

**ABSTRACT**

This paper analyzes the optimal set of suppliers in the presence of supplier and/or production failure risks. We incorporate the importance of cash-flow variability in the supplier selection and production planning process. The financial loss caused by disasters, the operating cost of working with multiple suppliers and unfilled demand cost are subject to uncertainty. We formulated optimization problems with different objective functions whose solution determines an optimal set of suppliers. Our result allows management to balance the two desirable but conflicting objectives of cost minimization and service levels achieved.

**Keywords:** Supply Chain, Risk Management, Suppliers' Failure, Production Failure, Unfilled Demand, Service Level.

**1. INTRODUCTION**

Recently, the interest of supply chain risk management (SCRM) has increased in purchasing, logistics and supply chain management research (e.g. Berger et al (2004) and Ruiz-Torres and Mahmoodi (2007), Lee (2008), Meena et al (2011)). Random customers' demand and unanticipated supply disruption or supply risk is a fact in business. Some of the undesirable outcomes associated to its occurrence are firm's inability to meet customers' demand or cause disruption to the operation of the organization. Clearly, supplier selection process is a critical component of supply chain risk management given the reliance of many organizations on suppliers to manage their inventories and logistics functions. Therefore, it is important for businesses to manage their supply risk.

Model that uses mean value analysis (e.g., expected total cost) is not suitable for analyzing the effect of traditional risk management tools such as insurance on the optimal suppliers selection process. This is due to the fact that insurance premium typically include some extra loading which account for administrative expenses and profit (e.g., Harrington and Niehaus (2004)). Thus, a model only consider expected total cost would consider buying insurance as costly since the premium cost more than the expected loss. On the other hand, the mean-variance approach allows us to recognize the effect of insurance coverage reduces the variability of the cash-flow. Effectively, buying insurance increases the expected cost but it reduces the variability of the cash-flow and the risk

reduction provided by insurance may allow the firm to increase its use of debt with tax-deductible interest payments. Furthermore, reducing variability of cash-flow increases the value of the firm. This is the trade-off that our model able to capture whereas models based on expected cost would not be able to do so.

This article aims to extend current SCRM knowledge by analyzing the impact of supplier failures as well as the effect of demand uncertainty on the supplier selection process. Our focused on SCRM lead us to measure the risk and uncertainty regarding the loss and operating cost associated with a set of suppliers. In particular, we obtained the mean and variance of the cost function. This result allows us to determine a set of optimal suppliers that minimized the cost incurred. Clearly, cost is only one factor in choosing a set of suppliers. Service levels provided by the suppliers is important as well. Cost minimization and service levels are two important yet conflicting factors. The proposed methodology also aims to balance these two conflicting objective. Thus, this paper represent a step toward achieving the goal of understanding how an organization can survive and thrive under unanticipated supply disruptions taking into considerations of important factors such as cost and service levels.

**2. THE MODEL**

The supply chain includes  $m$  groups of suppliers and a production system. There are  $n_i$  suppliers in group  $i$ ;  $i = 1, 2, \dots, m$ . Each of the  $m$  group supplies a unique type of input to the production system. That is any suppliers in group  $i$  can only supply input  $i$ . The production process requires *all*  $m$  types of input in order to produce products to satisfy demand. In this paper, we are interested in analyzing the impact of supply disruption risk and the operations risk impose by uncertain production process as well as random demand for the finish goods.

To model supply disruption risk, we assume that there are

*potential super-events* that can occur and affecting all

suppliers simultaneously. That is, events such as terrorism or a widespread airline action that put all suppliers down. Let  $S$  denote the indicator random variable associated with

the super-events. We assume that  $S$  equal to 1 if a

super-event occur during the supply cycle and 0 otherwise.

