INTEGRATING THE MECHANISM OF THREE-PART TARIFFS PRICING TO THE PROVISION OF INTRA-SITE SEARCH ENGINE ADVERTISING SERVICES

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ABSTRACT

Intra-site search engines (ISEs) dedicated in private electronic markets become popular with the fast-growing electronic markets. Among several research facets regarding the advertising services of ISEs, we focus on how to optimize the ISE-based advertising market mechanism by improving the pricing model. Current pricing model, Flat Fee (FF), fails to differentiate ISE advertising services among subscribers. Such inefficiency results in the loss of the subscribers, and the decline of the provider’s revenue. We design a new pricing scheme to be implemented by a two-stage contract. The proposed pricing model integrates the mechanism of three-part tariffs (3PT) to the provision of ISE advertising services. The conceived pricing model extracts more consume surplus through differentiating services among subscribers. It also develops an incentive for suppliers, and increases the stickiness of subscribers.

Keywords: search engine marketing; proprietary search engines; Three-part Tariff; Flat fee; pricing model

INTRODUCTION

In the era of information explosion, search engines play an important role in electronic commerce. For more than one decade, search engine marketing has been providing enormous business opportunities in electronic markets, and creating many successful stories, such as Google (AdWords), Yahoo! (Search Marketing), and Microsoft (adCenter). For instance, Google’s total advertising revenues were about $21 million in 2008, and grew at 8% in 2009 to hit about $23 million [1]. While public search engines (PSEs), the publicly accessible Internet information search services for users without any membership, such as Google, and Bing, have been well studied, we notice that another type of search engines is getting increasing popular but less researched. They are dedicated in proprietary electronic markets and normally require membership. The cases can be found in various forms of electronic markets, such as, eBay (Consumer-to-Consumer), Amazon (Business-to-Consumer), and Alibaba (Business-to-Business). We define this kind of search engines as intra-site search engines (ISEs).

Presently the advertising services in ISEs form a particular advertising market, denoted as Intra-site search engine advertising market (ISEAM). Users of ISE advertising need subscribe to the ISEAM services and pay the service fee. Such suppliers, called subscribers, have the privilege to use the advertising resources, mainly the clicks of clients. Commonly, ISE advertising services have been contained into a customizable premium package for suppliers.

As ISEAM is growing quickly in B2B or B2C market, relevant issues emerged in ISEAM need the effort of researchers, since many well accepted ideas in PSEs may not fit ISEs because of the differentiated features. There is a broad range of research topics in ISEAM, including search engine indexing, search result ranking from technical aspect, and keywords auction, search engine ecosystem from economic aspect. Some researchers have noticed these relevant issues. Yu et al utilized search logs of an ISE to reveal user search patterns on ISEs, which provides a validated basis for academic to conduct further research in relevant area [2]. Li at el proposed a new pricing model for ISEs advertising services [3].

Our work focuses on the pricing mechanism of advertising services on ISEAM.

Normally, the providers of electronic markets adopt FF pricing model to finance the customizable premium package, which contains ISE advertising services. FF pricing model is a fixed price for a given period of time as paid inclusion in ISEAM. FF scheme can reduce risk and administrative costs, and provide predictable revenue for providers. FF scheme, however, fails to differentiate the quality of ISEs.
advertising services among subscribers. Products of high-profile (strong competent) subscribers usually rank higher than those of low-profile subscribers, and hence receive more clicks from clients. As a result, those low-profile subscribers tend to quit the services package in the next period. This implies the decline of revenue of the providers. Alibaba B2B operator has met a similar situation in its member services. Alibaba B2B is the largest B2B electronic market in China. Therefore, improving the pricing mechanism is critical. The challenge for this issue is how to improve the current strategy in order to 1) extend the satisfaction of subscribers 2) increase the stickiness of subscribers 3) attract more suppliers into ISEAM to extend the revenue for providers.

