Abstract: Air-cargo capacity is random and affected by the number of passengers carried, because both cargo shipments and passenger bags are carried in a belly of a plane. The fewer the passengers carried, the higher the cargo capacity. Current seats sold usually provide some information on the passengers carried, and consequently the cargo capacity. Records of the passengers carried and the seats sold are readily available in a passenger revenue management system. We propose mathematical models to evaluate monetary benefits, if different levels of information in the passenger revenue management system are shared by the cargo revenue management system. At a minimum level, an airline constructs a prior distribution of random cargo capacity from a historical record of passengers carried. At a higher level, the airline updates the distribution of cargo capacity based on the number of seats sold. A numerical example that illustrates the proposed methodology is also provided.

Keywords: Air-Cargo; Revenue Management; Stochastic Model Applications

I. Introduction

Air-cargo operations are a significant source of revenue for passenger airlines, most of which carry cargo shipments in the belly of their aircraft. The air-cargo industry is expected to grow five percent annually during 2007–2027 [6]. The growth is propelled by global free trade, and the emerging implementation of supply chain management strategies, which emphasize on short lead times. Despite its importance, revenue management (RM) systems for managing air-cargo spaces are much less developed than those for controlling prices and availability of passenger tickets.

Air-cargo RM is more complex than passenger RM. Cargo capacity is a multi-dimensional quantity; two important dimensions are weight and volume. The flight may be full with respect to the weight capacity but not the volume capacity, or vice versa. In contrast, the total passengers on board are constrained by a one-dimensional quantity, the number of seats on the plane. Moreover, cargo capacity is random and may be affected by various factors, such as the amount of fuel, the length of the runway, the weather condition at the departure time, and the number of passengers carried as well as their bags. The belly of the plane carries both passenger bags and air-cargo shipments.
shipments throughout the cargo booking horizon minus any penalty costs if available cargo capacities are insufficient to accommodate these shipments. The problem is formulated as a finite-horizon Markov decision process (MDP); in each decision epoch, the carrier determines whether to accept or reject the booking request. We assume that the cargo capacity depends on the number of seats occupied at the flight departure time. The more seats occupied, the tighter the cargo volume and weight capacity constraints.

The cargo booking horizon is shorter than the passenger booking horizon. A plane ticket can be sold one year in advance, whereas cargo bookings are often made within two weeks before the departure. With information sharing, the carrier is assumed to know the current number of passenger reservations at the beginning of the cargo booking horizon, and this information is used to update the distribution of the random cargo capacity. We refer to the particular level of information sharing the imperfect information case. At a lower level, the carrier receives only the historical record of passengers on board, and constructs the belief regarding the cargo capacity, but does not update the distribution. We call this the base case. The difference between the expected contribution under the imperfect information and that in the base case represents the monetary value of information sharing. We also consider the case of perfect information, in which we assume that the number of passengers carried were available to the airline when it starts accepting cargo bookings.

There is a vast literature on passenger RM (e.g., [14] for survey) and a number of articles that discuss complexities of air-cargo RM (e.g., [3] [5] [11]). However, fewer journal papers present mathematical models for air-cargo RM. Articles on the network air-cargo RM can be found in e.g., [2] [4] [13]. Below, we briefly review some articles on the single-leg flight.

Formatulate a short-term capacity planning problem for air-cargo space as an MDP. In each time period, the carrier needs to determine how much short-term space to acquire given the long-term contract space in order to minimize the total cost, which includes the costs of short-term space and the penalty costs of backlogged shipments over the entire planning horizon. Their article considers a sequence of repeat flights; shipments may be sent via some later flights if the current flight is full. Putting it differently, all shipments are accepted and sent on the first-come, first-served basis. Our paper considers a single flight, and our objective is to determine an optimal booking control policy (rather than adopting the first-come, first-served heuristic).