The probability of one of these super-events occurring

during the supply cycle is denoted by  $P(S = 1) = p$ . We

introduce a *unique*-event scenario for each supplier; that is,

an event uniquely associated with a particular supplier that puts it down during the supply cycle. Let  $D_{ij}$  denote the indicator random variable associate with the unique event of  $i^{th}$  supplier in group  $j$ . That is  $D_{ij} = 1$  if the  $i^{th}$  supplier in group  $j$  is down during the supply cycle and 0 otherwise. We designate the probability that  $i^{th}$  supplier in group  $j$  is down as  $P(D_{ij} = 1) = d_{ij}$ . By definition of

super-events and unique-event, the  $S$  and  $D'_{ij}$ s are assume

to be independent random variables. We also *define*

*semi-super* events as events that affect a subset of all

suppliers but not all suppliers. Basically, one implicitly assumed that if some (but not all) of the suppliers in a group are down then the other suppliers in that group would be utilized to fill the needs. Effectively, for the suppliers, we restrict ourselves to three possible states of nature: (i) at least one group of suppliers is down (i.e., the production system can't engage in production due to lack of input), (ii) at least one supplier is up for each of the  $m$  groups and some suppliers are down and (iii) no suppliers are down. Notice that in the latter two cases, the production system is productive. However, the costs incurs for the two cases are not the same. Specifically, case (ii) should include the extra cost of getting the up suppliers to fill the order that are unable to deliver by the fail suppliers.

Given that the production system received all  $m$  types of input, the production system will engage in production to satisfy demands. To model operations risk for the production system, we let  $Z$  denote the indicator random variable associated with the event that some demand is not met during the production cycle given that the required supplies was delivered. Notice that demand is not met during the production cycle may be due to a variety of reason (e.g., inefficient/inadequate production during the production cycle, uncertainty of demand, etc). We assume that  $Z$  equal to 1 if some demand is not met during the production cycle and 0 otherwise. The probability of some demand is not met during the production cycle is denoted by  $P(Z = 1) = z$ . We can and do interpret  $z$  as the stock-out probability during the cycle.

For our model, the demand will not be met if any of the following scenarios occurs

- (1) The super-event or at least one group of suppliers is down (i.e., the production system can't engage in production due to lack of input),
- (2) some suppliers are down; the production is system is productive and demand is not completely filled and
- (3) no suppliers are down; the production system is productive and demand is not completely filled.

We let  $l_i: i = 1, 2, 3$  denote the financial loss to the

decision-making company associated with scenario  $i$ . For

example, we can and do interpret One may interpret  $l_1$  as *the total* loss suffers by the company when the production system can't engage in production due to failure of some group of suppliers in delivering the require input. One may interpret  $l_2$  as *the partial* loss in comparison to the total loss suffers when there is minor supply disruption (i.e., at least one supplier is up from each of the  $m$  groups and at least one supplier is down). One may interpret  $l_3$  as the financial loss due to inability to meet demand given that the production system engages in production. We assume that for each  $i = 1, 2, 3$ ;  $l_i$  is a positive random variable with first and second moments given by  $E(L_i) = l_i$  and  $E(L_i^2) = l_i^{(2)}$ . By definition, we have  $L_1 \geq L_2 \geq L_3$ . Thus, it is natural to assumed that  $l_1 \geq l_2 \geq l_3$ .

For each  $i \in \{1, 2, \dots, m\}$ , let us define  $J_i \equiv \{1, 2, \dots, n_i\}$  as the set of all available supplier in group  $i$ ,  $|J_i| \equiv n_i$  and

$$J \equiv \{J_1, J_2, \dots, J_m\}. \quad (1)$$

Suppose we have  $K = \{K_1, K_2, \dots, K_m\} \subseteq J$  (i.e.,  $K_i \subseteq J_i$  for all  $i$ ). Let us denote the cost of operating or working with suppliers set  $K$  by  $C_o(K)$ . Let  $G_{ij}$  denote the cost of including or maintaining supplier  $i$  in group  $j$  as part of the supply chain. We assume that  $G_{ij}^i$ s are independent nonnegative random variables with  $h(G_{ij}) = g_{ij}$  and  $E(G_{ij}^2) = g_{ij}^{(2)}$ . We also assume that for each supplier set  $K \subseteq J$ ,

$$C_o(K) \equiv a + \sum_{j=1}^m \sum_{i \in K_j} G_{ij}. \quad (2)$$

**Remark 1:** The functional form of  $C_o(K)$  given by the above equation is assume to simplify the analysis to follows. In general,  $C_o(K)$  is an increasing function of  $G_{ij}^i$ s; for all  $i, j$ .