This paper is intended to design a new pricing scheme to deal with the challenge. The pricing scheme is implemented by a two-stage contract through integrating the mechanism of 3PT. 3PT is defined by an access price, a usage allowance, and a surcharge for any usage in excess of the usage allowance. We introduce a twist into the 3PT to implement the two-stage contract. According to the new pricing model, a supplier paying an access price is entitled to demand to x clicks. If consuming more than x clicks, he needs to pay per additional click. If consuming less than x clicks, he can gain compensation per less click. This fee is realized by adding to or deducting from his charge for the access price in the next period. The new model develops an incentive to suppliers, and increases the stickiness of subscribers in ISEAM.

LITERATURE REVIEW

There are proliferated online advertising researches outcomes. For example, Hoffman and Novak introduced a CD now case to present the trend that per-click pricing and pay for performance displace the traditional impression model in Internet advertising marketing [5]. Moon and Kwon proposed a hybrid pricing policy which integrates two most popular pricing model, cost-per-impression and cost-per-click [6]. Regarding sponsored search, Weber & Zheng exploited the way to maximize the revenue of PSE providers through designing rankings strategy of sponsored slots [8]. For another, Yang and Ghose analyzed the relationship between different sponsored search factors in 2009, and then in 2010 they extend to discuss the relationship between organic and sponsored search results [9] [10]. Chatterjee and Hoffman modeled the commercial “clickstream” at an advertiser supported Web site to predict consumers interacting with advertising stimuli [11]. There are proliferated studies tackling the pricing model involving search engine advertising. For example, Sen et al compared three pricing strategies, Pay-Per-Click (PPC), Pay-Per-Purchase (PPP), and FF for paid placement on search engine services [12]. Li et al proposed a new pricing model for ISEs advertising services [3].

Presently, ISEs advertising services are integrated into a customizable premium package for suppliers. The providers commonly use FF pricing to finance the package. FF scheme is widely utilized in the previous development stage of services, such as the ticket, the telecommunication and phone. However, researchers noticed that the inefficiency of FF in financing services [13]. For example, McKnight and Boroumand discussed the inefficiency of FF pricing for internet services, and proposed new service pricing models [14]. Therefore, researchers put forward usage-based pricing models, which are able to avoid the inefficiency of FF pricing and to extend the revenue of service providers. For instance, Sundararaja suggested that firms should transform from low fixed-fee penetration pricing in nascent information market to an optimal pricing mix including usage-based pricing options as these markets mature [15].

As noted earlier, we adopt 3PT to the new pricing scheme. 3PT have been widely used in information industries, including cellular phone plans, Internet access service, data center hosting. 3PTs are often analogized to a two-part tariff (2PT). The two-part tariff is composed of two parts, a lump-sum fee and a per-unit charge. Oi first mentioned 3PT in his classical article “Disneyland Dilemma”, and interpreted it as the price policy adopted by IBM [18]. Bagh and Bhargava revealed that 3PT structure produced higher profit, and lowered manageral and decision complexity compared with 2PT [20].

In a proprietary electronic market, the websites, search engine, and so on are so-called “public goods”. Once they are supplied, every supplier, even those who did not pay to their consumption, will benefit from them. Most literature for the public goods problem is to design mechanisms to incent agents to reveal their true preference. Groves and Ledyard extended the general equilibrium model with the competitive behavioral assumption for consumers. In equilibrium, it is each individual reveals his true demand of the public goods [21]. Aspremont and G’erard-Varet introduced two approaches to incomplete information. One of them is to utilize Bayesian-Nash approach to construct an efficient, budget-balancing mechanism that the beliefs of all the agents are common knowledge [22]. Crémer and Riordan extended the results of the Bayesian-Nash approach. They introduced “Stackelberg” mechanisms with truth-telling a dominant strategy for agents except for the first. According to the
"Stackelberg", the first agent plays before the others maximizing his expected utility on the assumption that others will reveal their true preferences [23].

