TRADEOFFS IN DECISION MAKING: A SHIPPING CHOICE EXAMPLE

Xuchuan Yuan, National University of Singapore, Singapore, g0800755@nus.edu.sg
H. Brian Hwarng, National University of Singapore, Singapore, hwarng@nus.edu.sg

ABSTRACT

Decision makers often struggle with balancing the short-term gain and the long-term benefit. In bulk transport, carriers face a shipping choice problem, either as a conveyance shipping high-value goods (HVG) with high freight but uncertain supply, or as a newsvendor shipping low-value goods (LVG) subjected to demand uncertainty. We investigate the shipping choice under various situations based on the tradeoff analysis on each choice. The shipping choice and the associated waiting-time decision illustrate the tradeoff between the short-term profit and the long-term benefit in decision making.

Keywords: decision making, bulk transport, newsvendor, martingale model of forecast evolution, Bayesian updating

I. INTRODUCTION

Decision making in the real world means making tradeoffs based on the best information available. A typical tradeoff is between the short-term interest and the long-term benefit. For instance, in the area of investment, decision makers need to decide whether investing in assets with certain but low returns or waiting for the opportunities with high but uncertain payoffs. In the area of human capital development, the choice can be investing in raising existing workers' capabilities for long-term competitiveness or harvesting the short-term gains by cost-cutting measures like retrenchment. In the area of productivitiy, the choice can be building foundations to improve the firm's productivity through sustaining capabilities or focusing on quick fixes which guarantee no future.

To make a wise choice, decision makers need to balance the short-term interest and long-term benefit inherent in each choice. The cost and benefit associated with a choice depend on the specific management decision and situation. In this paper, we focus on a concrete choice problem in operations management, specifically, the shipping choice faced by a *carrier* in *bulk transport* under various situations. We consider the shipping choice problem both in a *single-period* situation and *multiple-period* situation. In a single-period situation, we concentrate on the one-time shipping choice between two goods, specifically, one with high profit and supply uncertainty v.s. the other with low profit and demand uncertainty. In a multiple-period situation where several shipping choices are made, we concentrate on the waiting-time decision in each period. We aim to investigate what will be the right (if not optimal) shipping decision that balances the short-term interest and long-term profit.

The structure of the paper is organized as follows. In section 2, we briefly introduce the shipping choice problem faced by a carrier in bulk transport. In section 3, we discuss the model and the assumptions used in the paper. In section 4, we focus on the shipping choice due to the supply uncertainty. In section 5, we concentrate on the impact of demand uncertainty on the shipping choice. In section 6, we investigate the shipping choice in terms of the waiting time decision in a multiple-period situation. Summary and managerial insight as well as future research directions are provided in section 7.

II. SHIPPING CHOICE PROBLEM IN BULK TRANSPORT

Bulk transport including ore, coal, cement, chamotte, steel, foodstuff, fertilizer and nonmetal mineral commands a large market share in the shipping industry, especially inland shipping. Bulk cargoes are transported mainly through *balk cargo ships* which are designed especially for particular bulk transport. Generally, bulk cargo ships are suitable for shipping a few types of goods with similar characteristics. During each trip, a cargo ship only carries one type of goods most of the time. Due to different characteristics of the goods, for the same distance and the same amount, the profit (or the freight) of each transportation is different depending on the type of goods. In general, the higher the value of the goods is, the higher the profit, in terms of per unit-distance, of the shipping transaction reaps. For example, the freight of steel (per unit-distance) is higher than that of ore. Ideally, the owner of the bulk cargo ship (denoted as the *carrier*) should choose only the most profitable goods for each shipping transaction. However, the transport demand of a particular goods, i.e., the business opportunity to the carrier, depends not only on the upstream supply but also on the downstream demand. As the shipping demand for different goods differ, the carrier, thus, faces the shipping choice decision that is to ensure the ship is deployed in an efficient and profitable way.