Noticed that loss occurred due to four sources, (i) the super-events, (ii) at least one group of suppliers is down

during the supply cycle, (iii) at least one supplier is up for all groups and some suppliers are down during the supply cycle and unable to meet demand, and (iv) no suppliers is down during the supply cycle and unable to meet demand. The first two cases dealt with total loss  $l_1$ . The third case dealt with partial loss  $l_2$  and the last case dealt with partial loss  $l_3$ . Thus, the loss cost associated with supplier set  $K$ ,  $C_l(K)$  can be represented by

$$\begin{aligned} C_l(K) &= L_1(S + (1 - S)[1 - \prod_{j=1}^m (1 - \prod_{i \in K_j} D_{ij})]) \\ &+ (L_2 + L_3 Z)(1 - S)[\prod_{j=1}^m (1 - \prod_{i \in K_j} D_{ij}) \\ &- \prod_{j=1}^m \prod_{i \in K_j} (1 - D_{ij})] \\ &+ L_3 Z(1 - S) \prod_{j=1}^m \prod_{i \in K_j} (1 - D_{ij}). \end{aligned} \quad (3)$$

For the special case of  $m = |J| = 1$  or the single supplier case, the second term in the above equation equal to 0. That is, there is no partial loss in the case of single supplier.

Combining equation (1)-(2), we see that for each set of

suppliers  $K \subseteq J$ , the total cost function can be written as

$$\begin{aligned} C(K) &= C_l(K) + C_o(K) \\ &= L_1(S + (1 - S)[1 - \prod_{j=1}^m (1 - \prod_{i \in K_j} D_{ij})]) \\ &+ (L_2 + L_3 Z)(1 - S) \\ &\times [\prod_{j=1}^m (1 - \prod_{i \in K_j} D_{ij}) - \prod_{j=1}^m \prod_{i \in K_j} (1 - D_{ij})] \\ &+ L_3 Z(1 - S) \prod_{j=1}^m \prod_{i \in K_j} (1 - D_{ij}) \\ &+ a + \sum_{j=1}^m \sum_{i \in K_j} G_{ij} \end{aligned} \quad (4)$$

It is part of the objective of this proposal to determine a set of supplier which would form part of the supply chain such that certain criterions are optimized. As a starting point, noticed that from equation (1)-(2), we get for each  $J \subseteq I_n$

$$\begin{aligned} E(C_o(K)) &= a + \sum_{j=1}^m \sum_{i \in K_j} g_{ij} \\ E(C_l(K)) &= l_1(p + (1 - p)[1 - \prod_{j=1}^m (1 - \prod_{i \in K_j} d_{ij})]) \\ &+ (l_2 + l_3 z)(1 - p)[\prod_{j=1}^m (1 - \prod_{i \in K_j} d_{ij}) \\ &- \prod_{j=1}^m \prod_{i \in K_j} (1 - d_{ij})] \\ &+ l_3 z(1 - p) \prod_{j=1}^m \prod_{i \in K_j} (1 - d_{ij}) \end{aligned}$$

$$\begin{aligned} Var(C_o(K)) &= \sum_{j=1}^m \sum_{i \in K_j} [g_{ij}^{(2)} - g_{ij}^2] \\ Var(C_l(K)) &= l_1^{(2)}(p + (1 - p)[1 - \prod_{j=1}^m (1 - \prod_{i \in K_j} d_{ij})]) \\ &+ [l_2^{(2)} + 2(l_2 l_3 + l_3^{(2)})z](1 - p) \\ &\times [\prod_{j=1}^m (1 - \prod_{i \in K_j} d_{ij}) \\ &- \prod_{j=1}^m \prod_{i \in K_j} (1 - d_{ij})] \\ &+ l_3^{(2)} z(1 - p) \prod_{j=1}^m \prod_{i \in K_j} (1 - d_{ij}) \\ &- E(C_l(K))^2 \\ E(C(K)) &= E(C_o(K)) + E(C_l(K)) \quad \text{and} \\ Var(C(K)) &= Var(C_o(K)) + Var(C_l(K)). \end{aligned} \quad (5)$$

To simplify the analysis to follows, for each group  $j$  let us order the failure probabilities  $d_{ij}^j$ s so that

$$d_{1j} \leq d_{2j} \leq \dots \leq d_{n_j j} \quad \text{for } j = 1, 2, \dots, m. \quad (5)$$