[1] Formulate an MDP for an air-cargo booking problem on a single-leg flight with the goal of maximizing the expected contribution. In each time period, the carrier needs to decide whether to accept or reject a booking request from a particular shipment type, which determines the distribution of volume and weight requirements and the expected freight charge. They show that an exact solution to the problem is impractical, because of its high-dimensional state space. They develop heuristics and compare their performances under various settings. To keep our problem tractable, we assume that the volume and weight requirements of each shipment are known at the time of the request. More importantly, the cargo capacity on the plane is random in our formulation, whereas it is fixed in theirs. Our optimal booking policy accounts for the uncertainty in the cargo supply.

Mathematical models that incorporate the randomness of the cargo capacity can be found in e.g., [12] [15] [9]. In these articles, the cargo capacity is assumed to be a univariate random variable. The first two articles consider a static (single-period) overbooking model; the carrier’s objective is to find an overbooking limit in order to maximize the expected contribution, which includes both oversale and spoilage costs. They are overbooking models, whereas ours is a booking control model, which attempts to achieve the best mix of demands from different shipment classes. The third article formulates a dynamic (multi-period) booking control problem as an MDP. The terminal value function is the expected offloading cost; the expectation is calculated with respect to the carrier’s prior belief regarding the cargo capacity. Their model is subsumed in ours. Furthermore, we allow the carrier to update the belief from additional information regarding the number of seats sold.

The rest of this paper is organized as follows. In Section 2, we describe the cargo booking control process and develop an MDP model for the problem in each of the three cases. We provide a numerical example to illustrate the calculations of the value of information in Section 3. A conclusion is presented in Section 4.

II. MDP Formulation

Assume that the cargo booking horizon for the single-leg flight is comprised of \( \tau \) time periods, numbered in reverse chronological order. More precisely, time period \( t = \tau \) corresponds to the beginning, and time period \( t = 0 \) corresponds to the end of the booking horizon. These intervals are small enough so that at most one booking request arrives in each period. Each request to book a shipment belongs to one of the \( m \) types, and the arrivals of booking requests are independent across time periods. Let \( p_{it} \in (0,1) \) be the probability that in time period \( t \) a type-i request arrives, and \( p_{it} = 1 - \sum_{i=1}^{m} p_{it} \) the probability that no requests arrive where \( \sum_{i=1}^{m} p_{it} \leq 1 \).

A request type determines the following three quantities: (volume, weight, contribution). If a type-i request is
accepted, then it generates a contribution of \( p_i \) and consumes \((v_i, w_i)\) units of volume and weight, respectively. For instance, a contribution for a shipment may be computed as \( \rho_i = \sum_{s} (\max\{w_i, v_i\} / \gamma) \) where \( \gamma = 167 \) kilograms per cubic meter [defined by the International Air Transportation Association (IATA) volumetric standard], and \( \sum_{s} \) is the contribution probability, the freight rate plus the ancillary contribution (e.g., special handling, insurance) minus the incremental costs (e.g., variable fuel costs). Note that two shipments which consume identical volume and weight may yield two different contributions, if one requires special handling and the other does not; the notion of types is simply a modeling device to capture the above situation.

Cargo capacity depends on the volume and weight of passenger bags, because both cargo and passenger bags are transported side-by-side in the cabin of the combi-aircraft or in the belly under the main deck of passenger planes. Let \((k_v(z), k_w(z))\) denote the volume and weight cargo capacity, if the number of seats occupied at the departure time (passengers carried) is \( s \). If the available capacity \((k_v(z), k_w(z))\) is not sufficient to accommodate booked shipments [i.e., \( x > k_v(z) \) or \( y > k_w(z) \) where \( x \) and \( y \) are the total booked volume and weight, respectively], then an oversold situation occurs, and the carrier incurs penalty cost.

Let \( c(x, y, z) = c_v (x - k_v(z))^2 + c_w (y - k_w(z))^2 \) where \( c_v \) (resp., \( c_w \)) be a unit penalty cost per volume (resp., weight) excess. This penalty function is previously assumed and discussed in [1].

