessel related data

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IV. Concluding Remarks

In this study, we focused on a variation of the integrated supply-distribution problem involving heterogeneous vessels, multiple customer ports, multiple time periods, the product availability, and the capacitated inventories. We reported and proved three observations on the properties of the problem and proposed a greedy heuristic search algorithm to solve it. We also demonstrated the use of the proposed heuristic search algorithm to solve a problem involving some real industry data. While the study focused on only a sub-problem of the collaborative planning and scheduling, it has potentials to be applied as an efficient decision support tool to assist the development of distribution plan in practice. The results reported in this study can also be used for developing the search algorithm for solving the general version of the integrated supply-distribution problem, where manufacturer’s capacity is included.

References

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Dr. Chunxing Fan is an assistant professor of supply chain management in the Business Administration Department, College of Business, Tennessee State University. He received his Doctoral degree in Supply Chain Management and MBA from Rutgers Business School, Rutgers University, NJ, USA. His research focuses on supply chain and operations management, integrated production and distribution optimization, logistics and transportation network design, and resource/project management. He published in Journal of Operational Society, Transportation Research Part E: Logistics and Transportation Review, International Journal of Management & Enterprise Development, etc.

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