**MODEL SETTING**

We focus on the scenario of two parties involved in the ISE advertising pricing scheme: ad suppliers and a service provider. There are two types of suppliers. One is service subscribers, those paying to the information services, mainly search engine advertising services, in ISEAM. The subscribers contract to a customizable premium package offered by the service provider, which consists of a series of services marketing products. Non-subscribers also are allowed to use lower prioritized search engine advertising services and other publicly available information services in the e-commerce platform operated by the service provider. Specifically, the benefit in ISE advertising services is the clicks of suppliers’ ads. By providing prioritized services to the subscribers, the service provider can resolve the free-rider problem in search engine advertising and make profit.

However, the FF pricing scheme incurs a new type of public good problem since the subscription fee is not justified by the services the subscribers receive. This can be treated as a new type of free riders problem, i.e. the fixed cost free-rider problem, in the e-commerce platform, since some better performed subscribers may overly exploit too much search engine advertising resources, leaving others much less satisfied since the advertising resource is limited.

To solve this problem, we conceive a new pricing scheme to be implemented by a two-stage contract.

Stage 1 (before service period 1):
A supplier pays a premium fee \( r \) to consume \( x \) amount of clicks, which changes his type to a subscriber.

Stage 2 (the end of the last service period and before the next service period):
1. If a subscriber consumed more than \( x \) clicks in the previous period, a price \( k \) per additional click will be added to his charge for the services in the next period. These subscribers are denoted as high-profile suppliers.
2. If a subscriber consumed less than \( x \) clicks in the previous period, a compensation \( w \) per less click will be deducted from his charge for the service in the next period. These subscribers are denoted as low-profile suppliers.

Let \( p_i \) as the additional fee for subscriber \( i \) due to the additional consumption. Let \( c_i \) as the compensation for subscriber \( i \).

**Profit of Suppliers**

We consider ISEAM with a finite set of suppliers, \( N=\{1,2,3…n\} \), and one provider. Denote the competent level of supplier \( i \) as \( a_i \). The competent level generally stays constant in given periods. Denote \( s_{ij} \) as the bundle of services that supplier \( i \) consumes in period \( j \) \((j>0)\). Let \( y_i \) as the cost that the supplier \( i \) pays in period \( j \) due to consuming services. Denote the revenue gained from the services package in period \( j \) as \( V_{ij} \). The profit \( U_{ij} \) that he gains in period \( j \) from the services package is defined as:

\[
U_{ij}(s_{ij}, a_i)=V_{ij}(s_{ij}, a_i) - y_i(s_{ij}, a_i)
\]

The profit of subscriber \( i \) is relevant to his competent level and the services he gains.

The competent level of subscribers has three main aspects. One is the ability of subscribers to exploit search engine advertising resources. Then, it is the ability to transfer a casual click into an actual sale. Finally, it is the profitability, which depends on the performance and the quantity of products traded in one transaction. For example, there are two subscribers, C and D. For one transaction, C sells 1000 LV bags, and each package is worth $1000. D just sells one bag, and the bag is worth $100. Thus, C can gain higher profit from a deal than D. We say C has higher profitability than D. Generally, the service provider does not know the exact competent level of each subscriber.

The services subscribers consume contain search engine advertising services and other services marketing products in the customizable premium package. The search engine advertising resources subscribers exploit is influenced by their competent levels. The \( n_{ij} \) is the number of clicks that subscriber \( i \) gains, where \( n_i \) as the number of impressions for all subscriber \( i \)’s ads in period \( j \). It determined by his competent level \( a_i \), i.e. \( n_{ij} = n_i(a_i) \). And \( t_i \) is his ads’ click-through rate (CTR) in period \( j \). CTR is defined as the number of clicks divided by the number of impressions. The CTRs of slots for search results is descend. The number of clicks is the common knowledge between the provider and subscriber \( i \).

Now, we will formulize the profit of subscribers under such mechanism.

