ON EFFECTIVENESS OF TWO REVERSE AUCTION PROCUREMENT MODELS FOR MULTIPLE BUYERS

Fu-Shiung Hsieh,Chaoyang University of Technology, Taiwan, fshsieh@cyut.edu.tw

ABSTRACT

Combinatorial reverse auction is a popular business model for procurement. If there are multiple buyers, the buyers may either hold multiple combinatorial reverse auctions independently. Alternatively, the buyers may delegate the auction to a group buyer who holds only one combinatorial reverse auction on behalf of all the buyers. In the existing literature, there is still a lack of comparative study on the efficiency of the aforementioned two different procurement models for multiple buyers. The goal of this paper is to study the effectiveness of these two different combinatorial reverse auction models. We first formulate the problems for these two combinatorial reverse auction models and then compare the performance as well as the computational efficiency for these two combinatorial reverse auction models. Our analysis indicates that the group-buying combinatorial reverse auction model outperforms multiple independent combinatorial reverse auctions not only in efficiency but also in performance.

Keywords: Business model, Procurement, Auction.

1. INTRODUCTION

Reverse auction is a popular business model that can be applied in corporations' procurement. Combinatorial reverse auction [7][25][32][34] enables a buyer to purchase multiple goods with the lowest prices from the sellers. Applying combinatorial reverse auctions in corporations' procurement processes can lead to significant savings [16][26][27]. In real world, multiple buyers may procure goods at the same time. If there are multiple buyers, group-buying may be applied to reduce the costs and benefit the buyers [17]. Group-buying is a popular business model employed by many companies in practice. The rationale of group-buying is due to demand aggregation, which benefits sellers, offering lower marketing costs and coordinated distribution channels, as well as buyers, who enjoy lower costs for product purchases [6]. From the perspective of buyers, quantity based discounts provide a huge incentive to form coalitions and take advantage of lower prices without ordering more than their actual demand. An interesting issue is to develop a business model for supporting multiple buyers' procurement by combining the concept of group-buying with combinatorial reverse auctions.

In existing literature, combinatorial auctions and group-buying have attracted considerable attention recently. An excellent survey on combinatorial auctions can be found in [5] and [28]. Combinatorial auctions [1] are notoriously difficult to solve from a computational point of view [30] due to the exponential growth of the number of combinations [18]. The combinatorial auction problem can be modeled as a set packing problem (SPP) [3][8][13][33]. Sandholm *et al*. mentions that determining the winners so as to maximize revenue in combinatorial auction is NP-complete [31][32]. Exact algorithms have been developed for the SPP problem, including iterative deepening A* search [31] and the direct application of available CPLEX IP solver [3]. Gonen and Lehmann proposed branch and bound heuristics for finding optimal solutions for multi-unit combinatorial auctions [10]. Jones and Koehler studied combinatorial auctions using rule-based bids [19]. In [11][14]15][16], the authors proposed a Lagrangian heuristic and a Lagrangian relaxation approach for combinatorial reverse auction problems.

There are also studies on group-buying in existing literature. Kauffman and Wang [20][21] examine group-buying as a dynamic pricing mechanism. They offer valuable insights for distributed group-buying mechanisms under uniform cost sharing. Chen et al. [4] analyze buyers' bidding strategies in a group-buying auction considering limited supply of an item and private information of buyers. Anand and Aron [2] analyze the value of group-buying and the optimal price curve from a seller's perspective. Cuihong Li, Katia Sycara and Alan Scheller-Wolf [24] introduce the concept of combinatorial coalition formation, which allows buyers to announce reserve prices for combinations of items. These reserve prices, along with the sellers' price-quantity curves for each item, are used to determine the formation of buying groups for each item. The objective is to maximize buyers' total surplus. Despite the aforementioned results on group-buying, there is still a lack of study on assessing the benefits and effectiveness of combining group-buying mechanism with combinatorial reverse auctions in multiple buyers' procurement.

In this paper, we study two different business models for multiple buyers' procurement based on combinatorial reverse auctions: (1) independent combinatorial reverse auctions: each buyer may hold a combinatorial reverse auction independently and (2) combinatorial reverse auctions based on group buying: multiple buyers delegate the auction to a group buyer and the group buyer holds only one combinatorial reverse auction for all the buyers. In developing an effective tool to support the decision of multiple buyers' procurement, a comparative study on the performance and efficiency of the two aforementioned business models is needed. This motivates us to study the effectiveness of these two different combinatorial reverse auction models, including performance and efficiency.

To compare the effectiveness of the aforementioned two combinatorial reverse auction models, we first illustrate the advantage of combining group-buying with combinatorial reverse auctions by an example. We then formulate the problems for these two combinatorial reverse auction models and propose solution algorithms for these two problems. For the two combinatorial reverse auctions, we formulate the corresponding winner determination problems. As the two winner determination problems are NP Complete, we adopt a Lagrangian relaxation approach to developing solution algorithms for finding approximate solutions. Based on the proposed algorithms for the two problems, we compare the performance as well as the computational efficiency for these two combinatorial reverse auction models. Our analysis indicates that combinatorial reverse auction with group-buying not only outperforms multiple independent combinatorial reverse auctions but also is more efficient than multiple independent combinatorial reverse auctions.

