EPQ FOR THE PRODUCT OF LIMITED PRODUCTION

Ping-Hui Hsu, De Lin Institute of Technology, Taiwan, tenghuim@ms11.hinet.net

ABSTRACT

Consumers' perceived value and purchase decisions are usually influenced by the scarcity. Limited production quantity is an important strategy for the marketing manner, which many industries often use in recent years. Exclusive outlet distribution and higher pricing are common practices for promoting business and increasing profit. In this study, we consider a newsvendor problem model with limited production quantity: both the unit selling price and customers' demand are influenced by the limited production quantity. An algorithm is developed to derive a production policy such that the expected profit is maximized. Numerical examples and sensitivity analysis are presented to illustrate the model.

Keywords: Limited production quantity, Newsvendor, Scarcity.

INTRODUCTION

Scarcity is a pervasive aspect of human life and is a fundamental precondition of economic behavior [1]. Consumers often consider scarce products as possessing higher values, which triggers them to desire these products even more. Accordingly, the manufacturers design their marketing strategies and shape the products to seem scarce, which would attract consumers and increase their interest in purchasing. This type of marketing strategy is very common among current industries. For example, department stores announce limited products on their promotional flyers during their anniversary sales, which require customers to take numbers and to wait in line on certain days.

Commodity theory [2] deals with the psychological effects of scarcity. The theory's principle claim is that "any commodity will be valued to the extent that it is unavailable". According to the theory, scarcity enhances the value (or desirability) of anything that can be possessed, is useful to its possessor, and is transferable from one person to another [1]. The need for uniqueness may vary across differing situations and persons; as such, a high need for uniqueness may be related to (a) forces in a given situation that promote an extreme sense of high similarity, and (b) dispositional factors that influence the high need for uniqueness across a variety of situations [3]. Sirgy [4] addressed the importance of scarcity in marketing strategy. Salespersons should apply such strategy while merchandising products or services, which may increase the motivation of the targeted customers to approach the promotional information. There are two strategies for price raise through scarcity: (1) direct result from the quality and symbolic interest, and (2) indirect result on quality and symbolic interest through the price. As a result, raising the prices of scarce products can make a positive impact, but also may backfire as customers perceive that they are paying more money [5]. Therefore, if we combine the commodity theory and the need for uniqueness theory, we can demonstrate that customers prefer possessing scarce product to show their uniqueness, compared to possessing common and easily-available products. We can also clearly demonstrate the reason why limited products can always trigger customers' desire to purchase.

Limited production quantity is an important strategy for the effect of scarcity which is usually single production. Recently many industries use this strategy frequently. This problem is well known as the "newsboy problem" or the "newsvendor problem". The newsboy problem, also known as the single period stochastic inventory model [6,7], is found to be a suitable tool for decision-making regarding stocking issues in today's supply chains [8]. Weng [9] analyzed the coordinated quantity decisions between the manufacturer and the buyer in a newsvendor model. Dominey and Hill [10] explored the effectiveness of a number of approaches for approximating a compound Poisson distribution in a newsboy model. Wang and Webster [11] used loss aversion to model manager's decision-making behavior in the single-period

newsvendor problem. Shi et al. [12] presented an extension to the multi-product newsvendor problem by incorporating the retailer's pricing decision, as well as considering supplier quantity discount. However, to the best of our knowledge, little researches had been found on the newsvendor problem including limited production quantity issues. For such reason, this study analyzes the newsvendor problem with limited production quantity.

In this study, the supplier manufactures the products and then sells to the retailer, or directly sells to the customers. The supplier has to consider the uncertainty in customers' need. Marketing the products with limited-edition and manufacturing an optimal quantity before the selling period of the product are vital to the supplier. An algorithm is presented to derive an economic production quantity and the unit selling price such that the expected profit is maximized.