Period 1, where \( j=1 \).

\[
U_{ij}(s_{ij}, a_i)=V_{ij}(s_{ij}, a_i)-y_{ij}
\]

\[
y_{ij}=r
\]

Where \( U_{ij}(s_{ij}, a_i) \) is the profit and \( V_{ij}(s_{ij}, a_i) \) is the
revenue if supplier $i$ subscribes to the services package in Period 1. And $y_{ij}$ is the cost in Period 1.

Period $j$, where $j \geq 2$.

The costs between high-profile and low-profile subscribers are different after Period 1. Thus, we formulize their profit functions separately.

For high-profile subscribers, i.e. $n_{ji}, t_{ji} > x$

The cost in Period $j$ is:

$$y_{ij}(n_{ji}, t_{ji}) = r + P_{ij} = k(n_{ji}, t_{ji} - x)$$

(3)

The profit of subscriber $i$ in Period $j$ is:

$$U_{ij}(s_{ij}, a_{ij}, n_{ji}, t_{ji}, j) = V_{ij}(s_{ij}, a_{ij}) - r - k(n_{ji}, t_{ji} - x)$$

(4)

Where $U_{ij}(s_{ij}, a_{ij}, n_{ji}, t_{ji}, j)$ is the profit if subscriber $i$ subscribes to the services package in Period $j$. $y_{ij}(n_{ji}, t_{ji})$ is the cost in this Period, and $p_{ij}$ is the additional fee for subscriber $i$ due to the additional consumption in Period $j$.

For low-profile subscribers, i.e. $n_{ji}, t_{ji} < x$

The cost is:

$$y_{ij}(n_{ji}, t_{ji}) = r + c_{ij}, c_{ij} = w(x - n_{ji}, t_{ji})$$

(5)

The profit of subscriber $i$ is:

$$U_{ij}(s_{ij}', a_{ij}', n_{ji}, t_{ji}, j) = V_{ij}'(s_{ij}', a_{ij}') - r - k(n_{ji}, t_{ji} - x)$$

(6)

Where $c_{ij}$ is the compensation for subscriber $i$ due to less consumption in Period $j$-1.

**Definition 1** In the new mechanism, commonly the subscriber $i$ would subscribe in Period $j$ if:

$$U_{ij}'(s_{ij}', a_{ij}', n_{ji}, t_{ji}, j) > 0$$

(7)

To simplify the model, we assume the profit of non-subscriber is zero.

For high-profile subscribers:

$$U_{ij}'(s_{ij}', a_{ij}', n_{ji}, t_{ji}, j) = V_{ij}'(s_{ij}', a_{ij}') - r - k(n_{ji}, t_{ji} - x)$$

(8)

For low-profile subscribers:

$$U_{ij}'(s_{ij}', a_{ij}', n_{ji}, t_{ji}, j) = V_{ij}'(s_{ij}', a_{ij}') - r + w(x - n_{ji}, t_{ji})$$

(9)

Where $U_{ij}'(s_{ij}', a_{ij}', n_{ji}, t_{ji}, j)$ is the expected profit and $V_{ij}'(s_{ij}', a_{ij}')$ is the expected revenue if subscriber $i$ subscribe to the service package in the next period. And $s_{ij}'$ is the expected bundle of services.

The two-part contract can increase the stickiness of subscribers, especially low-profile subscribers. The expected profit in the next period can incent them to subscribe to the service package in the next period.

**Profit of the Provider**

Now, let us model the profit of the provider in the new pricing scheme. The profit structure of the provider contains three additional portions: the revenue from charging mechanism incomes, $P_j$, the costs for compensation for the subscribers with low clicks, $C_j$, and subscribers’ fixed fee from subscribers, $R_j$. Let $D_{ij} = I, ..., N$, be the strategy space of supplier $i$ in period $j$. And $D_{ij} = 1$ mean supplier $i$ subscribes to the services package in period $j$; otherwise $D_{ij} = 0$. We can formulize the profit in period $j$ as follows:

$$Q_j = R_j + P_j - C_j$$

(10)

$$R_j = \sum_{i=1}^{N} r \cdot D_{ij}$$

(11)

$$P_j = \sum_{i=1}^{N} p_{ij}$$

(12)

$$C_j = \sum_{i=1}^{N} c_{ij}$$

(13)

The additional charge directly increases the profit of the provider through extracting the consumer surplus from the high-profile subscribers. The compensation increases the number of subscribers, and thus extends the fixed fee for the service provider. Properly choosing $x$, $w$ and $k$, the provider will have an increasing positive $Q_j$.