This paper is different from the previous study on single buyer's combinatorial reverse auction in [16] as it focuses on comparative study of two combinatorial reverse auction models for multiple buyers. The remainder of this paper is organized as follows. In Section 2, we first illustrate the advantage of group buying combinatorial reverse auction over multiple independent combinatorial reverse auctions by an example. In Section 3, we formulate the optimization problems for the aforementioned combinatorial reverse auction models and study the property of the solutions for the two problems. We propose the solution algorithms in Section 4. In Section 5, we study the efficiency of the two business model. Theoretical lower and upper bounds on the achievable reduction in cost are derived. We also compare the performance of the two combinatorial reverse auction models based on the results of numerical examples. We conclude this paper in Section 6.

2. MEETING BUYERS' REQUIREMENTS WITH COMBINATORIAL REVERSE AUCTIONS

In this section, we illustrate different ways to meet multiple buyers' requirements with combinatorial reverse auctions. We introduce two different combinatorial reverse auction models. In the first model, each buyer holds a combinatorial reverse auction independently. The solution that meets all the buyers' requirements can be obtained by solving N combinatorial reverse auction subproblems. Each subproblem is solved by applying any existing combinatorial reverse auction algorithm. In the second model, a virtual group buyer is created. The group buyer's requirements consolidate all the buyers' requirements. The bids placed by the potential bidders in the first model are regarded as the bids placed to the group buyer. The solution that meets the group buyer's requirements (and hence all the buyers' requirements) is found by solving the combinatorial reverse auction subproblem for the group buyer.

Consider an application scenario in which Buyer 1 wants to purchase at least a bundle of items 1A, 1B and 1C from the market and Buyer 2 wants to purchase a bundle of items 1C and 1D.

Buyer 1 and Buyer 2 may apply acquire the desired items based on combinatorial reverse auction using two different business models.

Model 1: Buyer 1 and Buyer 2 hold two independent combinatorial reverse auctions.

Suppose there are four bidders, Seller 1, Seller 2, Seller 3 and Seller 4 who place bids in the system. Suppose Seller 1 places the bid (1A,1C, p1) on Buyer 1, where p11 denotes the prices of the bid. Seller 2 places the bid (1B, 1C, p2) on Buyer 1. Seller 3 places the bid (1D, p3) on Buyer 2. Seller 4 places the bid (1C, 1D, p4) on Buyer 2. We assume that all the bids entered the auction are recorded. A bid is said to be active if it is in the solution. We assume that there is only one bid active for all the bids placed by the same bidder. For this example, the solution for this combinatorial reverse auction problem is Seller1: (1A, 1C, p1), Seller 2: (1B, 1C, p2) and Seller 4: (1C,1D, p4). The overall cost of this solution is p1+p2+p4.

Model 2: Buyer 1 and Buyer 2 delegate the procurement to a group buyer that holds only one combinatorial reverse auction

In this business model, the group buyer consolidates all the requirements of Buyer 1 and Buyer 2 and holds only one combinatorial reverse auction. Suppose there are four bidders, Seller 1, Seller 2, Seller 3 and Seller 4 who place bids on the Group Buyer. Suppose Seller 1 places the bid: (1A, 1C, p1), where p11 denotes the prices of the bid. Seller 2 places the bid: (1B, 1C, p2). Seller 3 places the bid: (1D, p3). Seller 4 places the bid: (1C, 1D, p4). We assume that all the bids entered the auction are recorded. A bid is said to be active if it is in the solution. We assume that there is only one bid active for all the bids placed by the same bidder. For this example, the solution for this combinatorial reverse auction problem depends on p3 and p4 as follows. If p3 $\leq$ p4, the winning bid is Seller1: (1A, 1C, p1), Seller 2: (1B, 1C, p2) and Seller 3: (1D, p3). The overall cost of this solution is p1+p2+p3, which is lower than that of Model 1 as p3 $\leq$ p4. If p3 $>$ p4, the winning bid is Seller1: (1A, 1C, p1), Seller 2: (1B, 1C, p2) and Seller 4: (1C, 1D, p4). Then the overall cost of the solution of Model 2 is the same as that of Model 1. This example illustrates that the overall cost of the combinatorial reverse auction based on group buying is no greater than that of two independent combinatorial reverse auctions.

3. PROBLEM FORMULATION FOR TWO COMBINATORIAL REVERSE AUCTION MODELS

Motivated by the above examples, it is interesting to compare the two models of combinatorial reverse auction from the cost and computation aspects. In the remainder of this paper, we first formulate the problem and then propose solution methodology for these two problems. We then compare the the cost and computation time by numerical examples.