NOTATIONS

The following notations are used in our analysis:

$E\pi$	the expected profit for the supplier
Q	the production quantity for the supplier; decision variable
Q^*	the economic production quantity (EPQ) for the supplier considering limited production quantity
Q_w^*	the economic production quantity for the supplier without considering limited production quantity
p_1	the selling price per unit without considering limited production quantity ; constant
p_2	the upper bound of selling price per unit when the production quantity is limited; constant
$P(Q)$	the selling price per unit with considering limited production quantity ; which is a function of production quantity
c_p	the production price per unit; $c_p < p(Q)$
s	the salvage value per unit $s < c_p$
r	the shortage cost per unit; represents costs of lost goodwill
x	the random demand with the PDF (Probability Density Function), $f(x)$, and CDF (Cumulative Distribution Function), $F(x)$; in this study, x is uniformly distributed

MODELING AND ASSUMPTIONS

In this section, we formulate the expected profit model for the supplier. Throughout this study, single production of this product-edition is assumed. The supplier manufactures a batch of the products, Q, and sells to the retailer or sells directly to customers. The unit production price of the product is c_p. The unit selling price is $P(Q)$. When the sale quantity is less than the batch Q, the leftover is sold with the unit salvage value s. When the demand is more than the batch, Q, the shortage occurs. In here, shortage backordered is not allowed and the shortage unit cost is r. If the selling price $P(Q)= p_1$ (that is, without considering variable selling price), and with the uncertainty in the customers' demand to be x, the supplier will manufacture an optimal batch of the products, Q_w^*, according to its optimal expected profit. This is the traditional newsboy problem. The suppliers' expected profit function $E\pi$ is:

$$E\pi(Q_w) = \int_0^{Q_w} \left\{ \left[p_1 - c_p \right] x - (c_p - s)(Q_w - x) \right\} f(x)dx + \int_{Q_w}^{\infty} \left\{ \left[p_1 - c_p \right] Q_w - (x - Q_w) r \right\} f(x)dx. \quad (1)$$

The suppliers' optimal production batch follows (Hadley and Whitin, 1963):

$$F(Q_w^*) = (p_1 - c_p + r)/(p_1 - s + r), \quad (2)$$

where $F(x)$ is the CDF of x. If the supplier manages the production batch of the products (that is, limited production quantity) for marketing and business purposes, then the consumers' perceived value and purchase decisions are usually influenced by the effect of scarcity. Furthermore, the unit selling price, $P(Q)$, of the limited quantity products can be increased, where $P(Q)$ is a decreasing function of Q. However, the customer demand will decrease due to the higher selling price simultaneously. Thus, in this study the

random demand of Uniformly distributed over the range 0 and $B(Q)$ is assumed, where $B(Q)$ is an increasing function of Q (It is because the higher production batch will decrease the selling price, while the lower selling price will increase the demand). That means the PDF, $f(x)$, of the random demand, x, depends on Q. The suppliers' expected profit function $E\pi$ is given as follows:

$$E\pi(Q)=\int_0^Q\left\{\left[p(Q)-c_p\right]x-(c_p-s)(Q-x)\right\}f(x)dx + \int_Q^{B(Q)}\left\{\left[p(Q)-c_p\right]Q-(x-Q)r\right\}f(x)dx. \quad (3)$$

Our problem can be formulated as:

$$Max: E\pi(Q). \quad (4)$$

For the maximum of the function, to prove its concavity is needed. Due to the complexity of $E\pi(Q)$, it is implausible to prove. We then investigate the model by an illustrative case study.

AN ILLUSTRATIVE CASE STUDY

In this section, the practical selling price and probability distribution are used to explain the results of the previous section. Since the selling price is always influenced by the limited production quantity (Wu & Hsing, 2006), therefore, the selling price per unit $P(Q)$ is assumed as

$$P(Q)=\frac{p_2-p_1}{\sqrt{Q}}+p_1,\ p_2>p_1>0,\ Q\geq 1. \quad (5)$$

Which means both $p_1<P(Q)<p_2$ and a decreasing function of Q. The random demand for the supplier is uniformly distributed over the range 0 and $B(Q)$, where

$$B(Q)=\frac{bp_1}{P(Q)}, \quad (6)$$

is a function of Q with positive constant b. It means that the higher selling price would decrease the demand. Thus, the PDF of the supplier's demand is

$$f(x)=\frac{1}{B(Q)}. \quad (7)$$

Two cases are discussed as follows.

