Optimal replenishment policy to mitigate hi-tech products risks under declining market

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■ **Abstract**

The risks on supply chain have increased continuously in recent years. One such risk for Hi-tech products is the result of rapid technological innovation which results in a significant decline in the component cost, the selling price and the demand (due to newer products introduction). The Hi-tech products include computers and communication consumer's products. From a practical standpoint, a more realistic replenishment policy is needed to consider the impact of the risk, especially when the shortage is partially backordered. In this paper, an economic lot size model with partial backordering in a finite planning horizon is developed for a buyer when the component cost, the selling price, and the demand rate to the end-consumer decline at a continuous rate. A numerical example is provided to illustrate two replenishment models with fixed and varying replenishment intervals. Sensitivity analysis is carried out to investigate the relationship between the decision variables and some important parameters.

Keywords: Hi-tech products; innovation; replenishment policy; risks; partial backordering

1. Introduction

Hi-tech products have the following characteristics: shorter product life cycle time, quicker responsive time, increasing need for globalization and massive customization. Moreover, the component cost, the selling price and demand rate usually decrease with time due to global recession and technological innovation. In some Hi-tech industries such as computers and communication consumer products, some component costs and selling prices are declining at about 1% per week [1]. This implies that purchasing or selling one-week earlier or later will result in about 1% loss. Lee [2] has made some comments on the importance of the above subject.

Many researchers such as Lev and Weiss [3], Goyal [4] and Gascon [5] have studied the ordering policy in the classic economic order quantity (EOQ) model for both finite and infinite horizon. Buzacott [6] assumed compound increasing price and setup cost with inflation in a finite horizon. Buzacott [6] and Erel [7] considered a continuous price increase due to inflation. Dave and Patel [8] considered the inventory of deteriorating items with time proportional demand. The consideration of exponentially decreasing demand was first analyzed by Hollier and Mak [9] who relaxed the fixed replenishment cycle assumption in Dave and Patel, and obtained optimal replenishment policies under both constant and variable replenishment intervals. Erel [7] considered a compound-increasing price EOQ model with inflation rate. Hariga and Benkherouf [10] derived an optimal and heuristic inventory replenishment policy for deteriorating items with exponential time-varying demand. Wee [11] derived a joint pricing and replenishment policy for deteriorating inventory with declining market. Yang et al. [12] and Yang and Wee [13] investigated the collaborative pricing and replenishment policy for Hi-tech industry with constant and varying demand respectively. Yang et al. [14] developed an inventory model with continuous change of cost and price without considering shortage and varying replenishment interval.

In this paper, a replenishment policy with finite planning horizon is developed for a buyer when the component cost, the demand rate and the selling price to the end-consumer decline at a continuous rate. Two cases of fixed and varying replenishment intervals are considered. Two mathematical models and its solution procedure are developed in the next two sections. A numerical example is then provided to demonstrate the difference between the two cases. Sensitivity analysis is carried out to derive the

sensitivity for the net profit of some important parameters. The concluding remark is given in the last section.

2. Mathematical modeling and analysis

The mathematical model in this paper is developed on the basis of the following assumptions:

(a) The replenishment rate is instantaneous.
(b) Component cost and product selling price to the end consumer decline at a continuous rate per unit time.
(c) Demand rate is continuous and exponentially decreasing.
(d) Entire planning horizon is finite.
(e) Two cases of both identical and different replenishment intervals are considered.
(f) Shortages are allowed except for the initial and final cycles.
(g) A fraction of the shortages is backlogged while the rest is lost sale.
(h) Purchase lead-time is constant.
(i) The order quantity, inventory level and demand are treated as continuous variables, and the number of replenishment is treated as discrete variable.

The decision variables are

n integral number of orders in the entire planning horizon

T_{i-1} replenishment time during ith cycle, $i=1, 2\ldots n$

t_i time point when the inventory level of ith cycle drops to zero

T replenishment interval for the case of identical replenishment interval

r a fraction in each replenishment cycle with stocks, defined as service level

The other related parameters are as follows:

$d(t)$ annual demand rate, where $d(t) = a\exp(-bt)$, a is the scale parameter and b is the sensitive parameter of demand

$C(t)$ unit component cost, where $C(t) = C_0(1-r_c)^t$, C_0 is the unit component cost when $t=0$, r_c is the annual decline-rate of component cost