The business opportunities for high-value goods, termed as HVG, are somewhat limited and uncertain. They are limited as HVG with higher profit per unit-distance is preferred by all carriers; they are uncertain as the supply and the demand of HVG fluctuate due to economic cycles. Therefore, the carrier can not always choose to transport HVG when the ship is available. While the business opportunities for low-value goods, termed as LVG, are ample, the carrier can ship the goods whenever available. In short, the business opportunities for LVG are predictable.

The uncertain business opportunities for HVG imply that if the carrier wants to ship HVG for high profit, he may need to wait at the port, which incurs *waiting cost*. Although the business opportunities of LVG are present, the profit from shipping LVG is relatively small. Therefore, there exists a significant tradeoff between profitability of a shipping choice and the uncertainty of business opportunities, i.e., shipping HVG for high profit with uncertainty v.s. shipping LVG for low profit with certainty.

As the transport cycle is usually long in the shipping industry, ranging from several days in inland shipping to several months in ocean shipping, the carrier needs to make a wise choice before committing the shipment. The transaction practices also differ depending on the types of goods. For HVG transport, the cargo ship is generally used as a *conveyance* and the carrier collects a fixed amount freight, i.e., the carrier only faces the *supply uncertainty of HVG*. For LVG transport, the carrier acts as a *retailer* and he needs to buy the goods at the port of shipment and sell the goods at the destination, i.e., the carrier faces *demand uncertainty of LVG*. Relative to the complete transport cycle, the selling period of LVG is short. Thus, in the case of shipping LVG, the carrier actually acts as a *newsvendor* and it is not profitable for the carrier to carry any unsold goods back. Besides the unfulfilled shipping demand of LVG is usually lost due to long transport cycles.

Therefore, this paper intends to evaluate the tradeoff and offer insights on if the carrier should ship HVG or LVG, and when waiting is a wise choice.

III. THE MODEL

Based on the above description, we adopt a simple mathematical model to analyze the shipping choice problem faced by the carrier. The follow notations are used throughout the paper. The capacity of the carrier is denoted as K. The random variables X on the support $[\underline{x}, \bar{x}]$, where $\bar{x}$ may be infinity and $\underline{x} \geq 0$, with probability density function (PDF) and cumulative distribution function (CDF) as $\phi(\cdot)$ and $\Phi(\cdot)$ is used to denote the arrival time of the uncertain business opportunity of HVG. The random variable D with PDF and CDF as $f(\cdot)$ and $F(\cdot)$ is used to denote the uncertain demand of LVG. For simplicity, we assume the transport distances of HVG and LVG are the same (while the managerial insight is not affected). If the carrier waits for HVG, the waiting cost is $C(x)$, which is assumed to be increasing and convex (the convexity can be used to reflect the impatience of carrier due to waiting). The fixed profit of HVG is h per unit. Therefore, the profit of shipping HVG is determined and fixed as $\pi_h = hK$. The unit purchasing cost and the selling price of LVG are w and l respectively. The salvage value of unsold LVG is s per unit and $s \leq w$. The expected profit of shipping LVG can be solved from the traditional newsvendor model [1] as

$$\begin{aligned}\pi_l &= \max_{Q \leq K} \left\{ lE\min(Q, D) + sE(Q-D)^+ - wQ \right\} \\ &= lE\min(Q_l, D) + sE(Q_l - D)^+ - wQ_l \qquad (1)\end{aligned}$$

where $Q_l = \min(Q^*, K)$ and Q^* is the traditional newsvendor solution, which satisfies $F(Q^*) = \Pr(D \leq Q^*) = \dfrac{l-w}{l-s}$.