With the above ordering, let us define

$$I_m \equiv \{(k_1, k_2, \dots, k_m) \mid 1 \leq k_i \leq n_j \text{ and } k_i \text{ is integer for all } i.\} \quad (6)$$

and for  $K = (k_1, k_2, \dots, k_m) \subseteq I_m$ , let

$$C_o(K) \equiv a + \sum_{j=1}^m \sum_{i=1}^{k_j} G_{ij} \quad (7)$$

$$\begin{aligned} C_l(K) \equiv & L_1(S + (1 - S)[1 - \prod_{j=1}^m (1 - \prod_{i=1}^{k_j} D_{ij})]) \\ & + (L_2 + L_3Z)(1 - S) \\ & \times [\prod_{j=1}^m (1 - \prod_{i=1}^{k_j} D_{ij}) - \prod_{j=1}^m \prod_{i=1}^{k_j} (1 - D_{ij})] \\ & + L_3Z(1 - S)[\prod_{j=1}^m \prod_{i=1}^{k_j} (1 - D_{ij})] \end{aligned} \quad (8)$$

$$C(K) \equiv C_o(K) + C_l(K) \quad (9)$$

Clearly,  $C_o(K)$ ,  $C_l(K)$  and  $C(K)$  are respectively the operating cost, loss cost and the total cost associated with the first  $k_j$  suppliers for all  $m$  groups under the ordering given by equation (6).

**Remark 1:** Equations (7)-(10) are define with respect to the ordering given by equation (6).

### 3. ANALYSIS

In this section, we shall present our analysis of the optimal suppliers' selection problem based on the model presented in section 2. In particular, we present a few formulations to our supply chain risk management problems under different performance criterions.

#### 3.1. Mean-Variance approach to finding an Optimal

##### Set of Suppliers

We aim to find a set of supplies which minimizes the cost incurred. Toward this goal, we formulate the following optimization problem (OP1)

$$\text{Min}_{K \subseteq J} E(C(K)) + \theta \sqrt{\text{Var}(C(K))} \quad (10)$$

where as usual,  $\theta$  represent the weight that measure our attitude towards risk.

**Remark 2:** (i) For the special case of  $m = 1$ ,  $\theta = L_2 = L_3 = z = 0$ ,  $n_1 = \infty$ , (OP1) reduces to the problem analyzed by Berger et al. (i.e., unlimited suppliers and mean value analysis without possibility of partial loss and no production/demand uncertainty). (ii) For the special case of  $m = 1$ ,  $L_3 = z = 0$ ,  $n_1 = \infty$ , (OP1) reduces to the problem analyzed by Lee (i.e., Lee 2008, unlimited available suppliers and single type of input with no production/demand uncertainty).

In this general form, (OP1) is a complicated combinatorial optimization problem. We propose to examine numerically a variety of cases aimed at understanding supplier failure risk versus total cost behavior. These numerical results also allow us to analyze the impact of risk and uncertainty on the optimal number of suppliers to be included in each of the  $m$  groups.

#### 3.2. Balancing Service Level and Cost approach to finding an Optimal Set of Suppliers

(OP1) aims to select an optimal set of suppliers so as to minimize cost taken into consideration the variability of cash flow. However, service level is of importance to the company as well. Thus, one is led naturally to balance two important and possibly conflicting objectives, service level and cost. This leads us to formulate the problem as one that maximize service level subject to a given budget constraint.

##### Suppliers' Service Level

The production process requires *all*  $m$  types of input in order to produce products to satisfy demand. Since total loss (i.e., the production system can't engage in production due to lack of input) occurs due to two sources, the

super-events  $S$  and the unique events that resulted in at

least one group of suppliers failed to deliver supply. Noticed that given a set of suppliers  $K = \{K_1, K_2, \dots, K_m\} \subseteq J$ , the probability that group  $i$  successfully deliver supply to the production/distribution system is equal to

$$\begin{aligned} & P(\text{suppliers in group } i \text{ deliver supply}) \\ & = 1 - P(\text{all suppliers in group } i \text{ fails}) = 1 - \prod_{j \in K_i} d_{ji}. \end{aligned}$$