**SURCHARGE RATE VS. USAGE ALLOWANCE**

**Surcharge Rate**

In 3PT, a rational operator generally sets the surcharge rate close to the margin cost of his productions. In information services, however, the margin cost is almost zero. But it is not reasonable to set the surcharge rate as zero in our mechanism.

Another characteristic is the demand of products. Lambrecht et al held that in 3PT the surcharge rate negatively impacts on the consumer demand [22]. In ISE advertising services, however, the demand for clicks does not change with the surcharge rate. This is because the click is random, and the number of clicks is determined by the clients. Therefore, we need not to consider the impact the surcharge rate exerts on the consumer demand. If the value of the surcharge rate is too high, subscribers would get negative revenue from the additional clicks. Therefore, it is reasonable that the surcharge rate should meet theorem 1.

**Theorem 1** The surcharge rate does not drive the subscribers to quit when it meets the condition as follows:

$$k < E(n_{ji}, t_{ji} - x)/ (n_{ji}, t_{ji} - x)$$

(14)

Denote the revenue from additional clicks is $E(n_{ji}, t_{ji} - x)$.

**Proof.** The profit of high-profile subscriber $i$ is:

$$U_{ij}(s_{ij}, a_{ij}, n_{ji}, t_{ji}) = V_{ij}(s_{ij}, a_{ij}) - r - k(n_{ji}, t_{ji} - x)$$

(15)
Separate revenue gained through the service package into two parts. One is the revenue from the additional clicks. Denote this revenue as $E(n_{ij},t_{ij},r-x)$. Another is the revenue from services excluding the additional clicks. Denote this revenue as $V_{ij}(s_{ij},a_i)$. The revenue can be formulated as below:

$$V_{ij}(s_{ij},a_i)=V_{ij}^*(s_{ij},a_i)+E(n_{ij},t_{ij},r-x) \quad (16)$$

Then,

$$U_{ij}(s_{ij},a_i)=V_{ij}^*(s_{ij},a_i)-r+E(n_{ij},t_{ij},r-x)-k(n_{ij},t_{ij},r-x) \quad (17)$$

If $E(n_{ij},t_{ij},r-x)-k(n_{ij},t_{ij},r-x) > 0$, the subscriber $i$ can gain extra profit through additional clicks. Otherwise, high-profile subscribers would resist the new mechanism. Therefore, the surcharge rate should meet:

$$k < E(n_{ij},t_{ij},r-x)/ (n_{ij},t_{ij},r-x) \quad (18)$$

Hence, the proof is complete.

### Usage Allowance

We utilize simulation to analyze the optimal usage allowance when CTR in ISEAM meets normal distribution. And we set surcharge rate as below:

$$k= 0.5 \cdot E(n_{ij},t_{ij},r-x)/ (n_{ij},t_{ij},r-x) \quad (19)$$

Here, we just consider the situation of high-profile suppliers. Therefore, we do not consider the effect of the compensation on the profit of the provider in the new pricing scheme. We developed two environments, Environment1 bases on FF scheme, and Environment2 relies on the new pricing scheme. The optimal fixed fee gained in the FF pricing model is introduced into the new pricing scheme as the access price. We control some parameters are same in two environments (Table 1). We introduce the optimal flat fee rate, $r=350$, found in the FF model into the new pricing scheme as the access price. We set usage allowance increase from 0 to 800 stepped by 20 to find the optimal value. Figure 2 presents the scatter plot for relationship between the profit of the provider and the usage allowance. The provider’ profit increases with the usage allowance, and reaches the maximum when the usage allowance is about 240. The optimal profit for the provider is 341,996. The green line is the optimal profit for the provider in FF pricing model. Figure 3 presents the relationship between the amount of the subscribers and the usage allowance. The amount of the subscribers increases, and then remains stable with the increase in usage allowance. That is to say, when the usage allowance is lower than a certain threshold, it has negative impact on the decision whether the suppliers subscribe to the service package. The negative impact disappears when the usage allowance is larger than the certain threshold.