Model 1: Combinatorial Reverse Auction for a Single Buyer

Let I denote the number of buyers in a combinatorial auction. Each $i \in \{1,2,3,....,I\}$ represents a buyer. Consider a buyer i who requests a set of items to be purchased, where $i \in \{1,2,3,....,I\}$. Let K denote the number of items requested. Let d_{ik} denote the desired units of the $k-th$ item requested by buyer i, where $k \in \{1,2,3,....,K\}$. In a combinatorial reverse auction, there are many bidders. Let N_i denote the set of bidders that take part in the combinatorial reverse auction of buyer i. That is, each $n \in N_i$ represents a bidder. To model the combinatorial reverse auction problem, the bid must be represented mathematically. We use a vector $b_{nj} = (q_{nj1}, q_{nj2}, q_{nj3}, ..., q_{njK}, p_{nj})$ to represent the $j-th$ bid submitted by bidder n, where q_{njk} is a nonnegative integer that denotes the quantity of the $k-th$ items and p_{nj} is a real positive number that denotes the price of the bundle. As the quantity of the $k-th$ items cannot exceed the quantity d_k, it follows that the constraint $0 \leq q_{njk} \leq d_k$ must be satisfied. The $j-th$ bid b_{nj} is actually an offer to deliver q_{njk} units of items for each $k \in \{1,2,3,....,K\}$ a total price of p_{nj}. Let J_n denote the number of bids placed by bidder $n \in N_i$. Let J denote the maximum number of bids that a bidder can place in each round of combinatorial reverse auction. That is, $J = \max_{n \in N_i} J_n$. To formulate the problem, we use the variable x_{inj} to indicate the $j-th$ bid placed by bidder n is active (x_{inj} =1) or inactive (x_{inj} =0). The winner determination problem can be formulated as an Integer Programming problem as follows.

Winner Determination Problem for Buyer i (WDP-i):

$$\min \sum_{n \in N_i} \sum_{j=1}^{J_n} x_{inj} p_{nj}$$

$$s.t. \sum_{n \in N_i} \sum_{j=1}^{J_n} x_{inj} q_{njk} \geq d_{ik} \forall k = 1,2,..., K \quad (a)$$

$$x_{inj} \in \{0,1\} \quad (b)$$

Constraints (a) in WDP assumes "free disposal" as the total quantity offered by the winners must be greater than or equal to the desired quantity of the buyer. If there are more quantities provided than needed, we can dispose of the surplus with no additional cost.

Model 2: Combinatorial Reverse Auction for Multiple Buyers based on Group Buying

Consider a buyer who requests a set of items to be purchased. Let K denote the number of items requested. Let I denote the number of buyers in a combinatorial auction. Each $i \in \{1,2,3,....,I\}$ represents a buyer. Let d_{ik} denote the desired units of the $k-th$ items, where $k \in \{1,2,3,....,K\}$. In a combinatorial reverse auction, there are many bidders to submit a tender. Let $N = \bigcup_{i \in I} N_i$ denote the set of bidders that take part in the combinatorial auction.

To model the combinatorial reverse auction problem, the bid must be represented mathematically. We use a vector $b_{nj} = (q_{nj1}, q_{nj2}, q_{nj3}, ..., q_{njK}, p_{nj})$ to represent the $j-th$ bid submitted by bidder n, where q_{njk} is a nonnegative integer that denotes the quantity of the $k-th$ items and p_{nj} is a real positive number that denotes the price of the bundle. As the quantity of the $k-th$ items cannot exceed the quantity d_{ik}, it follows that the constraint $0 \le q_{njk} \le \sum_{i=1}^{I} d_{ik}$ must be satisfied. The $j-th$ bid b_{nj} is actually an offer to deliver q_{njk} units of items for each $k \in \{1,2,3,....,K\}$ a total price of p_{nj}. Let J_n denote the number of bids placed by bidder $n \in \{1,2,3,....,N\}$. To formulate the problem, we use the variable x_{gnj} to indicate the $j-th$ bid placed by bidder n is active (x_{gnj} =1) or inactive (x_{gnj} =0). The winner determination problem can be formulated as an Integer Programming problem as follows.

Winner Determination Problem for Group Buyer (WDP-G)

$$\min \sum_{n \in N} \sum_{j=1}^{J_n} x_{gnj} p_{nj}$$

$$s.t. \sum_{n \in N} \sum_{j=1}^{J_n} x_{gnj} q_{njk} \ge \sum_{i=1}^{I} d_{ik} \, \forall k = 1,2,..., K \quad (c)$$

$$x_{nj} \in \{0,1\} \quad (d)$$

Inequalities (c) of WDP-G are the demand constraints that need to be satisfied by the solution.

To compare the performance of Model 1 and Model 2, we develop solution algorithms for them.

4. SOLUTION ALGORITHMS

One way to reduce the computational burden in solving the WDP is to adopt Lagrangian relaxation approach to set up a fictitious market to determine an allocation and prices in a decentralized way to adapt to dynamic environments where bidders and items may change from time to time. The buyer announces which sets of items and sets prices for them. If two or more agents compete for the same item, the buyer adjusts the price vector. This saves bidders from specifying their bids for every possible combination and the buyer from having to process each bid function. The bundle associated with the bid is tentatively assigned to that bidder only if the price of the bid is the lowest.

