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Optimal replenishment policy to mitigate hi-tech products risks under declining market

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Abstract

The risks on supply chain have increased continuously in recent years. One such risk for Hi-tech products is the result of rapid technological innovation which results in a significant decline in the component cost, the selling price and the demand (due to newer products introduction). The Hi-tech products include computers and communication consumer’s products. From a practical standpoint, a more realistic replenishment policy is needed to consider the impact of the risk, especially when the shortage is partially backordered. In this paper, an economic lot size model with partial backordering in a finite planning horizon is developed for a buyer when the component cost, the selling price, and the demand rate to the end-consumer decline at a continuous rate. A numerical example is provided to illustrate two replenishment models with fixed and varying replenishment intervals. Sensitivity analysis is carried out to investigate the relationship between the decision variables and some important parameters. Keywords: Hi-tech products; innovation; replenishment policy; risks; partial backordering

1. Introduction

Hi-tech products have the following characteristics: shorter product life cycle time, quicker responsive time, increasing need for globalization and massive customization. Moreover, the component cost, the selling price and demand rate usually decrease with time due to global recession and technological innovation. In some Hi-tech industries such as computers and communication consumer products, some component costs and selling prices are declining at about 1% per week [1]. This implies that purchasing or selling one-week earlier or later will result in about 1% loss. Lee [2] has made some comments on the importance of the above subject.


In this paper, a replenishment policy with finite planning horizon is developed for a buyer when the component cost, the demand rate and the selling price to the end-consumer decline at a continuous rate. Two cases of fixed and varying replenishment intervals are considered. Two mathematical models and its solution procedure are developed in the next two sections. A numerical example is then provided to demonstrate the difference between the two cases. Sensitivity analysis is carried out to derive the
sensitivity for the net profit of some important parameters. The concluding remark is given in the last section.

2. Mathematical modeling and analysis

The mathematical model in this paper is developed on the basis of the following assumptions:
(a) The replenishment rate is instantaneous.
(b) Component cost and product selling price to the end consumer decline at a continuous rate per unit time.
(c) Demand rate is continuous and exponentially decreasing.
(d) Entire planning horizon is finite.
(e) Two cases of both identical and different replenishment intervals are considered.
(f) Shortages are allowed except for the initial and final cycles.
(g) A fraction of the shortages is backlogged while the rest is lost sale.
(h) Purchase lead-time is constant.
(i) The order quantity, inventory level and demand are treated as continuous variables, and the number of replenishment is treated as discrete variable.

The decision variables are $n$ integral number of orders in the entire planning horizon $T_{i+1}$ replenishment time during $i$th cycle, $i=1, 2, \ldots, n$ $t_i$ time point when the inventory level of $i$th cycle drops to zero

$T$ replenishment interval for the case of identical replenishment interval $r$ a fraction in each replenishment cycle with stocks, defined as service level

The other related parameters are as follows:

$d(t)$ annual demand rate, where $d(t) = a \exp(-bt)$, $a$ is the scale parameter and $b$ is the sensitive parameter of demand

$C(t)$ unit component cost, where $C(t) = C_0(1-r_c)^t$, $C_0$ is the unit component cost when $t=0$, $r_c$ is the annual decline-rate of component cost

$S(t)$ unit selling price, where $S(t) = S_0(1-r_s)^t$, $S_0$ is the unit selling price when $t=0$, $r_s$ is the annual decline-rate of selling price to the end-consumer

$H$ length of the planning horizon by year

$C_1$ ordering cost per order

$C_2$ holding cost per unit per year

$C_3$ backlogged shortage cost per unit per year

$C_4$ lost sale shortage penalty cost per unit

$B$ fraction of shortages backordered

$NP$ net profit in the planning horizon

![Figure 1. A graphical representation of inventory system with decreasing demand](image)

A graphical representation of inventory system with decreasing demand is depicted in Figure 1. Without loss of generality, $T_{i+1}$ for $i=1, 2, \ldots, n$ are the replenishment times over the entire period $H$. Initial time ($T_0 = 0$) and final time ($T_n = H$) inventories are both zero. Inventory in $i$th cycle drops to zero at point $t_i$ for $i=1, 2, \ldots, n$. The purpose of this problem is to obtain optimal values of $n, r, T$ and $T_{i+1}$ such that the total net profit over the finite horizon is a maximum value.