Comparing the profits of the two shipping choices, if $\pi_l \geq \pi_h$, the carrier will choose to ship LVG all the times. Therefore, to make the choice of shipping HVG admissible, we assume $\pi_h \geq \pi_l$. Thus, the profit difference is $\Delta = \pi_h - \pi_l \geq 0$. While if $\beta^{\bar{x}}\left(\pi_h - C(\bar{x})\right) \geq \pi_l$, where $0 < \beta \leq 1$ is the discount factor, the carrier will ship HVG for sure since the profit from shipping HVG in the worst situation is larger. Therefore, we assume $\pi_l \leq \pi_h \leq \pi_l \beta^{-\bar{x}} + C(\bar{x})$. Since the right hand side is increasing in $\bar{x}$, if $\bar{x}$ is larger enough, the right hand side of the above assumption can be always satisfied. For $\beta = 1$, i.e., we do not consider the discount effect and the waiting cost is linear as $C(x) = cx$, where c is the unit waiting cost, we assume $\pi_l \leq \pi_h \leq \pi_l + c\bar{x}$.

We define the strategy in terms of the shipping choice made at the beginning of the horizon (the time when the carrier is ready for transport) in each shipping cycle. There are three strategies, namely two pure strategies and a mixed strategy. The two pure strategies are shipping HVG only and shipping LVG only, denoted as H and L respectively for simplicity. The expected profits of strategy H are L are denoted as Π_H and Π_L. Clearly, $\Pi_H = \int_{\underline{x}}^{\bar{x}} \left(\pi_h - C(x)\right) f(x)dx$, and $\Pi_L = \pi_l$. The mixed strategy is a *threshold* policy controlled by a waiting-time decision y, denoted as $M(y)$, i.e., if the arrival of HVG comes on or before y, the carrier will ship HVG, otherwise ship LVG immediately at time y. Therefore, the expected profit of $M(y)$ is

$$\begin{aligned}\Pi_M(y) &= \int_{\underline{x}}^{y} \beta^x \left(\pi_h - C(x)\right) f(x)dx \\ &+ \int_{y}^{\bar{x}} \beta^y \left(\pi_l - C(y)\right) f(x)dx \qquad (2)\end{aligned}$$

Clearly, the two pure strategies are special mixed strategies, i.e., $\Pi_H = \Pi_M(\bar{x})$ and $\Pi_L = \Pi_M(0)$. The above formulation is based on the assumption that the carrier knows the arrival distributions of HVG and the demand of LVG, and the shipping choice is made statically. Thus, under this situation, the carrier concerns about what the optimal shipping choice will be.

The carrier may not know the exact arrival distribution of HVG, and a dynamic shipping strategy will be adopted, where the carrier will ship HVG if it comes or faces an optimal stopping problem, i.e., whether continuing to wait for HVG at the sacrifice of increased waiting cost or shipping LVG immediately. The tradeoff faced by the carrier is between the increased waiting cost and the updated arrival information about HVG. Therefore, the carrier needs to decide the optimal stopping time if HVG has not come yet.

Since the carrier is a newsvendor when shipping LVG, the demand information affects the order quantity. The carrier can update the demand information about LVG when waiting for HVG, or even strategically wait to update the demand information before loading LVG. Therefore, the question is with this capability, what should be a proper shipping decision.

The carrier may not know the exact arrival distribution of HVG, such as the parameter of the distribution. If the carrier has multiple shipping decisions, he can estimate the arrival distribution based on previous experience. Waiting-time decision during each transportation affects the estimation of the arrival distribution. Longer waiting time increase the chances of observing more realizations of the arrival of HVG, which will benefit the future decisions. However, waiting longer decreases the expected profit in each single transportation. Therefore, the optimal waiting-time decision should balance the tradeoff between the short-term profit and the long-term benefit.

IV. SUPPLY UNCERTAINTY AND SHIPPING CHOICE

In this section, we assume the demand distribution of LVG is static and fixed, i.e., we assume the demand distribution of LVG can be considered as i.i.d variables in every time point, and focus on the one-time choice problem due to the uncertain arrival time of HVG. We first assume the arrival distribution of X is known and derive the conditions for different choices. We then assume the arrival distribution of X is unknown, and a dynamic model is formulated to investigate the characteristic of the shipping decision.