Thus, we get the probability that all  $m$  groups successfully deliver supply to the production/distribution system is

$$P(\text{all groups deliver supply}) = \prod_{i=1}^m (1 - \prod_{j \in K_i} d_{ji}) \quad (12)$$

and at least one group of suppliers failed to deliver supply is equal to

$$\begin{aligned} & P(\text{at least one group of suppliers failed to deliver supply}) \\ & = 1 - \prod_{i=1}^m (1 - \prod_{j \in K_i} d_{ji}). \end{aligned} \quad (13)$$

Thus, given a set of suppliers  $K = \{K_1, K_2, \dots, K_m\} \subseteq J$ , the probability that total loss occurs is given by

$$\begin{aligned}
 & P(\text{Suppliers' Loss}(K)) \\
 &= P(\text{Super-event}) + P(\text{no Super-event}) \\
 &\times P(\text{at least one group of suppliers failed to deliver supply}) \\
 &= p + (1 - p)[1 - \prod_{j=1}^m (1 - \prod_{i \in K_j} d_{ij})].
 \end{aligned} \tag{14}$$

Therefore, we may define service level associate with a given set of supplier  $K \subseteq J$  as

$$\begin{aligned}
 SSL(K) &\equiv 1 - P(\text{Suppliers' Loss}(J)) \\
 &= (1 - p)\prod_{j=1}^m (1 - \prod_{i \in K_j} d_{ij}).
 \end{aligned} \tag{15}$$

Notice that the suppliers' service level is a product of two probabilities; probability that the super-event did not occur during the cycle and probability that the production system received the required inputs to engage in production.

### System's Service Level

We define system's service level as the probability of meeting demand in a (supply + production) cycle. The production system needs to receive the required inputs from all  $m$  groups of suppliers. Given that the production system receives the require supply, the probability that demand is not filled is  $z$ . Thus, we see that there are two possibilities where the demand may not be filled; (1) The production system did not received the require inputs from at least one of the suppliers' group, and (2) the production system received the require inputs from the suppliers and the demand is not filled. The probability that the production system did not received the require inputs from at least one of the suppliers' group is given by

$$\begin{aligned}
 & P(\text{Super-event}) \\
 &+ P(\text{no Super-event}) \\
 &\times P(\text{at least one group of suppliers failed to deliver}) \\
 &= p + (1 - p)[1 - \prod_{j=1}^m (1 - \prod_{i \in K_j} d_{ij})].
 \end{aligned}$$

The probability that the production system received the require inputs from the suppliers and the demand is not filled is given by the product of three probabilities; probability that super-event did not occur, probability that all group of suppliers deliver supply and probability that production system failed to meet demand given that all group of suppliers deliver necessary supplies

$$(1 - p)\prod_{j=1}^m (1 - \prod_{i \in K_j} d_{ij})z.$$

Thus, given a set of suppliers  $K \subseteq J$  we see that probability of not meeting demand in a given cycle is

$$\begin{aligned}
 & P(\text{not meeting demand}(K)) \\
 &= p + (1 - p)[1 - \prod_{j=1}^m (1 - \prod_{i \in K_j} d_{ij})] \\
 &+ (1 - p)\prod_{j=1}^m (1 - \prod_{i \in K_j} d_{ij})z \\
 &= 1 - (1 - p)(1 - z)\prod_{j=1}^m (1 - \prod_{i \in K_j} d_{ij}).
 \end{aligned} \tag{16}$$

Therefore, given a set of suppliers  $K \subseteq J$  we may set the total system service level as the capability of meeting demand

$$\begin{aligned}
 TSSL(K) &\equiv 1 - P(\text{not meeting demand}(K)) \\
 &= (1 - p)[\prod_{j=1}^m (1 - \prod_{i \in K_j} d_{ij})](1 - z) \\
 &= SSL(K)(1 - z)
 \end{aligned} \tag{17}$$

Notice that the suppliers' service level is a product of three probabilities; probability that the super-event did not occur during the cycle, probability that the production system received the required inputs to engage in production and the probability that demand will be met during the cycle given that the production system engage in production.