Case A. For Fixed replenishment interval

For the case of fixed replenishment interval, the replenishment time can be expressed as

$$T = \frac{H}{n}$$

Since stock is depleted by demand, the differential equations of inventory levels during time intervals $[(i-1)T_i, (i-1+r)T_i]$ and $[(i-1+r)T_i, iT_i]$ are

$$\frac{dI(t)}{dt} = -ae^{-bt}, (i-1)T_i \leq t \leq (i-1+r)T_i$$

and

$$\frac{dI(t)}{dt} = -Be^{-at}, (i-1+r)T_i \leq t \leq iT_i$$

respectively.

Using the boundary condition $I(t) = 0$ when $t = (i-1+r)T_i$, the inventory levels during time intervals $[(i-1)T_i, (i-1+r)T_i]$ and $[(i-1+r)T_i, iT_i]$ are

$$I(t) = \frac{a}{b}(e^{-bt} - e^{-(i-1+r)bt}), (i-1)T_i \leq t \leq (i-1+r)T_i$$

and

$$I(t) = \frac{B}{b}(e^{-bt} - e^{-(i-1+r+1)bt}), (i-1+r)T_i \leq t \leq iT_i$$

respectively.

The lot size during $i$th cycle, $Q_{i-1}$ when $t = (i-1)T_i$, is the filled demand during time interval $[(i-2+r)T_i, (i-1+r)T_i]$. It is expressed as follows:

$$Q_{i-1} = B \int_{(i-2+r)T_i}^{(i-1+r)T_i} e^{-bt} dt + \sum_{i=0}^{i-1} e^{-bt} dt, i = 2, 3, \ldots, n - 1$$
The initial and final lot sizes are 
\[ Q_0 = \int_0^T a e^{-tu} dt \]
and 
\[ Q_{n-1} = B \int_{T-1}^T e^{-r_1 u} dt + \int_{T-1}^T a e^{-r_1 u} dt \] respectively.

During the \( i^{th} \) cycle, the annual holding cost per unit cost is assumed as a constant value \( C_2 \), the time-weighted inventory is \( \int_{i-1}^{i+1} I(t) dt \) during the time interval \( (i-1)T, (i-1+r)T \), and the holding cost, \( HC_i \), is
\[ HC_i = C_2 \int_{i-1}^{i+1} I(t) dt, i = 1, 2, n-1 \] (7)

In the entire horizon, the holding cost \( HC \) is
\[ HC = \sum_{i=1}^{n} HC_i + C_2 \int_{1}^{T} I(t) dt \] (8)

During the \( i^{th} \) cycle, the annual backordering cost per unit is assumed as a constant value \( C_3 \), the time-weighted shortage is \( \int_T^{T+T} I(t) dt \) during the time interval \( (i-1)T, iT \), the backordering shortage cost, \( BC_i \), is
\[ BC_i = -C_3 \int_{i-1}^{i} d(t) dt \] (9)

In the entire horizon, the backordering cost \( BC \) is
\[ BC = \sum_{i=1}^{T} BC_i \] (10)

During the \( i^{th} \) cycle, the lost sale penalty cost, \( LC_i \), is the unfilled demand during the time interval \( (i-1)T, iT \). That is
\[ LC_i = C_1 (1-B) \int_{i-1}^{i} d(t) dt \] (11)

In the entire horizon, the lost sales cost \( LC \) is
\[ LC = \sum_{i=1}^{T} LC_i \] (12)