$S(t)$ unit selling price, where $S(t) = S_0(1-r_s)^t$, S_0 is the unit selling price when $t=0$, r_s is the annual decline-rate of selling price to the end-consumer

H length of the planning horizon by year

C_1 ordering cost per order

C_2 holding cost per unit per year

C_3 backlogged shortage cost per unit per year

C_4 lost sale shortage penalty cost per unit

B fraction of shortages backordered

NP net profit in the planning horizon

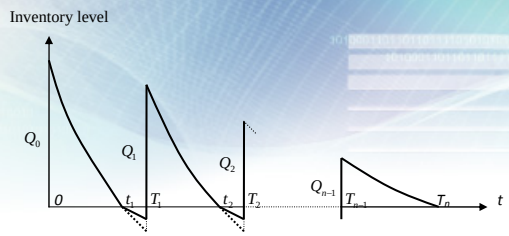

Figure 1. A graphical representation of inventory system with decreasing demand

A graphical representation of inventory system with decreasing demand is depicted in Figure 1. Without loss of generality, T_{i-1} ($i= 1, 2\ldots n$) are the replenishment times over the entire period H. Initial time ($T_0 = 0$) and final time ($T_n = H$) inventories are both zero. Inventory in ith cycle drops to zero at point t_i ($i= 1, 2\ldots n$). The purpose of this problem is to obtain optimal values of n, r and T_{i-1} such that the total net profit over the finite horizon is a maximum value.

Case A. For Fixed replenishment interval

For the case of fixed replenishment interval, the replenishment time can be expressed as

$$T = \frac{H}{n} \tag{1}$$

Since stock is depleted by demand, the differential equations of inventory levels during time intervals $[(i-1)T, (i-1+r)T]$ and $[(i-1+r)T, iT]$ are

$$\frac{dI(t)}{dt} = -ae^{-bt}, (i-1)T \le t \le (i-1+r)T \tag{2}$$

and

$$\frac{dI(t)}{dt} = -Bae^{-bt}, (i-1+r)T \le t \le iT \tag{3}$$

respectively.

Using the boundary condition $I(t) = 0$ when $t = (i-1+r)T$, the inventory levels during time intervals $[(i-1)T, (i-1+r)T]$ and $[(i-1+r)T, iT]$ are

$$I(t) = \frac{a}{b}(e^{-bt} - e^{-b(i-1+r)T}), (i-1)T \le t \le (i-1+r)T \tag{4}$$

and

$$I(t) = \frac{Ba}{b}(e^{-bt} - e^{-b(i-1+r)T}), (i-1+r)T \le t \le iT \tag{5}$$

respectively.

The lot size during ith cycle, Q_{i-1} when $t = (i-1)T$, is the filled demand during time interval $[(i-2+r)T, (i-1+r)T]$. It is expressed as follows:

$$Q_{i-1} = B\int_{(i-2+r)T}^{(i-1)T} ae^{-bt}dt + \int_{(i-1)T}^{(i-1+r)T} ae^{-bt}dt, i = 2, 3\ldots n-1 \tag{6}$$

The initial and final lot sizes are $Q_0 = \int_0^{rT} ae^{-bt}dt$ and $Q_{n-1} = B\int_{(n-2+r)T}^{(n-1)T} ae^{-bt}dt + \int_{(n-1)T}^{H} ae^{-bt}dt$ respectively.

During i[th] cycle, the annual holding cost per unit cost is assumed as a constant value C_2, the time-weighted inventory is $\int_{(i-1)T}^{(i-1+r)T} I(t)dt$ during time interval [$(i-1)T, (i-1+r)T$], and the holding cost, HC_i is

$$HC_i = C_2\int_{(i-1)T}^{(i-1+r)T} I(t)dt,\ i = 1, 2..n-1 \tag{7}$$

In the entire horizon, the holding cost is $HC = \sum_{i=1}^{n-1} HC_i + C_2\int_{(n-1)T}^{H} I(t)dt$.

During i[th] cycle, the annual backordering cost per unit is assumed as a constant value C_3, the time-weighted shortage is $\int_{(i-1+r)T}^{iT} I(t)dt$ during time interval [$(i-1+r)T, iT$], the backordering shortage cost, BC_i is

$$BC_i = -C_3\int_{(i-1+r)T}^{iT} I(t)dt \tag{8}$$

In the entire horizon, the backordering cost is $BC = \sum_{i=1}^{n-1} BC_i$.