Let  $B$  represent our budget allocated to operate the suppliers and handle the loss incurs. The following optimization problem (OP2) aims to select a set of suppliers that maximized the total system service level and stay within the allocated budget as well as achieved minimum suppliers' service level

$$\begin{aligned}
 \text{Max}_{K \subseteq J} \quad & TSSL(K) \\
 \text{s.t.} \quad & E(C(K)) + \theta\sqrt{\text{Var}(C(K))} \leq B \\
 & SSL(K) \geq \beta
 \end{aligned} \tag{18}$$

### 3.3. Target Service Level approach to finding an Optimal Set of Suppliers

The constraint of (OP2) is really a probabilistic constraint. It is possible that the constraint of (OP II) is satisfied but the real cost still exceed  $B$ . This is due to the risk and uncertainty associated with the loss amount or the operating cost. Furthermore, it is natural for a firm to impose a minimum acceptable service level. Thus, as an alternative to (OP2), one may seek a set of suppliers that minimized cost and achieved certain target service level. Let  $\delta$  be the minimum acceptable or targeted service level. The resulting optimization problem is (OP3)

$$\begin{aligned}
 \text{Min}_{K \subseteq J} \quad & E(C(K)) + \theta\sqrt{\text{Var}(C(K))} \\
 \text{s.t.} \quad & TSSL(K) \geq \delta
 \end{aligned} \tag{19}$$

Clearly, the maximum service level is obtained by using all the available suppliers. Thus, from equations (1), (16)-(17), we see that maximum total system service level

is given by

$$MTSSL \equiv (1 - p) \left[ \prod_{j=1}^m (1 - \prod_{i \in J_j} d_{ij}) \right] (1 - z). \tag{20}$$

Therefore, a given target service level  $\delta$  can be achieved if and only if

$$(1 - p) \left[ \prod_{j=1}^m (1 - \prod_{i \in J_j} d_{ij}) \right] (1 - z) \geq \delta. \tag{21}$$

Thus, (OP3) is feasible if and only if equation (21) holds.

#### 4. NUMERICAL RESULTS – IDENTICAL SUPPLIERS

In this section, we assume the case of identical suppliers with 5 groups of 3 suppliers. Specifically,

$$m = 5, n_1 = n_2 = n_3 = n_4 = n_5 = 3, \text{ and } |J| = \sum_{i=1}^m n_i = 15. \tag{22}$$

Furthermore, we assume deterministic loss and operating cost. Therefore, we also have

$$g_{ij} = g_1, g_{ij}^{(2)} = g_1^2, l_1^{(2)} = l_1^2, l_2^{(2)} = l_2^2, l_3^{(2)} = l_3^2 \text{ for all } i \text{ and } j. \tag{23}$$

Thus, the problem of locating optimal set of suppliers reduces to finding the optimal number of suppliers. We use the following base case parameter values

$$l_1 = 500, l_2 = 150, l_3 = 50, g_1 = 10, p = 0.01, d = d_j = 0.05 \text{ and } z = 0.05. \tag{24}$$

When one of the parameter's value is changed, it is assumed that all of the other parameters stay at the base value, unless otherwise noted. The entry of x in Tables 1-8 means that x suppliers are selected for each of the 5 groups.

Tables 1-4 tabulate our numerical results given by solving

(OP1) and Tables 5-8 tabulate our numerical results given by solving (OP3). Table 1 consider sensitivity analysis when the value of  $l$  changes. Table 2 consider sensitivity analysis when the value of  $g_1$  changes. Table 3 consider

sensitivity analysis when the value of  $p$  changes. Table 4 consider sensitivity analysis when the value of  $d$  changes. For each of these 4 tables, we tabulate the value of optimal number of suppliers  $n^*$  with three values of  $\theta = 0, 1, 2$ . The case  $\theta = 0$  represents the case of mean value analysis.

TABLE 1-Optimal  $k_j$ : Sensitivity Analysis of the value of loss,  $l_1$

$d = 0.05$				$d = 0.2$			
$l_1$	$\theta = 0$	$\theta = 1$	$\theta = 2$	$l_1$	$\theta = 0$	$\theta = 1$	$\theta = 2$
350	1	2	2	350	2	3	3
800	2	2	2	800	3	3	3
1200	2	2	3	1200	3	3	3
1600	2	3	3	1600	3	3	3
4000	2	3	3	4000	3	3	3
5000	2	3	3	5000	3	3	3