The component purchase cost during the \( i^{th} \) cycle, \( PC_i \), is the product of \( Q_{i-1} \) and unit component cost \( C_4 (1-r_1) \) at \( t = (i-1)T \). One has
\[ PC_i = Q_{i-1} C_4 (1-r_1), i = 1, 2, n \] (13)

In the entire horizon, the purchase cost \( PC \) is
\[ PC = \sum_{i=1}^{n} PC_i \] (14)

The sales revenue during the \( i^{th} \) cycle, \( SR_i \), is the integration of the product of the unit selling price and the filled demand quantity. One has
\[ SR_i = \int_{T-1}^{T} S(t) d(t) dt + B \int_{T-1}^{T} S(t) d(t) dt \] (15)

In the entire horizon, the sales revenue \( SR \) is
\[ SR = \sum_{i=1}^{n} SR_i + \int_{T-1}^{T} S(t) d(t) dt \] (16)

The net profit, \( NP \), is expressed as:
\[ NP(n,r) = SR - HC - BC - LC - PC - nC_1 \] (17)

In (12), the term \( nC_1 \) is the setup cost in the entire horizon.

Using transformation from (1) for fixed replenishment interval, the net profit function in (12) has two independent decision variables: \( n \) and \( r \).

**Case B. For varying replenishment interval**

For the case of varying replenishment interval, the relation among \( t_0, T_{r-1} \) and \( T_i \) is
\[ t_i = (1-r)T_{r-1} + rT_{i-1}, i = 1, 2, n-1 \] (18)

where the final value is \( t_n = H \).

Since stock is depleted by demand, the differential equations of inventory levels during time intervals \( [T_{r-1}, T_i] \) and \( [T_i, T_{i+1}] \) are
\[ \frac{dI(t)}{dt} = -ae^{-rt}, \quad T_{r-1} \leq t \leq T_i \] (19)

and
\[ \frac{dI(t)}{dt} = -Bae^{-bt}, \quad T_i \leq t \leq T_{i+1} \] (20)

respectively.

Using the boundary condition \( I(t) = 0 \) when \( t = T_i \), the inventory levels during time intervals \( [T_{r-1}, T_i] \) and \( [T_i, T_{i+1}] \) are
\[ I(t) = \frac{a}{b} (e^{-rt} - e^{-bt}), \quad T_{r-1} \leq t \leq T_i \] (21)

and
\[ I(t) = \frac{aB}{b} (e^{-rt} - e^{-bt}), \quad T_i \leq t \leq T_{i+1} \] (22)

respectively.

The lot size during the \( i^{th} \) cycle, \( Q_i \), when \( t = T_{r-1} \), is the filled demand during time interval \( [T_{r-1}, T_i] \). It is expressed as follows:
\[ Q_{i-1} = B \int_{T_{r-1}}^{T_i} e^{-r_1 u} dt + \int_{T_{r-1}}^{T_i} a e^{-r_1 u} dt, i = 2, 3, n-1 \] (23)