In this paper, we develop solution algorithms based on Lagrangian relaxation. The basic idea of Lagrangian relaxation is to relax some of the constraints of the original problem by moving them to the objective function with a penalty term. That is, infeasible solutions to the original problem are allowed, but they are penalized in the objective function in proportion to the amount of infeasibility. The constraints that are chosen to be relaxed are selected so that the optimization problem over the remaining set of constraints is in some sense easy. In WDP-i, we observe that the coupling among different operations is caused by the demand constraints (a). Let λ denote the vector with λ_k representing the Lagrangian multiplier for the $k-th$ items. We define

$$L_i(\lambda) = \sum_{k=1}^{K} \lambda_k d_{ik} + \sum_{n \in N_i} L_{in}(\lambda), \text{ with}$$

$$L_{in}(\lambda) = \min \sum_{j=1}^{J_n} x_{inj} (P_{nj} - \sum_{k=1}^{K} \lambda_k q_{njk})$$

$$s.t. \sum_{j=1}^{J_n} x_{inj} \le 1$$

$$x_{inj} \in \{0,1\}$$

$L_{in}(\lambda)$ defines a bidder's subproblem (BS).

For a given Lagrange multiplier λ, the relaxation of constraints (a) decomposes the original problem into a number of bidders' subproblems (BS). Lanrange multipliers are determined by solving the following dual problem.

$$\max_{\lambda \ge 0} L_i(\lambda)$$

Our methodology for finding a near optimal solution of WDP consists of three parts: (1) an algorithm for solving subproblems, (2) a subgradient method for solving the dual problem and (3) a heuristic algorithm for finding a near-optimal feasible solution.

(1)An algorithm for solving subproblems

Given λ, the optimal solution to BS subproblem $\mathrm{L}_n(\lambda)$ can be solved as follows.

Let $j^* = \arg \min_{j \in \{1,2,...,n_i\}} (P_{nj} - \sum_{k=1}^{K} \lambda_k q_{njk})$. The optimal solution to $\mathrm{L}_n(\lambda)$ is as follows.

$$x_{inj} = \begin{cases} 0 & \forall j \in \{1,2,...,J_n\} \setminus \{j^*\} \\ 1 & if\ P_{nj^*} - \sum_{k=1}^{K} \lambda_k q_{nj^*k} < 0 \\ 0 & if\ P_{nj^*} - \sum_{k=1}^{K} \lambda_k q_{nj^*k} >= 0 \end{cases}$$

(2)A subgradient method for solving the dual problem $\max_{\lambda \geq 0} L_i(\lambda)$

Let x^l be the optimal solution to the subproblems for given Lagrange multipliers λ^l of iteration l. We define the subgradient of $L_i(\lambda)$ as $g^l(k) = d_{ik} - \sum_{n \in N_i} \sum_{j=1}^{J_n} x_{inj}^l q_{njk}$, where $k \in \{1,2,...,K\}$.

The subgradient method proposed by Polyak [9] is adopted to update λ as follows

$$\lambda_k^{l+1} = \begin{cases} \lambda_k^l + \alpha^l g^l(k)\ if\ \lambda_k^l + \alpha^l \lambda_k^l \geq 0; \\ 0\ otherwise. \end{cases}$$

where $\alpha^l = c \dfrac{\overline{L}_i - L_i(\lambda)}{\sum_k (g^l(k))^2}$, $0 \leq c \leq 2$ and $\overline{L}_i$ is an estimate of the optimal dual cost. The iteration step terminates if α^l is smaller than a threshold. Polyak proved that this method has a linear convergence rate.

The solution obtained by applying the subgradient method may not be a feasible. If it is not feasible, it could be adjusted to a feasible solution without a great increase in objective function value. To adjust the solution of the dual problem to a feasible one, one must identify the set of demand constraints violated K^0 and the set of bidders I^0 that is not a winner in solution of the dual problem. Then we pick the bidders from the set I^0 according to the rule of minimal cost first to fulfill the insufficient quantity required by the set of violated demand constraints K^0.

(3)Heuristic Algorithm for Finding a Feasible Solution

Step 0: $\bar{x}_i \leftarrow x_i^*$

Step 1: Find the set of demand constraints violated.

Find $K^0 =$

$\{k | k \in \{1,2,3,....,K\}, \sum_{n \in N_i} \sum_{j=1}^{J_n} x_{inj} q_{njk} < d_{ik}\}$

.

Step 2: Find the set of bidders that is not a winner in solution x_i^*.

Find $N^0 = \{n | n \in n \in N_i, x_{inj}^* = 0\}$.