(i). General case

In this case, $B(Q)=\frac{bp_1}{P(Q)}$, one has

$$E\pi'(Q)=\int_0^Q[P'(Q)x-c_p+s]f(x)dx + \int_Q^{B(Q)}[P'(Q)Q+P(Q)-c_p+r]f(x)dx + B'(Q)f(B(Q))\left\{\left[P(Q)-c_p\right]Q-\left[B(Q)-Q\right]r\right\}. \quad (8)$$

$$\begin{aligned}E\pi''(Q)=&\,p''(Q)\int_0^Q xf(x)dx \\ &+\left[p''(Q)Q+2p'(Q)\right]\int_Q^{B(Q)}f(x)dx-\left[p(Q)-s+r\right]f(Q). \\ &+\left\{\left[p(Q)-c_p\right]Q-\left[B(Q)-Q\right]r\right\}\left[B'(Q)\right]^2f'(B(Q)) \\ &+\{2B'(Q)\left[p'(Q)Q+p(Q)-c_p+r\right] \\ &+B''(Q)\left[p(Q)Q-c_pQ-rB(Q)+rQ\right]-r\left[B'(Q)\right]^2\}f(B(Q)).\end{aligned} \quad (9)$$

From (10), it is implausible to prove the concavity of $E\pi(Q)$. The numerical examples are provided to illustrate the model.

Example 1. Given p_2=250, p_1=130, c_p=100, b=1500, s=50, and r=5.

The concavity of $E\pi(Q)$ is illustrated in Figures 1, 2, and 3. Figure 1 presents the shape of $E\pi(Q)$, which reaches the maximum in the interval [500, 1000]. Figure 3 presents the shape of $E\pi''(Q)$, which obtains its negative value. That means the concavity of $E\pi(Q)$. Figure 2 presents the shape of $E\pi'(Q)$, which means the root of $E\pi'(Q)$=0 is located in the interval [500, 1000]. Set $E\pi'(Q)$ equals to zero, using Software Maple 8, Q^*=614 is derived, the selling price per unit is $P(Q^*)$=\$134.8, and the optimal expected profit for the supplier is $E\pi(Q^*)$ =\$9137. When limited production quantity is not considered, Q^*=618 (using Eq.(2)), $E\pi(Q_w^*)$ =\$7059 (using Eq.(1)), and the percentage profit increase is $\frac{E\pi(Q^*)}{E\pi(Q_w^*)}-1$ =29.4%.

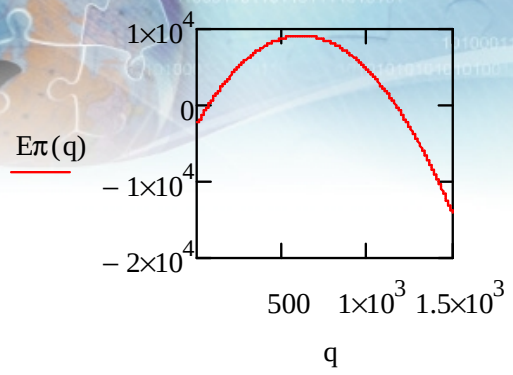

FIGURE 1. The shape of $E\pi(Q)$ in example 1.

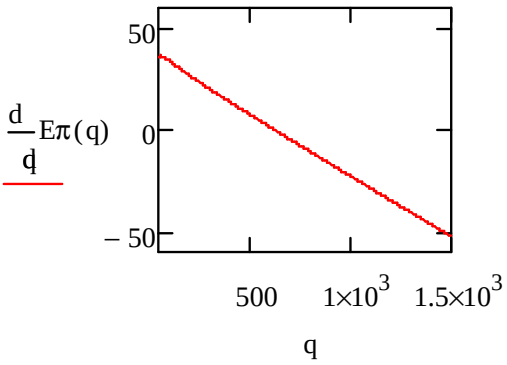

FIGURE 2. The shape of $E\pi'(Q)$ in example 1.