The initial and final lot sizes are \( Q_0 = \int_0^T a e^{-tu} dt \) and
\[ Q_{n-1} = B \int_{T-2}^{T} e^{-ru} dt + \int_{T-2}^T a e^{-ru} dt \] respectively.
During i\textsuperscript{th} cycle, the annual holding cost per unit cost is assumed as a constant value \( C_r \), the time-weighted inventory is \( \int_{t_{i-1}}^{T_i} I(t)dt \) during time interval \([T_{i-1}, T_i]\), and the holding cost, \( HC_i \), is
\[
HC_i = C_r \int_{t_{i-1}}^{T_i} I(t)dt, \quad i = 1, \ldots, n - 1
\]
In the entire horizon, the holding cost is
\[
HC = \sum_{i=1}^{n} HC_i = C_r \int_{t_{i-1}}^{T_i} I(t)dt.
\]
During i\textsuperscript{th} cycle, the annual backordering cost per unit is assumed as a constant value \( C_r \), the time-weighted shortage is \( \int_{t_{i-1}}^{T_i} I(t)dt \) during time interval \([t_i, T_i]\), the backordering shortage cost, \( BC_i \), is
\[
BC_i = -C_r \int_{t_{i-1}}^{T_i} I(t)dt.
\]
In the entire horizon, the backordering cost is
\[
BC = \sum_{i=1}^{n} BC_i.
\]
During i\textsuperscript{th} cycle, the lost sale penalty cost, \( LC_i \), is the unfilled demand during time interval \([t_i, T_i]\). That is
\[
LC_i = C_r (1 - B) \int_{t_{i-1}}^{T_i} I(t)dt.
\]
In the entire horizon, the lost sale penalty cost is
\[
LC = \sum_{i=1}^{n} LC_i.
\]
The component purchase cost during i\textsuperscript{th} cycle, \( PC_i \), is the product of \( Q_{i,j} \) and unit component cost \( C_0 (1 - r)^j \) at \( t = T_{i,j} \). One has
\[
PC_i = Q_{i,j} C_0 (1 - r)^j.
\]
In the whole horizon, the purchase cost is
\[
PC = \sum_{i,j} PC_i.
\]
The sales revenue during i\textsuperscript{th} cycle, \( SR_i \), is the integration of the product of the unit selling price and the filled demand quantity. One has
\[
SR_i = \int_{t_{i-1}}^{T_i} S(t)I(t)dt + B \int_{t_{i-1}}^{T_i} S(t)I(t)dt, \quad i = 1, \ldots, n - 1
\]
In the entire horizon, the sales revenue is
\[
SR = \sum_{i=1}^{n} SR_i = \int_{t_{i-1}}^{T_i} S(t)I(t)dt.
\]
The net profit, \( NP \), is expressed as:
\[
NP(n, r, T_i) = SR - HC - BC - PC - nC_i
\]
Subject to (3) and \( 0 \leq r \leq 1 \).

Using transformation from (13) for varying replenishment interval, there are \( n + 1 \) independent decision variables: \( n, r \) and \( T_i \) where \( i = 1, \ldots, n - 1 \).

3. Solution procedure

Our aim is to derive the optimal values of the decision variables and maximize the total net profit in the entire planning horizon.

Case A. For fixed replenishment interval
The solution procedure is as follows:
Step 1. Let \( n \) be a fixed positive integer.
Step 2. Equate the first derivatives of \( NP \) in (12), denoted by \( \frac{dNP(r)}{dr} \) with respect to \( r \) to zero and solve the \( r \) value. Check for concavity. The sufficient optimality condition for maximizing \( NP(r) \) is
\[
\frac{dNP(r)}{dr} = 0,
\]
(i) If \( r > 1 \) (example in Figure 2 and 3), let \( r = 1 \), then calculate \( NP \) in (12).
(ii) If \( r < 0 \) (example in Figure 4 and 5), let \( r = 0 \), then calculate \( NP \) in (12).
(iii) If \( 0 \leq r \leq 1 \) (example in Figure 6 and 7), then calculate \( NP \) in (12).
Step 3. Repeat Step 1 to Step 2 by assuming different positive integer values of \( n \). The optimal solution \((n^*, r^*)\) must satisfy the following condition:
\[
NP(n^* - 1, r) \leq NP(n^*, r) \geq NP(n^* + 1, r)
\]
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Figure 4. NP(n, r) vs. (n & r) when
\( C_1 = 40, C_2 = 50 \)
\( B = 1, n = 5.15, r = -2.1 \)

Figure 5. NP(r) vs r when
\( C_1 = 40, C_2 = 50 \)
\( B = 1, n = 10, r = -2.1 \)

Figure 6. NP(n, r) vs. (n & r) when
\( C_1 = 80, C_2 = 120 \)
\( B = 0.95, n = 11, r = -1.2 \)

Figure 7. NP(r) vs r when
\( C_1 = 80, C_2 = 120 \)
\( B = 0.95, n = 5.20, r = 0.1 \)

Case B. For varying replenishment interval

The solution procedure is as follows:

Step 1. Let the optimal value of Case A as the starting value of \( n \). The net profit (24), denoted by \( \text{NP}(T, T_1, \ldots, T_{n-1}, r) \), has \( n \) decision variables with known \( n \) and \( T_0 = H \).