Step 3: While $K^0 \neq \Phi$

Select

$$k = \arg \min_{k \in K^0} d_{ik} - \sum_{n \in N_i} \sum_{j=1}^{J_n} x_{inj} q_{njk} \text{ from } K^0$$

Select $n \in N^0$ and $j \in \{1,2,..., J_n\}$ with $j = \arg \min_{j \in \{1,2,...J_n\}, q_{njk} > 0} p_{nj}$

Set $\bar{x}_{inj} = 1$

$N^0 \leftarrow N^0 \setminus \{n\}$

End While

The effectiveness of the solution algorithms can be evaluated based on the duality gap, which is the ratio of the difference between primal and dual objective values divided by the primal objective value. That is, duality gap of the solution for WDP- i of buyer i is defined by $\dfrac{f_i(\bar{x}_i) - L_i(\lambda^*)}{f_i(\bar{x}_i)}$ with $f_i(\bar{x}_i) = \sum_{n \in N_i} \sum_{j=1}^{J_n} \bar{x}_{nj} p_{nj}$.

Let $L_I(\lambda_I^*) = \sum_{i=1}^{I} L_i(\lambda_i^*)$, $f_I(x_I^*) = \sum_{i=1}^{I} f_i(x_i^*)$ and $f_I(\bar{x}_I) = \sum_{i=1}^{I} f_i(\bar{x}_i^*)$. We have

$$L_I(\lambda_I^*) \leq f_I(x_I^*) \qquad (e).$$

By applying a similar procedure to WDP -*G* for Group Buyer. We define the following dual problem.

$$\max_{\lambda \geq 0} L(\lambda),$$

where
$$\mathrm{L}(\lambda) = \sum_{k=1}^{K} \lambda_k (\sum_{i=1}^{I} d_{ik}) + \sum_{n \in N} \sum_{j=1}^{\mathrm{J_n}} \mathrm{L_{nj}}(\lambda), \text{ with}$$
$$\mathrm{L_{nj}}(\lambda) = \min x_{gnj}(p_{nj} - \sum_{k=1}^{K} \lambda_k q_{njk})$$
$$s.t.\ x_{gnj} \in \{0,1\}$$

$L_{\mathrm{nj}}(\lambda)$ defines a bidder's subproblems (BS). Our methodology for finding a near optimal solution of WDP consists of three parts as follows.

(1) An algorithm for solving subproblems

Given λ, the optimal solution to BS subproblem $L_{\mathrm{nj}}(\lambda)$ can be solved as follows.

$$x_{gnj} = \begin{cases} 1\ if\ P_{nj} - \sum_{k=1}^{K} \lambda_k q_{njk} < 0 \\ 0\ if\ P_{nj} - \sum_{k=1}^{K} \lambda_k q_{njk} \geq 0 \end{cases}$$

(2) A subgradient method for solving the dual problem $\max_{\lambda \geq 0} L(\lambda)$

Let x^l be the optimal solution to the subproblems for given Lagrange multipliers λ^l of iteration l. We define the subgradient of $L(\lambda)$ as

$$g_k^l = \frac{\partial L(\lambda)}{\partial \lambda_k}\bigg|_{\lambda_k^l} = \sum_{i=1}^{I} d_{ik} - (\sum_{n \in N} \sum_{j=1}^{J_n} x_{gnj} q_{njk}) \quad ,$$

where $k \in \{1,2,..., K\}$.

The subgradient method proposed by Polyak [9] is adopted to update λ as follows $\lambda_k^{l+1} = \begin{cases} \lambda_k^l + \alpha^l g_k^l \ \ if \ \ \lambda_k^l + \alpha^l \lambda_k^l \geq 0; \\ 0 \ otherwise. \end{cases}$

where $\alpha^l = c \dfrac{\overline{L} - L(\lambda)}{\sum_k (g_k^l)^2}$, $0 \leq c \leq 2$ and $\overline{L}$ is an estimate of the optimal dual cost. The iteration step terminates if α^l is smaller than a threshold. Polyak proved that this method has a linear convergence rate.

By iteratively applying the algorithms in (1) and (2), it will converge to an optimal dual solution (x_g^*, λ_g^*).

(3)A heuristic algorithm for finding a near-optimal $\overline{x}_g$, feasible solution based on the solution (x_g^*, λ_g^*) of the relaxed problem. The solution (x_g^*, λ_g^*) may result in one type of constraint violation due to relaxation: assignment of the quantity of items less than the demand of the items. Our heuristic scheme first checks all the demand constraints $\sum_{n \in N} \sum_{j=1}^{J_n} x_{gnj} q_{njk} \geq \sum_{i=1}^{I} d_{ik} \ \forall k = 1,2,..., K$ that have not been satisfied. Let K^0 = $\{k | k \in \{1,2,3,...., K\}, \sum_{n \in N} \sum_{j=1}^{J_n} x_{gnj} q_{njk} < \sum_{i=1}^{I} d_{ik}\}$.