During i[th] cycle, the lost sale penalty cost, LC_i is the unfilled demand during time interval [$(i-1+r)T, iT$]. That is

$$LC_i = C_4(1-B)\int_{(i-1+r)T}^{iT} d(t)dt \tag{9}$$

In the entire horizon, the lost sale cost is $LC = \sum_{i=1}^{n-1} LC_i$.

The component purchase cost during i[th] cycle, PC_i is the product of Q_{i-1} and unit component cost $C_0(1-r_c)^t$ at $t = (i-1)T$. One has

$$PC_i = Q_{i-1}C_0(1-r_c)^{(i-1)T},\ i = 1, 2...n \tag{10}$$

In the entire horizon, the purchase cost is $PC = \sum_{i=1}^{n} PC_i$

The sales revenue during i[th] cycle, SR_i is the integration of the product of the unit selling price and the filled demand quantity. One has

$$SR_i = \int_{(i-1)T}^{(i-1+r)T} S(t)d(t)dt + B\int_{(i-1+r)T}^{iT} S(t)d(t)dt \tag{11}$$

In the entire horizon, the sales revenue is $SR = \sum_{i=1}^{n-1} SR_i + \int_{(n-1)T}^{H} S(t)d(t)dt$.

The net profit, *NP* is expressed as:

$$NP(n,r) = SR - HC - BC - LC - PC - nC_1 \tag{12}$$
$$\textit{Subject to } (1) \textit{ and } 0 \le r \le 1$$

In (12), the term nC_1 is the setup cost in the entire horizon.

Using transformation from (1) for fixed replenishment interval, the net profit function in (12) has two independent decision variables: *n* and *r*.

Case B. For varying replenishment interval

For the case of varying replenishment interval, the relation among t_i, T_{i-1} and T_i is

$$t_i = (1-r)T_{i-1} + rT_i,\ i = 1, 2...n-1 \tag{13}$$

where the final value is $t_n = H$.

Since stock is depleted by demand, the differential equations of inventory levels during time intervals [T_{i-1}, t_i] and [t_i, T_i] are

$$\frac{dI(t)}{dt} = -ae^{-bt},\quad T_{i-1} \le t \le t_i \tag{14}$$

and

$$\frac{dI(t)}{dt} = -Bae^{-bt},\quad t_i \le t \le T_i \tag{15}$$

respectively.

Using the boundary condition $I(t) = 0$ when $t = t_1$, the inventory levels during time intervals [T_{i-1}, t_i] and [t_i, T_i] are

$$I(t) = \frac{a}{b}(e^{-bt} - e^{-bt_i}),\quad T_{i-1} \le t \le t_i \tag{16}$$

and

$$I(t) = \frac{aB}{b}(e^{-bt} - e^{-bt_i}),\quad t_i \le t \le T_i \tag{17}$$

respectively.

The lot size during i[th] cycle, Q_{i-1} when $t = T_{i-1}$, is the filled demand during time interval [t_{i-1}, t_i]. It is expressed as follows:

$$Q_{i-1} = B\int_{t_{i-1}}^{T_{i-1}} ae^{-bt}dt + \int_{T_{i-1}}^{t_i} ae^{-bt}dt,\ i = 2, 3...n-1 \tag{18}$$

The initial and final lot sizes are $Q_0 = \int_0^{rT} ae^{-bt}dt$ and

$Q_{n-1} = B\int_{(n-2+r)T}^{(n-1)T} ae^{-bt}dt + \int_{(n-1)T}^{H} ae^{-bt}dt$ respectively.

During i[th] cycle, the annual holding cost per unit cost is assumed as a constant value C_2, the time-weighted inventory is $\int_{T_{i-1}}^{t_i} I(t)dt$ during time interval $[T_{i-1}, t_i]$, and the holding cost, HC_i is

$$HC_i = C_2 \int_{T_{i-1}}^{t_i} I(t)dt,\ i = 1, 2...n-1 \tag{19}$$

In the entire horizon, the holding cost is $HC = \sum_{i=1}^{n-1} HC_i + C_2 \int_{(n-1)T}^{H} I(t)dt$.