Step 2. Equate the first partial derivatives of the net profit (24) with respect to \( T_i \) and \( r \) to zero as follows:

\[
\frac{\partial \text{NP}(T, T_1, \ldots, T_{n-1}, r)}{\partial T_i} = 0, \quad i = 1, 2, \ldots, n-1
\]

(26)

and

\[
\frac{\partial \text{NP}(T, T_1, \ldots, T_{n-1}, r)}{\partial r} = 0
\]

(27)

Step 3. Solve \((T, T_1, \ldots, T_{n-1}, r)\) by using the \( n \) simultaneous equations from (26) and (27). (i) If \( r > 1 \), let \( r = 1 \). Derive \( T_i \) and NP by solving the following \( n-1 \) simultaneous equations:

\[
\frac{\partial \text{NP}(T, T_1, \ldots, T_{n-1}, r)}{\partial T_i} = 0, \quad i = 1, 2, \ldots, n-1
\]

(28)

(ii) If \( r < 0 \), let \( r = 0 \). Derive \( T_i \) and NP by solving the \( n-1 \) simultaneous equations in (28).

(iii) If \( 0 \leq r \leq 1 \), calculate NP.

Step 4. Repeat Step 1 through Step 3 by different values of \( n \).

The optimal value of \( n \), denoted by \( n^* \), must satisfy the following condition:

\[
\text{NP}(T, T_1, \ldots, r^*) - \text{NP}(T, T_1, \ldots, r) \geq \text{NP}(T, T_1, \ldots, r^* + 1)
\]

(29)

4. A numerical example

The preceding theory can be illustrated by the following numerical example. The parameters are given as follows: Unit component cost, \( C = C_0(1-r)^{y} \), where \( C_0 = $200 \) and \( r = 0.4 \) per year (approximately, \( r = 0.01 \) per week); Unit selling price, \( S = S_0(1-r)^{y} \), where \( S_0 = $400 \) and \( r = 0.4 \) per year; Entire planning horizon considered, \( H = 4 \) years; Demand rate, \( d(t) = a \exp(-bt) \), \( 0 \leq t \leq H \), \( a = 200 \), \( b = 0.1 \); Ordering cost per order, \( C_I = $400 \); Holding cost per unit per year, \( C_2 = 40 \); Backlogging cost per unit per year, \( C_3 = 80 \); Lost sale penalty cost. Per unit, \( C_4 = 120 \); Fraction of shortages backordered, \( B = 0.9 \). Applying the solution procedure, the computational results are given as follows:

For Case A and B, the optimal replenishment times are both 17 in the planning horizon of 4 years. The net profits are $46,255 and $46,531. The percentage of net profit increase (PNPI) between Case A and Case B is defined as:

\[
\text{PNPI} = \frac{(\text{NP for Case B} - \text{NP for Case A})}{\text{NP for Case A}}
\]

From Table 1 and Figure 8, Case B’s net profit is larger than Case A’s net profit; the value of PNPI is +0.60%. However, the computational process for Case A is easier than that for Case B.

The values for 17 replenishment cycles are shown in Table 2. With identical replenishment intervals in Case A, the lot size decreases with time due to decreasing demand. For Case B, the lot size increases with time because the effect of increasing replenishment time interval counteracts the effect of decreasing demand.