Let \( S \) be the number of passengers carried, and \( R \) the number of seats that have been reserved from the beginning of the passenger booking horizon to that of the cargo booking horizon. Assume that they have a common state space \{a, a + 1, ..., b, b - 1, b\} where \( 0 < a < b \). Note that if the airline overbooks, \( b \) can be greater than the number of seats on the plane. Recall that the cargo bookings normally start after the passenger bookings; a needs not be zero. If the beginning of the cargo booking horizon is two weeks before the departure, then \( S \) is equal to \( R \) plus additional seats sold during the last two weeks minus cancellations and no-shows.

Let \( \gamma(\mathbf{S}|\mathbf{R}) = P(\mathbf{S} = \mathbf{S}|\mathbf{R} = \mathbf{R}) \) the conditional probability that the number of passengers carried is \( \mathbf{S} \), given that the total seats sold at the beginning of the cargo booking horizon is \( \mathbf{R} \).

Assume that cash flows are not discounted, since cargo bookings occur over a short period. Furthermore, assume that the carrier is risk neutral, since the flights are repeated many times in each season. The carrier wants to determine a booking policy so that its expected total contribution is maximized. Suppose that the total seats sold at the beginning of the cargo booking horizon is \( \mathbf{R} \). Let \( g_{\mathbf{R}}(x, y|\mathbf{S}) \) be the maximum expected total contribution that can be obtained from time period \( t \) until the departure time, given that the total volume (resp., weight) sold is \( x \) (resp., \( y \)). We refer to the real-valued function \( g_{\mathbf{R}} \) as the value function at time \( t \).

The carrier believes that the passengers carried are \( \mathbf{S} \) with contribution \( \gamma(\mathbf{S}|\mathbf{R}) \). The optimality equations are as follows:

\[
g_{\mathbf{R}}(x, y|\mathbf{S}) = \sum_{s=0}^{b} p_{\mathbf{R}} (x + v_s y + w_s | s) \cdot \gamma(s) \cdot \max \{ \rho_s + g_{\mathbf{R} - 1}(x + v_s y + w_s | s), g_{\mathbf{R} - 1}(x, y|\mathbf{S}) \}
\]

\[
g_{\mathbf{R} - 1}(x, y|\mathbf{S}) = - \sum_{s=0}^{b} c(x, y, s) \cdot \gamma(s) \cdot \gamma(s)
\]

In period \( t \) when the carrier has already sold \( x \) units of volume and \( y \) units of weight, the request to book a type-\( i \) shipment is accepted, if \( \rho_i \geq g_{\mathbf{R} - 1}(x, y|\mathbf{S}) \). An optimal policy states that we accept the request if its contribution \( \rho_i \) exceeds the expected loss from future contribution (i.e., the opportunity cost from accepting the request).

Let \( \alpha = [\alpha_a, \alpha_{a + 1}, ..., \alpha_{b - 1}, \alpha_b] \) be the distribution of random variable \( R \). If the number of seats occupied at the beginning of the horizon is \( \mathbf{R} \), then the conditional expectation of the carrier’s maximum total contribution is \( g_{\mathbf{R}}(0, 0|\mathbf{R}) \). The (unconditional) expectation can be found by the law of total probability:

\[
\xi^{(C)} = E[g_{\mathbf{R}}(0, 0|\mathbf{R})] = \sum_{\mathbf{R}} \alpha_\mathbf{R} \cdot g_{\mathbf{R}}(0, 0|\mathbf{R})
\]

We refer to this as the expected total contribution under imperfect information.