During i[th] cycle, the annual backordering cost per unit is assumed as a constant value C_3, the time-weighted shortage is $\int_{t_i}^{T_i} I(t)dt$ during time interval $[t_i, T_i]$, the backordering shortage cost, BC_i is

$$BC_i = -C_3 \int_{t_i}^{T_i} I(t)dt \tag{20}$$

In the entire horizon, the backordering cost is $BC = \sum_{i=1}^{n-1} BC_i$.

During i[th] cycle, the lost sale penalty cost, LC_i is the unfilled demand during time interval $[t_i, T_i]$. That is

$$LC_i = C_4(1-B)\int_{t_i}^{T_i} d(t)dt \tag{21}$$

In the entire horizon, the lost sale penalty cost is $LC = \sum_{i=1}^{n-1} LC_i$.

The component purchase cost during i[th] cycle, PC_i is the product of Q_{i-1} and unit component cost $C_0(1-r_c)^t$ at $t = T_{i-1}$. One has

$$PC_i = Q_{i-1} C_0 (1-r_c)^{T_{i-1}} \tag{22}$$

In the whole horizon, the purchase cost is $PC = \sum_{i=1}^{n} PC_i$.

The sales revenue during i[th] cycle, SR_i is the integration of the product of the unit selling price and the filled demand quantity. One has

$$SR_i = \int_{T_{i-1}}^{t_i} S(t)d(t)dt + B\int_{t_i}^{T_i} S(t)d(t)dt,\ i = 1, 2...n-1 \tag{23}$$

$$SR_i = \int_{T_{i-1}}^{t_i} S(t)d(t)dt + B\int_{t_i}^{T_i} S(t)d(t)dt,\ i = 1, 2...n-1 \tag{23}$$

In the entire horizon, the sales revenue is $SR = \sum_{i=1}^{n-1} SR_i + \int_{(n-1)T}^{H} S(t)d(t)dt$

The net profit, *NP* is expressed as:

$$NP(n, r, T_i) = SR - HC - BC - LC - PC - nC_1 \tag{24}$$
$$\textit{Subject to } (13) \textit{ and } 0 \le r \le 1$$

Using transformation from (13) for varying replenishment interval, there are $n+1$ independent decision variables: *n*, *r* and T_i, where *i*=1, 2...*n*-1.

3. Solution procedure

Our aim is to derive the optimal values of the decision variables and maximize the total net profit in the entire planning horizon.

Case A. For fixed replenishment interval

The solution procedure is as follows:

Step 1. Let *n* be a fixed positive integer.

Step 2. Equate the first derivatives of *NP* in (12), denoted by $NP(r|n)$ with respect to *r* to zero and solve the *r* value. Check for concavity. The sufficient optimality condition for maximizing $NP(r|n)$ is $\frac{d^2 NP(r|n)}{dr^2} < 0$

(i) If $r > 1$ (example in Figure 2 and 3), let $r = 1$, then calculate NP in (12).
(ii) If $r < 0$ (example in Figure 4 and 5), let $r = 0$, then calculate NP in (12).
(iii) If $0 \le r \le 1$ (example in Figure 6 and 7), then calculate NP in (12).

Step 3. Repeat Step 1 to Step 2 by assuming different positive integer values of *n*. The optimal solution (n^*, r) must satisfy the following condition:

$$NP(n^*-1, r) \le NP(n^*, r) \ge NP(n^*+1, r) \tag{25}$$

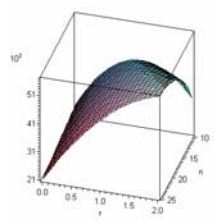

Figure 2. *NP*(*n*,*r*) vs. (*n* & *r*) when $C_3 = 80, C_4 = 120$ $B = 0.8, n = 10..25, r = 0..2,$

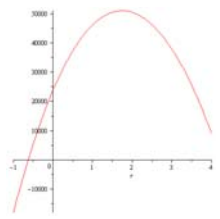

Figure 3. *NP*(*r*) vs. *r* when $C_3 = 80, C_4 = 120$ $B = 0.8, n = 17, r = -1..4$

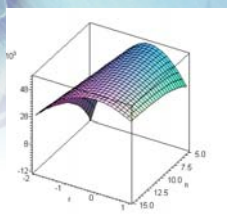

Figure 4. NP(n,r) vs. (n & r) when $C_3 = 40, C_4 = 50$ $B = 1, n = 5..15, r = -2..1$