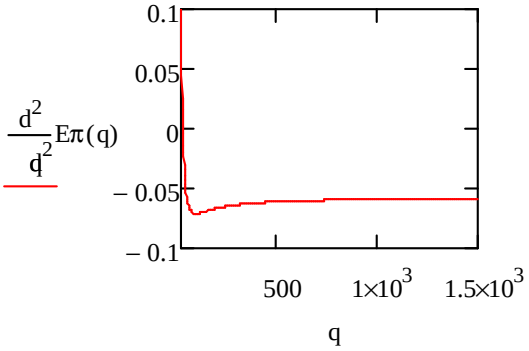

FIGURE 3. The shape of $E\pi''(Q)$ in example

(ii). Special case

In this case, without considering variable selling price, that is, $P(Q)=p_1$ and $B(Q)=b$. That is, $f(x)=\frac{1}{b}$. One has

$$E\pi'(Q)=\int_0^Q [P'(Q)x-c_p+s]f(x)dx$$

$$+\int_Q^b [P'(Q)Q+P(Q)-c_p+r]f(x)dx$$

$$=P'(Q)\int_0^Q F(x)dx+[P(Q)-s+r]F(Q)$$

$$-[P(Q)-c_p+r]-QP'(Q). \tag{10}$$

Where $F(x)$ is the CDF of r.v.x.

$$E\pi''(Q)=p''(Q)\int_0^Q xf(x)dx$$

$$+\left[p''(Q)+2p'(Q)\right]\int_Q^b f(x)dx-\left[p(Q)-s+r\right]f(Q). \tag{11}$$

The concavity and optimal solution of $E\pi(Q)$ are derived in Theorem 1.

Theorem 1. For the random demand $X\sim U(0,b)$ with the selling price function $P(Q)=\frac{p_2-p_1}{\sqrt{Q}}+p_1$, $Q\geq 1$, the expected profit function $E\pi(Q)$ is concave.

Proof: (Please see Appendix A)

From Theorem 1, the economic production quantity Q^* can be derived by setting

$$E\pi'(Q)=0. \tag{12}$$

Example 2. Given p_2=250, p_1=130, c_p=100, b=1500, s=50, and r=5.

Set $E\pi'(Q)$ equals to zero, using Software Maple 8, Q^*=633 is derived, the selling price per unit is $P(Q^*)$=\$134.8. Using Eq.(3), the optimal expected profit for the supplier is $E\pi(Q^*)$ =\$9434. When limited production quantity is not considered, using Eq.(2),

$F(Q^*)=\frac{Q^*}{b}=\frac{Q^*}{1500}=\frac{p_1-c_p+r}{p_1-s+r}$ =0.412, Q^*=618 is derived.

Using Eq.(1), $E\pi(Q_w^*)$ =$7059, and the percentage profit increase is ($\frac{E\pi(Q^*)}{E\pi(Q_w^*)}-1$)*100%=33.7%.

CONCLUSION

This study derives a newsvendor problem model with limited production quantity. Analyzing the system provides managerial insights on how to develop strategies for making the greatest profit. An illustrative case study, numerical examples, and sensitivity analysis are presented to demonstrate our argument. Numerical examples show that the percentage profit increase is fairly significant. Further researches are suggested to consider different selling period and the distribution of customers' demand.

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Appendix A

Proof of Theorem 1

$$E\pi''(Q) = p''(Q)\int_0^Q xf(x)dx + \left[p''(Q) + 2p'(Q)\right]$$

$$\times\int_Q^\infty f(x)dx - \left[p(Q) - s + r\right]f(Q)$$

$$= \frac{-1}{8bQ^{(2/3)}}[8Q^{(2/3)}(p_1 - s + r)$$

$$+3Q(p_2 - p_1) + 2b(p_2 - p_1)].$$

Since $p_1 - s$ >0, and, $p_2 - p_1$ >0, therefore, $E\pi''(Q)$ <0, the proof is completed.