We also consider two other cases, namely the base case and the perfect information case. In the case of perfect information, we assume that the number of passengers carried becomes available to the carrier, when it starts accepting cargo booking requests. In other words, the carrier has perfect foresight and correctly forecasts the total seats occupied. Suppose that the number of passengers (the realization of \( S \)) is \( \mathbf{S} \). Let \( f_{\mathbf{S}}(x, y|\mathbf{S}) \) be the maximum expected revenue that can be obtained from time period \( t \) until the departure time, given that the total volume (resp., weight) sold is \( x \) (resp., \( y \)). The value function can be computed recursively via the following optimality equations:

\[
f_{\mathbf{S}}(x, y|s) = \sum_{s=0}^{b} p_{\mathbf{S}} (x + v_s y + w_s | s) \cdot \gamma(s) \cdot \max \{ \rho_s + f_{\mathbf{S} - 1}(x + v_s y + w_s | s), f_{\mathbf{S} - 1}(x, y|s) \}
\]

\[
f_{\mathbf{S} - 1}(x, y|s) = -c(x, y, s)
\]

The expected total contribution under perfect information is obtained by weighting each scenario with the associated probability:

\[
\xi^{(P)} = E[f_{\mathbf{S}}(0, 0|\mathbf{S})] = \sum_{\mathbf{S}} \gamma(\mathbf{S}) \cdot f_{\mathbf{S}}(0, 0|\mathbf{S})
\]
where \( \pi(z) = \sum_{x \in A} \pi(s|x) \alpha(x) \) is the (unconditional) probability that the number of seats occupied at the end of the horizon is \( z \). We call vector \( \pi = [\pi(x), \pi(x+1), \ldots, \pi(b-1), \pi(b)] \) the carrier’s prior belief regarding the number of passengers carried. In the base case, we assume that the carrier does not know the number of seats sold at any time periods. However, it has the prior belief regarding the number of passengers carried. In contrast to the case of imperfect information, the carrier’s belief has never been updated, since it does not receive any additional information regarding the number of seats sold. The optimality equations in the base case become

\[
h_t(x, y) = \sum_{t=1}^{\infty} p_t \max \left( \alpha_t + h_{t-1}(x + v_t, y + w_t), h_{t-1}(x, y) \right) + \rho_t \theta_{t-1}(x, y)
\]

\[
h_t(x, y) = -\sum_{t=1}^{\infty} \pi(z) \pi(x, y, z)
\]

The expected contributions in the base case is

\[\xi^\Omega = h_t(0,0)\]

The total contribution obtained in the case of perfect information corresponds to the best possible outcome. The carrier could not do better than this. The expected total contribution under imperfect information is bounded above by that under perfect information and bounded below by that in the base case:

\[\xi^\Omega \leq \xi^{\Omega_0} \leq \xi^{\Omega_2}\]

The difference \(\xi^{\Omega_0} - \xi^{\Omega_2}\) is sometimes referred to as the expected value of perfect information (EVPI), and \(\xi^{\Omega_0} - \xi^{\Omega_2}\) the expected value of imperfect information (EVPII); see Chapter 12 in Clemen and Reilly (2001). Calculations of these quantities are illustrated via a numerical example in the next section.

### III. Illustrative Example

We divide the booking horizon into \( t = 57 \) time periods and categorize the shipment requests into \( m = 10 \) types. The request probabilities for shipment type are shown in Table 1, and the volume and weight requirements for each shipment type are shown in Table 2. With this choice of parameters, the expected number of requests for the entire booking horizon is around 142, the expected total volume and weight are 84 cubic meters and 15000 kilograms. These volume and weight capacities are approximately equal to those of Airbus A330-300. The freight rate is 50 per kilogram; i.e.,

\[\rho_t = 50 \max \{ 100 w_t, v_t / 167 \}.\]

The penalty costs are \( c_v = 10 \) per one unit volume oversold and \( c_w = 10000 \) per one unit weight oversold.