The case with known arrival distribution

We assume a linear waiting cost, i.e., $C(x) = cx$ first. Assume the distribution of X is known with mean and variance as μ and σ^2 respectively. Therefore, the expected profit of strategy H is $\Pi_H = \int_{\underline{x}}^{\bar{x}} \beta^x (\pi_h - cx) f(x)dx$. Without discount effect, i.e., $\beta = 1$, the expected profit of strategy H is $\Pi_H = \pi_h - c\mu$, i.e., only the mean of the arrival distribution matters for a risk neutral carrier. Clearly, if $\Pi_H > \Pi_L$, it is optimal to adopt strategy H; otherwise, strategy L is a better choice. The expected profit of mixed strategy $M(y)$ is

$$\Pi_M(y) = \int_{\underline{x}}^{y} \beta^x \left(\pi_h - cx\right) f(x)dx + \int_{y}^{\bar{x}} \beta^y \left(\pi_l - cy\right) f(x)dx .$$

Intuitively, since $\pi_h \geq \pi_l$, there may exists an optimal waiting time where the expected profit of the mixed strategy is larger than any pure strategies. Therefore, it is interesting to see whether there exists an optimal waiting time in the mixed strategy.

To derive the optimal waiting time in the mixed strategy, we take the first order derivative of $\Pi_M(y)$ as

$$\Pi_M'(y) = \beta^y \left(1 - F(y)\right) G(y) \quad (3)$$

where $\lambda(y) = \dfrac{f(y)}{1 - F(y)}$ is the failure rate of X at y and $G(y) = \Delta\lambda(y) + \left(\pi_l - cy\right)\log\beta - c$.

From the above result, it turns out that the characteristic of the failure rate of the arrival distribution X plays an important role in the choice decision. We have the first order derivative $G'(y) = \Delta\lambda'(y) - c\log\beta$. We focus on three types of distributions in terms of the failure rate characteristics, namely increasing failure rate (IFR), decreasing failure rate (DFR) and constant failure rate (CFR). The following result summarizes the impact of the failure rate in the shipping choice:

Proposition 1. *For IFR and CFR distributions, the optimal choice is either strategy H or strategy L. For DFR distributions, if $G(\underline{x}) \geq 0$ and $G(\bar{x}) < 0$, there exists an optimal waiting time y^*, where the mixed strategy $M(y^*)$ is optimal.*

Proof: For IFR distributions, we have $G'(y) \geq 0$. Therefore, if $G(\underline{x}) \geq 0$, $\forall y \in [\underline{x}, \bar{x}], \Pi_M'(y) \geq 0$, the optimal choice is strategy H; if $G(\bar{x}) \leq 0$, $\forall y \in [\underline{x}, \bar{x}], \Pi_M'(y) \leq 0$, the optimal choice is strategy L; if $G(\underline{x}) < 0$ and $G(\bar{x}) > 0$, the optimal choice is the one with higher expected profit, denoted as $\max(\Pi_H, \Pi_L)$. For CFR distributions, $G'(y) = -c\log\beta \geq 0$ is constant. Therefore, the situation is the same as IFR distributions. Thus, there is no feasible mixed strategy which is optimal for IFR and CFR distributions.

For DFR distributions, if $\forall y \in [\underline{x}, \bar{x}], G'(y) \geq 0$, the situation is the same as IFR and CFR distributions; if $G(\underline{x}) \geq 0$ and $G(\bar{x}) < 0$, there exists y^*, such that $\Pi_M'(y) = 0$ and $\Pi_M(y)$ is unimodal, first increasing and then decreasing. Therefore, we can see the existence of an optimal mixed strategy only occurs under DFR distributions. ■

The critical role of the failure rate is not difficult to understand. The failure rate reflects the conditional probability of the arrival of HVG if it has not come yet. An IFR or CFR distribution means the conditional probability the HVG will arrive increases if it has not come yet. The expression $G(y)$ reflects the potential profit margin of waiting additional time given the condition that the business opportunity of HVG has not come yet at time y, since $\Delta\lambda(y)$ is the potential profit gain of shipping HVG, and $\left(\pi_l - cy\right)\log\beta - c$ is the potential loss due to additional waiting. Therefore, if $G(y)$ is always positive, it is profitable to wait for HVG. Otherwise, waiting is dominated by shipping LVG immediately. Under DFR distributions, the potential profit gain of shipping HVG $\Delta\lambda(y)$ will decrease, which may cause the potential profit margin of waiting $G(y)$ to decrease. Therefore, the optimality of mixed strategies only occurs under DFR distributions.