K^0 denotes the set of demand constraints violated. Let $N^0 = \{n | n \in N, x_{gnj}^* = 0\}$. N^0 denotes the set of bidders that is not a winner in solution x^*. To make the set of constraints K^0 satisfied, we first pick $k \in K^0$ with

$$k = \arg \min_{k \in K^0} \sum_{i=1}^{I} d_{ik} - \sum_{n \in N} \sum_{j=1}^{J_n} x_{gnj}^* q_{njk} .$$

The heuristic algorithm proceeds as follows to make constraint k satisfied. Select $n \in N^0$ with $\mathrm{n} = \arg \min_{n \in \{1,2,...,N\}, q_{njk} > 0} \mathrm{p_{nj}}$ and set x_{gnj}^* =1. After performing the above operation, we set $N^0 \leftarrow N^0 \setminus \{n\}$. If the violation of the k -th constraint cannot be completely resolved, the same procedure repeats. Eventually, all the constraints will be satisfied. We use $\overline{x}$ to denote the resulting feasible solution obtained from the above heuristics.

That is, duality gap of the solution for WDP -*G* of the group buyer is defined by $\dfrac{f_g(\overline{x}_g) - L(\lambda_g^*)}{f_g(\overline{x}_g)}$ with $f_g(\overline{x}_g) = \sum_{n \in N} \sum_{j=1}^{J_n} \overline{x}_{gnj} p_{nj}$.

5. NUMERICAL RESULTS AND ANALYSIS

In this section, we will verify the benefit of group-buying mechanism. Based on the proposed algorithms, we compare the effectiveness of the two combinatorial reverse auction models by examples.

Example 1: Suppose there are four buyers, Buyer 1, Buyer 2, Buyer 3 and Buyer 4, who want to purchase the required goods by applying combinatorial reverse auctions. We compare Model 1 and Model 2 for this example as follows.

Model 1 for Example 1: Suppose Buyer 1, Buyer 2, Buyer 3 and Buyer 4 hold four combinatorial reverse auctions independently. There are ten bidders (sellers). The bids placed by the sellers are listed in Table 2 through Table 9.

The data for Buyer 1's combinatorial reverse auction are as follows:

$N_1 = \{1,5\}, \quad J = 2, \quad K = 4$

$d_{11} = 2, d_{12} = 2, d_{13} = 3, d_{14} = 1.$

The four bids submitted by the two bidders are as follows:

$q_{111} = 2, q_{112} = 2, q_{113} = 3, q_{114} = 0,$

$q_{511} = 0, q_{512} = 0, q_{513} = 0, q_{514} = 1,$

$q_{121} = 1, q_{122} = 1, q_{123} = 1, q_{124} = 0,$

$q_{521} = 0, q_{522} = 0, q_{523} = 0, q_{524} = 2,$

$P_{11} = 140, \quad P_{51} = 36,$

$P_{12} = 63, \quad P_{52} = 83.$

By applying our algorithm, we find the solution: $\overline{x}_{112}$ =0, $\overline{x}_{151}$ =1, $\overline{x}_{152}$ =0.

The cost is: 176.

CPU time: 62.

The data for Buyer 2's combinatorial reverse auction are as follows:

$N_2 = 2, \quad J = 2, \quad K = 4$

$d_{21} = 1, d_{22} = 2, d_{23} = 2, d_{24} = 3.$

The four bids submitted by the two bidders are as follows:

$q_{211}=0, q_{212}=2, q_{213}=2, q_{214}=3,$
$q_{611}=1, q_{612}=0, q_{613}=0, q_{614}=0,$
$q_{221}=0, q_{222}=1, q_{223}=1, q_{224}=1,$
$q_{621}=2, q_{622}=0, q_{623}=0, q_{624}=0,$
$P_{21}=212, \quad P_{61}=8,$
$P_{22}=95, \quad P_{62}=21.$

By applying our algorithm, we find the solution: $\bar{x}_{221}=1$, $\bar{x}_{222}=0$, $\bar{x}_{261}=1$, $\bar{x}_{262}=0$.

The cost is: 220.
CPU time: 47.

The data for Buyer 3's combinatorial reverse auction are:

$N_3=\{3,7,9,10\}, \quad J=2, \quad K=4$
$d_{31}=3, d_{32}=3, d_{33}=1, d_{34}=1.$

The eight bids submitted by the four bidders are as follows:

$q_{311}=3, q_{312}=3, q_{313}=0, q_{314}=1,$
$q_{711}=0, q_{712}=0, q_{713}=1, q_{714}=0,$
$q_{911}=1, q_{912}=1, q_{913}=0, q_{914}=0,$
$q_{10,11}=0, q_{10,12}=0, q_{10,13}=1, q_{10,14}=1,$
$q_{321}=1, q_{322}=1, q_{323}=0, q_{324}=1,$
$q_{721}=0, q_{722}=0, q_{723}=2, q_{724}=0,$
$q_{921}=1, q_{922}=0, q_{923}=1, q_{924}=0,$
$q_{10,21}=0, q_{10,22}=1, q_{10,23}=0, q_{10,24}=1.$
$P_{31}=212, \quad P_{71}=8, P_{91}=124, \quad P_{10,1}=78,$
$P_{32}=95, \quad P_{72}=21, P_{92}=42, \quad P_{10,2}=65.$

By applying our algorithm, we find the solution: $\bar{x}_{331}=1$, $\bar{x}_{332}=0$, $\bar{x}_{371}=1$, $\bar{x}_{372}=0$, $\bar{x}_{391}=0$, $\bar{x}_{392}=0$, $\bar{x}_{3,10,1}=0$, $\bar{x}_{3,10,2}=0$.