### Table 1 Request probabilities for shipment type

<table>
<thead>
<tr>
<th>period type</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-19</td>
<td>.27</td>
<td>.20</td>
<td>.10</td>
<td>.07</td>
<td>.10</td>
<td>.05</td>
<td>.05</td>
<td>.00</td>
<td>.00</td>
<td></td>
</tr>
<tr>
<td>20-38</td>
<td>.30</td>
<td>.15</td>
<td>.05</td>
<td>.05</td>
<td>.06</td>
<td>.03</td>
<td>.04</td>
<td>.02</td>
<td>.02</td>
<td></td>
</tr>
<tr>
<td>39-57</td>
<td>.10</td>
<td>.08</td>
<td>.08</td>
<td>.08</td>
<td>.09</td>
<td>.09</td>
<td>.08</td>
<td>.05</td>
<td>.06</td>
<td></td>
</tr>
</tbody>
</table>

Assume that the flight contains 290 passenger seats. The volume and weight cargo capacities given this particular number of seats occupied are 75 cubic meters and 10400 kilograms. The fewer seats occupied at the departure time, the more cargo capacities on the plane. The additional capacities are chosen such that one passenger seat is converted approximately into 90 kilograms and 0.1 cubic meter. The expected contributions in the base case is

\[230 \text{ per-flight expected contribution around } 22000\]

### Table 2 Volume and weight requirements for each type

<table>
<thead>
<tr>
<th>Class</th>
<th>Volume (cubic meter)</th>
<th>Weight (hundred kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
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<tr>
<td>4</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

### Table 3 Capacities for different numbers of seats occupied

<table>
<thead>
<tr>
<th>Seats</th>
<th>290</th>
<th>270</th>
<th>250</th>
<th>230</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume</td>
<td>75</td>
<td>77</td>
<td>80</td>
<td>82</td>
</tr>
<tr>
<td>Weight</td>
<td>104</td>
<td>122</td>
<td>140</td>
<td>150</td>
</tr>
</tbody>
</table>

### Table 4 Conditional probabilities \( P(S = s | R = r) \)

<table>
<thead>
<tr>
<th>rs</th>
<th>290</th>
<th>270</th>
<th>250</th>
<th>230</th>
</tr>
</thead>
<tbody>
<tr>
<td>290</td>
<td>0.9</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>270</td>
<td>0.7</td>
<td>0.4</td>
<td>0.3</td>
<td>0.0</td>
</tr>
<tr>
<td>250</td>
<td>0.2</td>
<td>0.6</td>
<td>0.2</td>
<td>0.0</td>
</tr>
<tr>
<td>230</td>
<td>0.0</td>
<td>0.5</td>
<td>0.4</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The expected contribution under perfect information is \(\xi^{\Omega_0} = 620000\), that in the base case \(\xi^{\Omega_2} = 520000\) (equivalently 83.1% of 626000), and that under imperfect information \(\xi^{\Omega_0} = 542000\) (equivalently 86.6% of 626000). The EVPI is 100000, which represents the maximum amount that the airline would be willing to pay for perfect information. The EVPII is 22000: the carrier could improve the per-flight expected contribution around 22000.
(equivalently 3.5% of 626000) by taking into account the number of seats sold at the beginning of the cargo booking horizon. We have demonstrated how our mathematical models can be used to quantify the monetary value of information sharing.

IV. Concluding Remarks

We present an MDP model for the single-leg air-cargo booking control problem under stochastic capacity. The cargo capacity is random but dependent on the number of passengers carried, which also depends on the number of seats sold at the beginning of the cargo booking horizon. The carrier can infer the distribution of the number of animals from the data on how many people are seated. We propose a mathematical model for evaluating the expected monetary gain when passenger RM shares information on the number of passengers with cargo RM.

Our study can be extended in several directions. From theoretical viewpoint, one could identify conditions on the model parameters (e.g., the initial distribution, the transition matrix, and the request probabilities) for which value of information is large. In the current study, we assume that information sharing is costless, and that the update is done only once at the beginning of the cargo booking horizon. From practical viewpoint, one could determine the optimal number of updates and the associated updating time periods in order to balance the operational cost and the incremental benefit.

References


Background of Authors

Kannapha received a bachelor degree in mathematics from Princeton University, had a master degree in operations research and industry engineering from University of California-Berkeley, and earned a Ph.D. in industrial engineering from University of Minnesota-Twin Cities. She is currently a lecturer in school of applied statistics at National Institute of Development Administration (NIDA) Thailand. Her research interests include stochastic models in supply chain and revenue management.