TABLE 2-Optimal  $k_j$ : Sensitivity Analysis of the value of  $g_1$

$d = 0.05$				$d = 0.2$			
$g_1$	$\theta = 0$	$\theta = 1$	$\theta = 2$	$g_1$	$\theta = 0$	$\theta = 1$	$\theta = 2$
1	2	2	3	1	3	3	3
4	2	2	2	4	3	3	3
10	1	2	2	10	2	3	3
16	1	2	2	16	2	3	3
25	1	2	2	25	2	2	2

TABLE 3-Optimal  $k_j$ : Sensitivity Analysis of the value of  $p$

$p$	$\theta = 0$	$\theta = 1$	$\theta = 2$
0.0001	1	2	2
0.001	1	2	2
0.01	1	2	2
0.1	1	2	2

0.5	1	1	2
0.6	1	1	1

TABLE 4-Optimal  $k_i$ : Sensitivity Analysis of the value of  $d$

$d$	$\theta = 0$	$\theta = 1$	$\theta = 2$
0.01	1	2	2
0.05	1	2	2
0.1	2	2	3
0.15	2	3	3
0.2	2	3	3
0.3	3	3	3

From Tables 1-4, we observe that the mean value analysis

consistently underestimated the optimal number of suppliers when one takes into consideration the variability of the cashflow.

Tables 5-8 tabulate optimal solution of (OP3) with targeted service level of  $\delta = 90\%$ . Using equation (22)-(24), and (20), we see that the maximum service level is 93.99%. Thus, (OP3) is infeasible for  $\delta \geq 94\%$ . In Tables 7-8,  $x = \text{inf}$  means that (OP3) is infeasible. In Tables 5-8 we find many cases where imposing the targeted service levels increase the optimal number of suppliers. Therefore, we see that imposing a targeted service level in choosing the optimal number of supplier could change the solution drastically.

TABLE 5-Optimal  $k_i$ : Sensitivity Analysis of the value of loss,  $l_1$

$d=0.05, \delta = 0.9$				$d=0.20, \delta = 0.9$			
$l_1$	$\theta = 0$	$\theta = 1$	$\theta = 2$	$l_1$	$\theta = 0$	$\theta = 1$	$\theta = 2$
350	2	2	2	350	3	3	3
800	2	2	2	800	3	3	3
1200	2	2	3	1200	3	3	3
1600	2	3	3	1600	3	3	3

400	2	3	3	4000	3	3	3
5000	3	3	3	5000	3	3	3

TABLE 6-Optimal  $k_i$ : Sensitivity Analysis of the value of  $g_1$

$d=0.05, \delta = 0.9$				$d=0.20, \delta = 0.9$			
$g_1$	$\theta = 0$	$\theta = 1$	$\theta = 2$	$g_1$	$\theta = 0$	$\theta = 1$	$\theta = 2$
1	2	2	2	1	3	3	3
4	2	2	2	4	3	3	3
10	2	2	2	10	2	3	3
15	2	2	2	15	2	3	3
25	2	2	2	25	2	2	3

TABLE 7-Optimal  $k_i$ : Sensitivity Analysis of the value of  $\mu$

$\mu$	$\delta = 0.9$		
	$\theta = 0$	$\theta = 1$	$\theta = 2$
0.0001	2	2	2
0.001	2	2	2
0.01	2	2	2
0.05	3	3	3
0.1	inf	inf	inf

TABLE 8-Optimal  $k_i$ : Sensitivity Analysis of the value of  $d$

$d$	$\delta = 0.9$		
	$\theta = 0$	$\theta = 1$	$\theta = 2$
0.01	2	2	2
0.05	2	2	2
0.1	3	3	3
0.15	3	3	3
0.2	3	3	3

0.25	inf	inf	inf
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## 5. SUMMARY

We have proposed a mean-variance approach to determine the optimal number of suppliers in the presence of supplier failure risks. In particular, we have provided three formulations for the optimal supplier group selection problem. Our formulation of the problem generalizes the analysis proposed by Lee (2008). It allows us to take into consideration the issues of multiple groups of suppliers, limited number of available suppliers, and the impact of meeting random demand. We also provided some numerical results for the special case of five groups of identical suppliers. The results are tabulated in Tables 1–4. We also show that imposing targeted service level could increase the optimal number of suppliers and in some cases render the problem infeasible. The results are tabulated in Tables 5–8.

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