The cost is: 153.
CPU time: 62.

The data for Buyer 4's combinatorial reverse auction are as follows:

$N_4=\{4,8\}, \quad J=2, \quad K=4$
$d_{41}=1, d_{42}=2, d_{43}=3, d_{44}=1.$

The four bids submitted by the two bidders are as follows:

$q_{411}=1, q_{412}=1, q_{413}=3, q_{414}=1,$
$q_{811}=0, q_{812}=1, q_{813}=0, q_{814}=0,$
$q_{421}=0, q_{422}=1, q_{423}=2, q_{424}=1,$
$q_{821}=0, q_{822}=2, q_{823}=0, q_{824}=0,$
$P_{41}=155, \quad P_{81}=16,$
$P_{42}=128, \quad P_{82}=43.$

By applying our algorithm, we find the solution: $\bar{x}_{441}=1$, $\bar{x}_{442}=0$, $\bar{x}_{481}=1$, $\bar{x}_{482}=0$.

The cost is: 171.
CPU time : 62.

The total cost of Model 1 is $f_I(\bar{x}_I)$ =176+220+153+171=720.

Model 2 for Example 1: A group buyer holds the combinatorial reverse auction for Buyer 1, Buyer 2, Buyer 3 and Buyer4.

The data for Group Buyer's combinatorial reverse auction are as follows:

$N=N_1 \cup N_2 \cup N_3 \cup N_4=\{1,2,3,4,5,6,7,8,9,10\},$
$J=2, \quad K=4, \quad I=4,$
$d_{11}=2, d_{12}=2, d_{13}=3, d_{14}=1,$
$d_{21}=1, d_{22}=2, d_{23}=2, d_{24}=3,$
$d_{31}=3, d_{32}=3, d_{33}=1, d_{34}=1,$
$d_{41}=1, d_{42}=2, d_{43}=3, d_{44}=1.$

The twenty bids submitted by the ten bidders are as follows:

$q_{111}=2, q_{112}=2, q_{113}=3, q_{114}=0,$
$q_{211}=0, q_{212}=2, q_{213}=2, q_{214}=3,$
$q_{311}=3, q_{312}=3, q_{313}=0, q_{314}=1,$
$q_{411}=1, q_{412}=1, q_{413}=3, q_{414}=1,$
$q_{511}=0, q_{512}=0, q_{513}=0, q_{514}=1,$
$q_{611}=1, q_{612}=0, q_{613}=0, q_{614}=0,$
$q_{711}=0, q_{712}=0, q_{713}=1, q_{714}=0,$
$q_{811}=0, q_{812}=1, q_{813}=0, q_{814}=0,$
$q_{911}=1, q_{912}=1, q_{913}=0, q_{914}=0,$
$q_{1011}=0, q_{1012}=0, q_{1013}=1, q_{1014}=1,$
$q_{121}=1, q_{122}=1, q_{123}=1, q_{124}=0,$
$q_{221}=0, q_{222}=1, q_{223}=1, q_{224}=1,$
$q_{321}=1, q_{322}=1, q_{323}=0, q_{324}=1,$
$q_{421}=0, q_{422}=1, q_{423}=2, q_{424}=1,$
$q_{521}=0, q_{522}=0, q_{523}=0, q_{524}=2,$
$q_{621}=2, q_{622}=0, q_{623}=0, q_{624}=0,$
$q_{721}=0, q_{722}=0, q_{723}=2, q_{724}=0,$
$q_{821}=0, q_{822}=2, q_{823}=0, q_{824}=0,$
$q_{921}=1, q_{922}=0, q_{923}=1, q_{924}=0,$
$q_{1021}=0, q_{1022}=1, q_{1023}=0, q_{1024}=1,$
$P_{11}=140, \quad P_{21}=212, \quad P_{31}=124, \quad P_{41}=155,$
$P_{51}=36,$
$P_{61}=8, \quad P_{71}=29, \quad P_{81}=16, \quad P_{91}=35,$
$P_{101}=78,$
$P_{12}=63, \quad P_{22}=95, \quad P_{32}=73, \quad P_{42}=128,$
$P_{52}=83,$
$P_{62}=21, \quad P_{72}=69, \quad P_{82}=43, \quad P_{92}=42,$
$P_{102}=65.$

By applying our algorithm, we find the solution: $\bar{x}_{g11}=1$, $\bar{x}_{g21}=1$, $\bar{x}_{g31}=1$, $\bar{x}_{g41}=1$, $\bar{x}_{g51}=1$, $\bar{x}_{g61}=1$, $\bar{x}_{g71}=1$, $\bar{x}_{g81}=1$, $\bar{x}_{g91}=0$, $\bar{x}_{g10,1}=0$, $\bar{x}_{g12}=0$, $\bar{x}_{g22}=0$, $\bar{x}_{g32}=0$, $\bar{x}_{g42}=0$, $\bar{x}_{g52}=0$, $\bar{x}_{g62}=0$, $\bar{x}_{g72}=0$, $\bar{x}_{g82}=0$, $\bar{x}_{g92}=0$, $\bar{x}_{g10,2}=0$.