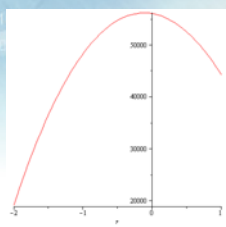

Figure 5. NP(r) vs r when $C_3 = 40, C_4 = 50$ $B = 1, n = 10, r = -2..1$

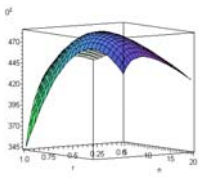

Figure 6. $NP(n,r)$ vs. (n & r) When $C_3 = 80, C_4 = 120$ $B = 0.95, n = 5..20, r = 0..1$

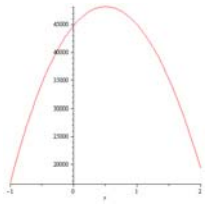

Figure 7. $NP(r)$ vs. r when $C_3 = 80, C_4 = 120$ $B = 0.95, n = 11, r = -1..2$

Case B. For varying replenishment interval

The solution procedure is as follows:

Step 1. Let the optimal value of Case A as the starting value of n. The net profit (24), denoted by $NP(T_1, T_2 ... T_{n-1}, r|n)$ has n decision variables with known n and $T_n = H$.

Step 2. Equate the first partial derivatives of the net profit (24) with respect to T_i.and r to zero as follows:

$$\frac{\partial NP(T_1, T_2 ... T_{n-1}, r|n)}{\partial T_i} = 0,\ i = 1, 2 ... n-1 \tag{26}$$

and

$$\frac{\partial NP(T_1, T_2 ... T_{n-1}, r|n)}{\partial r} = 0 \tag{27}$$

Step 3. Solve $(T_1, T_2 ... T_{n-1}, r)$ by using the n simultaneous equations from (26) and (27).

(i) If $r > 1$, let $r = 1$. Derive T_i and NP by solving the following n-1 simultaneous equations:

$$\frac{\partial NP(T_1, T_2 ... T_{n-1}|r, n)}{\partial T_i} = 0,\ i = 1, 2 ... n-1 \tag{28}$$

(ii) If $r < 0$, let $r = 0$. Derive T_i and NP by solving the n-1 simultaneous equations in (28).

(iii) If $0 \le r \le 1$, calculate NP.

Step 4. Repeat Step 1 through Step 3 by different values of n.

The optimal value of n, denoted by n^*, must satisfy the following condition:

$$NP(T_1, T_2 .. T_{n^*-2}, r|n^* - 1) \le NP(T_1, T_2 .. T_{n^*-1}, r|n^*) \ge NP(T_1, T_2 .. T_{n^*}, r|n^* + 1) \tag{29}$$

4. A numerical example

The preceding theory can be illustrated by the following numerical example. The parameters are given as follows: Unit component cost, $C = C_0(1 - r_c)^t$, where $C_0 = \$200$ and $r_c = 0.4$ per year (approximately, $r_c = 0.01$ per week); Unit selling price, $S = S_0(1 - r_s)^t$, where $S_0 = \$400$ and $r_s = 0.4$ per year; Entire Planning horizon considered, $H = 4$ years; Demand rate, $d(t) = a \exp(-bt)$, $0 \le t \le H$, a= 200, b= 0.1; Ordering cost per order, C_1= \$400; Holding cost per unit per year, C_2= 40, Backlogging cost per unit per year, C_3=80; Lost sale penalty cost. Per unit, C_4=120; Fraction of shortages backordered, B=0.9. Applying the solution procedure, the computational results are given as follows:

For Case A and B, the optimal replenishment times are both 17 in the planning horizon of 4 years. The net profits are \$46,255 and \$46,531.The percentage of net profit Increase (PNPI) between Case A and Case B is defined as:

PNPI = (NP for Case B - NP for Case A)/ NP for Case A

From Table 1 and Figure 8, Case B's net profit is larger than Case A's net profit; the value of PNPI is +0.60%. However, the computational process for Case A is easier than that for Case B.

The values for 17 replenishment cycles are shown in Table 2. With identical replenishment intervals in Case A, the lot size decreases with time due to decreasing demand. For Case B, the lot size increases with time because the effect of increasing replenishment time interval counteracts the effect of decreasing demand.