Table 1. Net profit with various replenishment times for Case A & B

<table>
<thead>
<tr>
<th>Case A</th>
<th>Case B</th>
<th>PNPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>NP for Case A</td>
<td>NP for Case B</td>
<td>(NP for Case B - NP for Case A) / NP for Case A</td>
</tr>
<tr>
<td>0.96%</td>
<td>1.00%</td>
<td>0.04%</td>
</tr>
<tr>
<td>1.02%</td>
<td>1.04%</td>
<td>0.02%</td>
</tr>
<tr>
<td>1.08%</td>
<td>1.10%</td>
<td>0.02%</td>
</tr>
<tr>
<td>1.14%</td>
<td>1.16%</td>
<td>0.02%</td>
</tr>
<tr>
<td>1.20%</td>
<td>1.22%</td>
<td>0.02%</td>
</tr>
<tr>
<td>1.26%</td>
<td>1.28%</td>
<td>0.02%</td>
</tr>
<tr>
<td>1.32%</td>
<td>1.34%</td>
<td>0.02%</td>
</tr>
<tr>
<td>1.38%</td>
<td>1.40%</td>
<td>0.02%</td>
</tr>
<tr>
<td>1.44%</td>
<td>1.46%</td>
<td>0.02%</td>
</tr>
<tr>
<td>1.50%</td>
<td>1.52%</td>
<td>0.02%</td>
</tr>
<tr>
<td>1.56%</td>
<td>1.58%</td>
<td>0.02%</td>
</tr>
<tr>
<td>1.62%</td>
<td>1.64%</td>
<td>0.02%</td>
</tr>
<tr>
<td>1.68%</td>
<td>1.70%</td>
<td>0.02%</td>
</tr>
<tr>
<td>1.74%</td>
<td>1.76%</td>
<td>0.02%</td>
</tr>
<tr>
<td>1.80%</td>
<td>1.82%</td>
<td>0.02%</td>
</tr>
<tr>
<td>1.86%</td>
<td>1.88%</td>
<td>0.02%</td>
</tr>
<tr>
<td>1.92%</td>
<td>1.94%</td>
<td>0.02%</td>
</tr>
<tr>
<td>1.98%</td>
<td>2.00%</td>
<td>0.02%</td>
</tr>
</tbody>
</table>
Figure 8. Net Profit with various \( n \) for Case A & B

Table 2. Results of the optimal replenishment cycles for Case A & B

<table>
<thead>
<tr>
<th>A</th>
<th>( r_{1} )</th>
<th>( Z_{1} )</th>
<th>( L_{1} )</th>
<th>( Z_{2} )</th>
<th>( L_{2} )</th>
<th>( \text{Optimal} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>3</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>4</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
</tbody>
</table>

5. Sensitivity analysis

If there exists decline-rates of component cost and product selling price, the net profits when \( (r_{c}, r_{s}) \) is considered compared with that when \( (r_{c}, r_{s}) \) is ignored are shown in Table 3. When the value of \( (r_{c}, r_{s}) \) increases from the basic value of 0.4 to \( \pm 5\% \), \( \pm 10\% \) and \( \pm 15\% \), the percentage of net profit increase (PNPI) is derived. The range of PNPI value is from 2.13\% to 4.06\%. Since the PNPI is significant, the changes of \( (r_{c}, r_{s}) \) cannot be ignored. When decline-rates of \( (r_{c}, r_{s}) \) do exist and is considered, the number of replenishment becomes larger to keep lower component purchase cost compared with that when \( (r_{c}, r_{s}) \) is ignored. While, when the values of \( (r_{c}, r_{s}) \) decrease, the number of replenishments become larger to reduce the purchase and shortage costs.

Table 3. PNPI when \( (r_{c}, r_{s}) \) is considered for identical replenishment interval \((B=0.9)\)

<table>
<thead>
<tr>
<th>( P_{c} )</th>
<th>0.40</th>
<th>0.44</th>
<th>0.45</th>
<th>0.46</th>
<th>0.47</th>
<th>0.48</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{s} )</td>
<td>0.05</td>
<td>0.10</td>
<td>0.15</td>
<td>0.20</td>
<td>0.25</td>
<td>0.30</td>
</tr>
<tr>
<td>( \text{PNPI} )</td>
<td>2.13%</td>
<td>4.06%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: PNPI = \(|\text{NP when } (r_{c}, r_{s})\text{ is considered} - \text{NP when } (r_{c}, r_{s})\text{ is ignored}| / \text{NP when } (r_{c}, r_{s})\text{ is ignored}\)