For Model 2, the cost is $f_g(\bar{x}_g)$ =651.

CPU time : 62.

The benefit achieved by our algorithm for Model 2 is $f_I(\bar{x}_I) - f_g(\bar{x}_g)$ =720-651=69.

Based on the results of Model 1 and Model 2 for Example 1, we observe that the cost of the solution of Model 2 is lower than that of Model 1. The total CPU time required for finding the solution for Model 2 is 62 whereas the total CPU time required for finding the solutions for Model 1 is 62 + 47 + 62 + 62=233. For this example, Model 2 not only outperforms Model 1 but is also more efficient than Model 1.

In addition to Example 1, we also compare the cost of Model1 and Model2 for several cases. The duality gap for all these cases are no greater than 3% for Model 1 and Model 2. This indicates our algorithms generate acceptable approximate solutions for all these cases. The cost of Model 1 is one for each case whereas the cost of Model 2 is less than one for each case. This means the solutions of Model 2 are better than Model 1.

We also conduct several experiments to study the computational efficiency of our proposed algorithms for Model 1 and Model 2. These experiments shows the growth of CPU time with respect to I and N, respectively.

This result justifies the fact that the CPU time to compute $L(\lambda)$ for a given λ grows approximately linearly with respect to I for model 1 of combinatorial reverse auction.

Figure 1 indicates that the increase in the CPU time is not significant as parameter I is increased. This is consistent with our expectation. The CPU time required for solving Model 1 is significantly longer than Model 2.

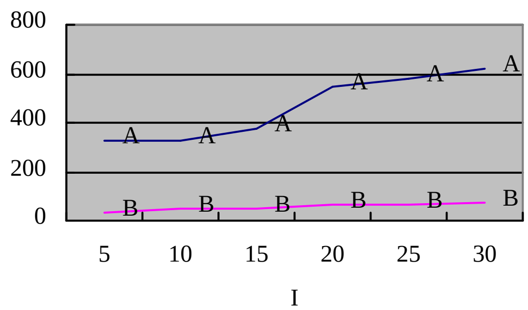

Figure 1 CPU time (in millisecond) respect to I.
A: Model1, B: Model2

6. CONCLUSION

In real world, multiple buyers may procure goods at the same time. If there are multiple buyers, group-buying may be applied to reduce the costs and benefit the buyers. The rationale of group-buying is due to demand aggregation, which benefits sellers, offering lower marketing costs and coordinated distribution channels, as well as buyers, who enjoy lower costs for product purchases. From the perspective of buyers, quantity based discounts provide a huge incentive to form coalitions and take advantage of lower prices without ordering more than their actual demand. Traditional group-buying mechanisms are usually based on a single item and uniform cost sharing. By holding a combinatorial reverse auction, it is possible to reduce the total cost to acquire the required items significantly due to complementarities between items. Therefore, combining group-buying with combinatorial reverse auctions has the potential advantage to achieve lower cost. However, combinatorial reverse auctions suffer from high computational complexity. In order to assess the advantage of combining group-buying with combinatorial reverse auctions, further study is required.

In this paper, we study two different combinatorial reverse auction models for multiple buyers. For multiple buyers that want to acquire goods by combinatorial reverse auctions, we consider two combinatorial reverse auction models: (1) Model 1: The buyers hold multiple combinatorial reverse auctions with each buyer holding a combinatorial reverse auction independently and (2) Model 2: The buyers delegate the combinatorial reverse auction to a group buyer and the group buyer holds only one combinatorial reverse auction for all the buyers. Our assessment of the advantage of combining group-buying with combinatorial reverse auctions is based on the comparative study of Model 1 and Model 2.

To compare the effectiveness of the two aforementioned combinatorial reverse auction models, we formulate two winner determination optimization problems for the two combinatorial reverse auction models and study the properties of the solutions for these two problems. By applying Lagrangian relaxation technique and subgradient method, the original optimization can be decomposed into a number of bidders' subproblems that can be solved efficiently and iteratively. Based on the proposed subgradient based algorithms for the two problems, we compare the efficiency and performance of the two combinatorial reverse auction models. Numerical results indicate that holding a combinatorial reverse auction based on group-buying yields better performance (lower cost) than holding multiple independent combinatorial reverse auctions. Moreover, the computational efficiency of combinatorial reverse auction based on group-buying is also significantly better than multiple independent combinatorial reverse auctions. The reduction in cost due to combinatorial reverse auction based on group-buying provides appropriate index to measure the value of introducing a group buyer. Our examples show significant reduction in cost due to group-buying. This justifies the value of the group buyer and encourages the combination of group-buying mechanicsm with combinatorial reverse auctions.

Acknowledgement

This paper is supported in part by National Science Council under Grant NSC97-2410-H-324-017-MY3.

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