Table 1. Net profit with various replenishment times for Case A & B

n	NP for Case A (I)	NP for Case B (II)	PNPI (II - I)/(I)
11	\$45,861	\$46,038	0.39%
12	\$46,003	\$46,203	0.43%
13	\$46,104	\$46,325	0.48%
14	\$46,174	\$46,416	0.52%
15	\$46,219	\$46,481	0.57%
16	\$46,245	\$46,525	0.61%
17	\$46,255*	\$46,531*	0.60%
18	\$46,252	\$46,493	0.56%
19	\$46,169	\$46,415	0.53%
20	\$46,072	\$46,306	0.51%
21	\$45,946	\$46,169	0.49%
22	\$45,795	\$46,007	0.46%

Note: * Optimal solution

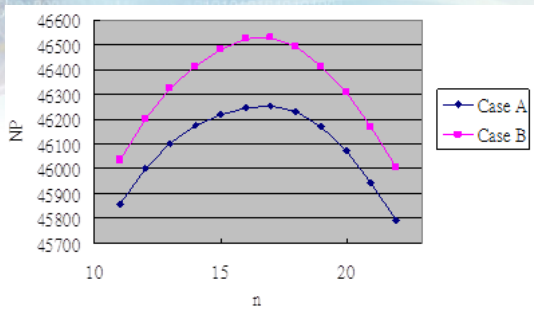

Figure 8. Net Profit with various *n* for Case A & B

Table 2. Results of the optimal replenishment cycles for Case A & B

i* cycle	Case A			Case B		
	T_i	Interval (t_i-t_{i-1})	Lot size Q_{i*}	T_i	Interval (t_i-t_{i-1})	Lot size Q_{i*}
1	0.2353	0.2353	46.5	0.1660	0.1660	32.9
2	0.4706	0.2353	45.4	0.3385	0.1725	33.6
3	0.7059	0.2353	44.4	0.5180	0.1795	34.4
4	0.9412	0.2353	43.3	0.7049	0.1869	35.2
5	1.1765	0.2353	42.3	0.8995	0.1946	35.9
6	1.4118	0.2353	41.3	1.1023	0.2028	36.7
7	1.6471	0.2353	40.4	1.3138	0.2135	37.5
8	1.8824	0.2353	39.4	1.5345	0.2207	38.3
9	2.1176	0.2353	38.5	1.7647	0.2302	39.0
10	2.3529	0.2353	37.6	2.0049	0.2402	39.8
11	2.5882	0.2353	36.8	2.2557	0.2508	40.5
12	2.8235	0.2353	35.9	2.5173	0.2616	41.2
13	3.0588	0.2353	35.1	2.7901	0.2728	41.9
14	3.2941	0.2353	34.3	3.0746	0.2845	42.4
15	3.5294	0.2353	33.5	3.3709	0.2963	42.9
16	3.7647	0.2353	32.7	3.6793	0.3084	43.4
17	4.0000	0.2353	31.9	4.0000	0.3207	43.7

5. Sensitivity analysis

If there exists decline-rates of component cost and product selling price, the net profits when (r_c, r_s) is considered compared with that when (r_c, r_s) is ignored are shown in Table 3. When the value of (r_c, r_s) increases from the basic value of 0.4 to $\pm 5\%$, $\pm 10\%$ and $\pm 15\%$, the percentage of net profit increase (PNPI) is derived. The range of PNPI value is from 2.13% to 4.06%. Since the PNPI is significant, the changes of (r_c, r_s) cannot be ignored. When decline-rates of (r_c, r_s) do exist and is considered, the number of replenishment becomes larger to keep lower component purchase cost compared with that when (r_c, r_s) is ignored. While, when the values of (r_c, r_s) decrease, the number of replenishments become larger to reduce the purchase and shortage costs.

Table 3. PNPI *when* (r_c, r_s) is considered for identical replenishment interval (*B*=0.9)

r_c, r_s	When (r_c, r_s) is ignored (r_c=r_s=0)			When (r_c, r_s) is considered			PNPI
	n	r	NP(I)	n	r	NP(II)	(II-I)/(I)
0.46	11	1	$37,421	15	0.8587	$38,941	4.06%
0.44	11	1	$39,836	16	0.9014	$41,273	3.61%
0.42	11	1	$42,341	17	0.9866	$43,709	3.23%
(0.40)	11	1	$44,941	17	1	$46,255	2.92%
0.38	11	1	$47,639	17	1	$48,880	2.61%
0.36	11	1	$50,439	17	1	$51,627	2.36%
0.34	11	1	$53345	17	1	$54,482	2.13%

Note: PNPI = [NP when (r_c, r_s) is considered - NP when (r_c, r_s) is ignored]/ NP when (r_c, r_s) is ignored

Optimal solutions for various *B* values ranging from 0 to 1 are shown in Table 4. The following observations are made from the results of the analysis:

(1) Under the condition of perfect competition (*B*=0), the optimal service level is 1 to reduce shortage cost.