Optimal solutions for various \( B \) values ranging from 0 to 1 are shown in Table 4. The following observations are made from the results of the analysis:

1. Under the condition of perfect competition \((B=0)\), the optimal service level is 1 to reduce shortage cost.
2. Under the condition of monopoly, the optimal service level and the number of replenishment times are much lower since the shortages can be backlogged completely.
3. When the value of decreases, the service level increases to reduce shortage cost.
4. The higher service level, the more the number of replenishment. It makes sense that JIT deliveries increase service level.
5. In perfect competition, the number of replenishments is large and the service level maintains at 100\% to reduce shortage.
6. In monopoly, since the shortage will be backordered completely, the service level maintains lower and the number of replenishments is less.
7. When \( B \leq 0.9 \) (the lower bound of \( B \)) the optimal solution is to allow no shortage \((i.e., r=1)\); this is due to higher shortage cost. An iterative approach to derive the value of the lower bound of \( B \) is described in Appendix A.

Table 4. Optimal solutions for various \( B \) values with identical replenishment interval

<table>
<thead>
<tr>
<th>( B )</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_{1} )</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
<td>0.9</td>
</tr>
<tr>
<td>( Z_{2} )</td>
<td>0.6</td>
<td>0.4</td>
<td>0.2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( L_{1} )</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
<td>0.9</td>
</tr>
<tr>
<td>( L_{2} )</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
<td>0.9</td>
</tr>
</tbody>
</table>

6. Concluding remarks

Models with partially backordered for fixed \((i.e., \text{Case A})\) and varying \((i.e., \text{Case B})\) replenishment intervals are developed to consider the risk of decreasing component cost, selling price and demand rate. The solution of Case A is sub-optimal, and Case B is optimal. The percentage of net profit increase is approximately 0.60\%. However, the solution procedure and the computational process of Case A are easier than that of Case B.

If the component cost decline-rate decreases, the service level increases. It results in more frequent deliveries and less purchase cost. If the
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selling price decline-rate decreases, the service level increases. It results in more frequent deliveries and less shortage cost. This study provides managerial insights to inventory practitioners in their replenishment planning.

In this study, the basic assumptions are instantaneous replenishment rate, exponentially decreasing demand, allowed shortage, constant purchase lead time. For the case of finite replenishment rate, other patterns of varying demand and varying purchase lead time are left for further research.

References


Appendix A. Search of lower bound of B value

The procedure of searching the lower of B is as follows:

Step 1. Set initial value \( B = 1 \).

Step 2. Let \( n \) be a fixed value.

Step 3. Equate the first derivatives of \( NP(r_n, B) \) with respect to \( r \) to zero and solve \( r \). Check for concavity. The sufficient optimality condition for maximizing \( NP(r_n, B) \) is

\[
0 \leq \frac{d^2 NP(r_n, B)}{dr^2} < 0
\]

(i) If \( r > 1 \), let \( r = 1 \). Calculate \( NP \) in (12).

(ii) If \( r < 0 \), let \( r = 0 \). Calculate \( NP \) in (12).

(iii) If \( 0 \leq r \leq 1 \), Calculate \( NP \) in (12).

Step 4. Repeat Step 2 through Step 3 by different values of \( n \) until the following condition is satisfied.

\[
NP(r_n - 1, B) \leq NP(r_n, B) \leq NP(r_n + 1, B)
\]

Step 5. Decrease the value of \( B \). Repeat Step 2 through Step 4.

The lower bound of \( B \) occurs at the least value of \( B \) that satisfies the sufficient optimality condition in Step 3. When the actual fraction of \( B \) is not larger than the lower bound, no shortage is allowed (i.e., \( r = 1 \)).