Therefore, properly choosing usage allowance $x$ when CTR is normal distribution, the provider can gain more profit and have more subscribers.

<table>
<thead>
<tr>
<th>Table 1 Parameter set</th>
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<tbody>
<tr>
<td><strong>Items</strong></td>
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<tr>
<td>Impression(n)</td>
</tr>
<tr>
<td>CTR</td>
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<tr>
<td></td>
</tr>
<tr>
<td>Click</td>
</tr>
<tr>
<td>Conversion rate</td>
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<tr>
<td>Value per transaction</td>
</tr>
<tr>
<td>Number of suppliers</td>
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<td>The utility as a free rider</td>
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Conversion rate is the ratio of visitors who convert casual clicks into desired transactions. The amount of impressions for all subscribers is equal to $n$.

First we compute the optimal revenue of the provider in FF pricing model. The flat fee rate increase from 0 to 1000 stepped by 50. The number of subscribers declines with the increase in the flat fee rate. The profit of the provider increases with the increase in the flat fee rate, and then decreases with it (Figure 1).

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THE VALUE OF THE COMPENSATION

To incent subscribers booking the services package in the next period, the provider needs to consider the value of the compensation carefully. If the compensation is too low to reach the expectation of the subscriber $i$, the compensation would fail to incent him subscribing to the services package in the next period. If the compensation is too high, the provider has to give up. These expected utilities are obtained only when both of them agree. We use the cooperative bargaining model to address the problem [24].

Let us analyze the specificity for our game situation. In a bilateral negotiation, the symmetry should be considered. Roth argued the property of symmetry requires that the solution of the bargaining game should not distinguish between the players [25]. For our game situation, it does not meet the property of symmetry. In order to deal with this issue, we utilize the Generalized Nash Bargaining (GNB). Nagarajan and Sosic extended the cooperative bargaining into GNB through considering the risk preference [26]. They connected risk preference with the negotiation power of the players. The negotiation power means the ability of one player influences his opponent action. Nagarajan and Sosic held that a player’s bargaining power increases as his opponent becomes more risk averse. The GNB from Nagarajan and Sosic (2008) is:

$$B(x_1, x_2; d_1, d_2; \alpha, \beta) = \operatorname{arg\, max} (x_1 - d_1)^\alpha (x_2 - d_2)^\beta$$  \hspace{1cm} (20)

Where the constants $\alpha$ and $\beta$ present the negotiation powers of the two players, and $\alpha + \beta = 1$. When an agreement is reached, players will have utilities of $x_1$ and $x_2$ respectively. And $d_1$, $d_2$ are the utilities when players fail to agree.

Commonly subscribers are risk aversion. The subscribers have lower negotiation power compared with the service provider. Utilizing GNB into the game between the provider and the low-profile subscriber $i$, we can determine the optimal compensation $c_{ij}$ ($i > 1$). We assume the provider and the subscriber $i$ are rational. The expected utilities when the players reach an agreement are:

For the provider,

$$x_1 = r - c_{ij}$$  \hspace{1cm} (21)

For the subscriber,

$$x_2 = U_j(s_j, a_j)$$  \hspace{1cm} (22)

$$U_j'(s_j, a_j, n_{ij}, t_{ij}) = V_j(s_j, a_j) + c_{ij} - r$$  \hspace{1cm} (23)

$$c_{ij} = w(x - n_{ij}, t_{ij})$$  \hspace{1cm} (24)