(2) Under the condition of monopoly, the optimal service level and the number of replenishment times are much lower since the shortages can be backlogged completely.

When the value of decreases, the service level increases to reduce shortage cost.

(3) In order to keep the service level high, lot size and holding cost must increase.

(4) The higher service level, the more the number of replenishment. It makes sense that JIT deliveries increase service level.

(5) In perfect competition, the number of replenishments is large and the service level maintains at100% to reduce shortage.

(6) In monopoly, since the shortage will be backordered completely, the service level maintains lower and the number of replenishments is less.

(7) When $B \le 0.9$ (the lower bound of *B*) the optimal solution is to allow no shortage (i.e., *r*=1); this is due to higher shortage cost. An iterative approach to derive the value of the lower bound of *B* is described in Appendix A.

Table 4. Optimal solutions for various *B* values with identical replenishment interval

B	0	0.5	0.9	0.91	0.95	1
n	17	17	17	14	11	10
r	1	1	1	0.8162	0.5023	0.2570
T	0.2353	0.2353	0.2353	0.2857	0.3636	0.4000
SR	119,592	119,592	119,592	117,786	116,849	119,592
HC	3,091	3,091	3,091	2,580	1,485	759
BC	0	0	0	216	2,074	5,339
LC	0	0	0	1,217	1804	0
PC	63,447	63,447	63,447	61,816	58,871	57,281
nC₁	6,800	6,800	6,800	5,600	4,400	4,000
NP	46,255	46,255	46,255	46,357	48,216	52,214
Remarks	No shortage is allowed (in perfect competition)	Shortage partially backordered (50%)	Shortage partially backordered (90%)	Shortage partially backordered (91%)	Shortage partially backordered (95%)	Shortage completely backordered (in monopoly)

6. Concluding remarks

Models with partially backordered for fixed (i.e., Case A) and varying (i.e., Case B) replenishment intervals are developed to consider the risk of decreasing component cost, selling price and demand rate. The solution of Case A is sub-optimal, and Case B is optimal. The percentage of net profit increase is approximately 0.60%. However, the solution procedure and the computational process of Case A are easier than that of Case B.

If the component cost decline-rate decreases, the service level increases. It results in more frequent deliveries and less purchase cost. If the

selling price decline-rate decreases, the service level increases. It results in more frequent deliveries and less shortage cost. This study provides managerial insights to inventory practitioners in their replenishment planning.

In this study, the basic assumptions are instantaneous replenishment rate, exponentially decreasing demand, allowed shortage, and constant purchase lead time. For the case of finite replenishment rate, other patterns of varying demand and varying purchase lead time are left for further research.

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Appendix A. Search of lower bound of *B* value

The procedure of searching the lower of *B* is as follows:

Step 1. Set initial value *B*=1.

Step 2. Let *n* be a fixed value.

Step 3. Equate the first derivatives of $NP(r|n,B)$ with respect to *r* to zero and solve *r*. Check for concavity. The sufficient optimality condition for maximizing $NP(r|n,B)$ is $\frac{d^2NP(r|n,B)}{dr^2}<0$.

(i) If $r>1$, let $r=1$. Calculate NP in (12).

(ii) If $r<0$, let $r=0$. Calculate NP in (12).

(iii) If $0\le r\le 1$, Calculate NP in (12).

Step 4. Repeat Step 2 through Step 3 by different values of *n* until the following condition is satisfied.

$$NP(n^*-1|r,B)\le NP(n^*|r,B)\ge NP(n^*+1|r,B)$$

Step 5. Decrease the value of *B*. Repeat Step 2 through Step 4.

The lower bound of *B* occurs at the least value of *B* that satisfies the sufficient optimality condition in Step 3. When the actual fraction of *B* is not larger than the lower bound, no shortage is allowed (i.e., $r=1$).