In sum, under IFR and CFR distributions, the carrier only needs to calculate the expected profit of each pure strategy and make the optimal choice; while under DFR, the choice may need further comparisons. There may exist an optimal waiting time, where the mixed strategy is optimal in terms of the expected profit. The following examples are special distributions to illustrate the impact of the failure rate of the arrival distribution on the choice decision.

Uniform distribution (IFR) We assume the arrival distribution is uniform in $[0,T]$ with $\lambda(y)=\frac{1}{T-y}$ which is increasing in $y\in[0,T)$. Therefore, we have the expected profit of strategy H as

$$\Pi_H=\begin{cases}\dfrac{(\pi_h+\frac{c}{\log\beta})(\beta^T-1)-cT\beta^T}{T\log\beta}, & \beta\in(0,1)\\ \pi_h-\dfrac{cT}{2}, & \beta=1\end{cases}\qquad(4)$$

which is decreasing in T, since the first order derivative is

$$\frac{\partial\Pi_H}{\partial T}=\begin{cases}\dfrac{\beta^T(\pi_h-cT)}{T}-\dfrac{\int_0^T\beta^x(\pi_h-cx)dx}{T^2}\le 0, & \beta\in(0,1)\\ -\dfrac{c}{2}<0, & \beta=1\end{cases}\qquad(5)$$

Therefore, if $\beta=1$, there exists a critical value $T_c=\frac{2\Delta}{c}$, such that if $T<T_c$, the optimal choice is strategy H; otherwise, strategy L is optimal. For $0<\beta<1$, using L'Hpital's rule, we have $\lim_{T\to\infty}\Pi_H=0$ and $\lim_{T\to 0}\Pi_H=\pi_h$, indicating there exists a critical value T_c, such that if $T<T_c$, the optimal choice is strategy H; otherwise, strategy L is optimal. In sum, if the arrival distribution of HVG is uniform, either shipping HVG or shipping LVG is optimal and there is no optimal mixed shipping strategy.

Exponential distribution (CFR) The exponential distribution with rate λ is CFR, since $\lambda(y)=\lambda$. The expected profit of strategy H is $\Pi_H=\int_0^\infty\beta^x(\pi_h-cx)\lambda e^{-\lambda x}dx$. To get a closed form result, we first assume there is no discount effect, i.e., $\beta=1$. So we have $\Pi_H=\pi_h-\frac{c}{\lambda}$ which is decreasing in λ, indicating, there exists a critical value $\lambda_{c1}=\frac{\Delta}{c}$, such that if $\lambda>\lambda_{c1}$, the optimal choice is strategy H; otherwise, strategy L is optimal. If the waiting cost can be ignored, i.e., c is relatively small compared with π_h, and the discount effect dominates the expected profit, we have $\Pi_H=\int_0^\infty\beta^x\pi_h\lambda e^{-\lambda x}dx=\frac{\pi_h\lambda}{\lambda-\log\beta}$, which indicating there exists a critical value $\lambda_{c2}=-\frac{\pi_l\log\beta}{\Delta}$, if $\lambda>\lambda_{c2}$, the optimal choice is strategy H; otherwise, strategy L is optimal.