We assume the subscriber $i$ expects the revenue in the next period is same as the revenue in current period. Denote $q$ as the real interest rate. The expected revenue in the next period as follows:

$$V_j'(s_j, a_j) = V_j(s_j, a_j)/(1+q)$$  \hspace{1cm} (25)

When the agreement reaches, the utility of subscriber $i$ is:

$$x_1 = V_j(s_j, a_j)(1+q) + c_{ij} - r$$  \hspace{1cm} (26)

The disutility when the players fail to reach an agreement is zero, i.e. $d_1 = d_2 = 0$. Thus, the compensation value is derived as follows:

$$c_{ij} = \operatorname{arg\, max} \eta = \operatorname{arg\, max} (V_j(s_j, a_j)/(1+q) + c_{ij} - r)/(r - c_{ij})$$  \hspace{1cm} (27)

Where $\beta_i$ is the relative negotiation power of the subscriber $i$, and $1 - \beta_i$ is the negotiation power of the provider. We can determine the value of compensation for the subscriber $i$ in Theorem 2.

Theorem 2 Compensation value determined by GNB is:

$$c_{ij} = r(1 - \beta_i) - V_j(s_j, a_j)/(1+q)$$  \hspace{1cm} (28)

Proof. We find an optimum compensation value $c_{ij}$ regarding to the first-order condition, and then check the value through second-order condition. First, we take a logarithm for $\eta$ to solve the nonlinear form:

$$\ln \eta = \beta_i \ln(V_j(s_j, a_j)/(1+q)+c_{ij} - r)/(1-\beta_i)$$  \hspace{1cm} (29)

From the first-order condition, we have:

$$\partial \ln \eta \partial c_{ij} = \beta_i/(1+q) + c_{ij} - r + (1 - \beta_i)/(r - c_{ij})$$  \hspace{1cm} (30)

$$r - c_{ij} = \beta_i/(1+q) + c_{ij} - r$$  \hspace{1cm} (31)

Check the second-order derivative:

$$\partial^2 \ln \eta \partial^2 c = -\beta_i/(1+q) + c_{ij} - r - (1 - \beta_i)/(r - c_{ij})^2 < 0$$  \hspace{1cm} (33)

For $\beta_i \in (0, 1)$. Hence, the proof is complete.

According to Theorem 2, the expected optimal profit for the subscriber $i$ is:

$$x_1 = \beta_i V_j(s_j, a_j)/(1+q)$$  \hspace{1cm} (34)

The expected optimal profit for the provider regarding to the subscriber $i$ is:

$$x_2 = (1 - \beta_i) V_j(s_j, a_j)/(1+q)$$  \hspace{1cm} (35)

In the optimal situation, the provider bases on its relative negotiation power to extract the revenue the subscriber $i$ gains by the services package in the next period.
According to Theorem 2, the optimal compensation rate $w$ for subscriber $i$ in period $j$ meets the condition as follows:

$$c_{ij} = w(x - n_{ij})$$  \hspace{1cm} (36)

$$w = c_{ij}/x - n_{ij} = [r - (1 - \beta) V_{ij}/V_{ij}^t + \alpha]/(1 + q)/[x - n_{ij}]$$  \hspace{1cm} (37)

The provider can integrate the situation of different subscribers to determine the common compensation rate $w$.

**CONCLUSION**

In this paper, we conceive a new pricing model to be implemented by a two-stage contract for ISE advertising services. The proposed pricing model integrates 3PT to the provision of ISE advertising services. This model can differentiate ISE advertising services among subscribers, and extract more consume surplus from high-profile ones. Meanwhile, it offers an incentive for suppliers, and increases the stickiness of subscribers.

There are several interesting ways to extend our current work for the future study. For instance, the new pricing scheme charges the additional usage based on the quantity. Thus, each click is treated equally in bringing the utility to subscribers. Generally, the utility of each click is different. Another, charging the additional usage based on the number of click takes the risk of click fraud, although rising sophisticated means of detection are used. Click fraud should be the issue in the near future.

**REFERENCE**