Taking both discounting effect and waiting cost into consideration, we have

$$\begin{aligned}\Pi_H&=\int_0^\infty\beta^x(\pi_h-cx)\lambda e^{-\lambda x}dx\\&\le\min\left(\pi_h-\frac{c}{\lambda},\frac{\pi_h\lambda}{\lambda-\log\beta}\right)\end{aligned}\qquad(6)$$

Therefore, there exists a critical value $\lambda_c\ge\max(\lambda_{c1},\lambda_{c2})$, such that if $\lambda>\lambda_c$, the optimal choice is strategy H; otherwise, strategy L is optimal. Due to the memoryless property of exponential distribution, the conditional probability of the arrival of HVG does not depend on the past waiting time. A high failure rate λ indicates the probability of the arrival of HVG in the same length of waiting time is high, which indicates the carrier only needs to wait a short time at a small waiting cost, while with high probability the arrival of HVG will come. Thus, waiting for HVG is more profitable.

Pareto distribution (DFR) The Pareto distribution is given as $\Phi(x)=1-\left(\frac{x_m}{x}\right)^\alpha,\phi(x)=\frac{\alpha x_m^\alpha}{x^{\alpha+1}},x\in[x_m,\infty),x_m>0,\alpha>1$, which is DFR, since $\lambda(y)=\frac{\alpha}{y}$ is decreasing in $y\in[x_m,\infty)$. The Pareto distribution indicates, the arrival of HVG only occurs after a certain time x_m.

The expected profit of strategy H is $\Pi_H=\int_{x_m}^\infty\beta^x(\pi_h-cx)\frac{\alpha x_m^\alpha}{x^{\alpha+1}}dx$. Without discount effect ($\beta=1$), the expected value is

$$\Pi_H=\int_{x_m}^\infty(\pi_h-cx)\frac{\alpha x_m^\alpha}{x^{\alpha+1}}dx=\pi_h-\frac{c\alpha x_m}{\alpha-1}\qquad(7)$$

Therefore, for fixed $x_m<\frac{\Delta}{c}$, there exists a critical value $\alpha_c^{'}=\frac{\Delta}{\Delta-cx_m}$, such that under $\beta=1$, if $1<\alpha\le\alpha_c^{'}$, $\Pi_H\le\Pi_L$; otherwise, $\Pi_H>\Pi_L$. Incorporating the discount effect, we have a critical value $\alpha_c\ge\alpha_c^{'}$, such that if $1<\alpha\le\alpha_c$, $\Pi_H\le\Pi_L$; otherwise, $\Pi_H>\Pi_L$. While if $x_m\ge\frac{\Delta}{c}$, we have $\forall\alpha>1$, $\Pi_H<\Pi_L$. Therefore, if the time x_m when the arrival of HVG will not come with certainty is long enough, strategy H is always dominated by strategy L.

For mixed strategy $M(y),y\ge x_m$, we have $G(y)=\Delta\frac{\alpha}{y}+(\pi_l-cy)\log\beta-c$ and $G^{'}(y)=-\frac{\Delta\alpha}{y^2}-c\log\beta$. Therefore, if $G^{'}(x_m)=-\frac{\Delta\alpha}{x_m^2}-c\log\beta\ge 0$, we have $\forall y\ge x_m,G^{'}(y)\ge 0$, and the optimal choice is $\max(\Pi_H,\Pi_L)$;

while if $G^{'}(x_m) = -\frac{\Delta\alpha}{x_m^2} - c\log\beta < 0$, since $G^{'}(y)$ is increasing, $G^{'}(y)$ is eventually positive, and $G(y)$ is increasing and positive, which indicates $\Pi_M(y)$ is increasing. We have $\lim_{y\to\infty}\Pi_M(y) = \Pi_H$. Therefore, the optimal choice is still $\max(\Pi_H, \Pi_L)$.

From the above examples, we can see under the linear waiting cost, the arrival distribution of HVG is critical in the shipping choice decision. Under IFR and CFR distributions, the carrier either waits for HVG or ships LVG immediately. Only under DFR distributions, there may exist an optimal mixed shipping strategy.

The convex curvature of the waiting cost may also impact the choice decision. However, the existence of the discount effect makes the analysis complicated. To simplify the burden of calculation, we assume $\beta = 1$ in the following section, which does not affect the managerial insight. We assume the waiting cost is increasing and convex as $C(x) = cx^2$ (the convex curvature can be used to incorporate the discount effect.). The expected profit of strategy H is $\Pi_H = \int_{\underline{x}}^{\overline{x}}(\pi_h - cx^2)f(x)dx = \pi_h - cE(X^2) = \pi_h - c(\mu^2 + \sigma^2)$. Therefore, the expected profit of strategy H depends on the mean and the variance of the arrival distribution of HVG. If we know the mean and variance of the arrival distribution, the comparison between the two pure strategies is easily to be determined. For example, for exponential distribution with rate λ, the expected profit of shipping HVG is $\Pi_H = \pi_h - \frac{2c}{\lambda^2}$. Thus, if $\lambda > \lambda_c = \sqrt{\frac{2c}{\Delta}}$, strategy H has a larger expected profit; otherwise, the expected profit of strategy L is larger.

For mixed strategy $M(y)$, we have the expected profit and the first order derivative with respect to the waiting time y as

$$\Pi_M(y) = \int_{\underline{x}}^{y}(\Pi_h - cx^2)f(x)dx + \int_{y}^{\overline{x}}(\Pi_l - cy^2)f(x)dx \qquad (8)$$

$$\Pi_M^{'}(y) = (1 - F(y))(\Delta\lambda(y) - 2cy) \qquad (9)$$

Based on the above assumptions, i.e., $C(x) = cx^2$ and $\beta = 1$, we can get the following result:

Proposition 2. *For CFR and DFR distributions, the optimal waiting time* $y^?$ *(if exists) satisfies* $\Delta\lambda(y^?) = 2cy^?$. *For IFR distributions, there may not exist an optimal waiting time* $y^?$.

Proof: It is obvious for CFR and DFR distributions, since $\Delta\lambda^{'}(y) - 2c < 0$ and there exists only one $y^?$ (if exists), such that $\Delta\lambda(y^?) = 2cy^?$. For IFR distributions, $\lambda^{'}(y) \geq 0$. Thus, there may exist several or none solutions that $\Delta\lambda(y) - 2cy = 0$. For convex $\lambda(y)$, $\Delta\lambda(y) - 2cy$ is also convex. Therefore, if $(\Delta\lambda(\underline{x}) - 2c\underline{x}) < 0, (\Delta\lambda(\overline{x}) - 2c\overline{x}) > 0$, the optimal choice is $\max(\Pi_H, \Pi_L)$; while if $(\Delta\lambda(\underline{x}) - 2c\underline{x}) > 0, (\Delta\lambda(\overline{x}) - 2c\overline{x}) < 0$, there exists an optimal waiting time y^*; otherwise, the optimal choice is $\max(\Pi_H, \Pi_L)$. For concave failure rate $\lambda(y)$, $\Delta\lambda(y) - 2cy$ is also concave. Therefore, if $(\Delta\lambda(\underline{x}) - 2c\underline{x}) < 0, (\Delta\lambda(\overline{x}) - 2c\overline{x}) > 0$, the optimal choice is $\max(\Pi_H, \Pi_L)$; while if $(\Delta\lambda(\underline{x}) - 2c\underline{x}) > 0, (\Delta\lambda(\overline{x}) - 2c\overline{x}) < 0$, there exists an optimal waiting time y^*; otherwise, the optimal choice is $\max(\Pi_H, \Pi_L)$. Therefore, one of the sufficient conditions for the existence of an optimal waiting time y^* is $(\Delta\lambda(\underline{x}) - 2c\underline{x}) > 0, (\Delta\lambda(\overline{x}) - 2c\overline{x}) < 0$. ■

For uniform distribution, the failure rate $\lambda(y) = \frac{1}{T - y}$ is convex. Therefore, if $\frac{2\Delta}{T^2} \geq c$, we have $\forall y \in [0, T), \Delta\lambda(y) - 2cy = \frac{\Delta}{T - y} - 2cy \geq 0$, indicating the optimal choice is strategy H. If $\frac{2\Delta}{T^2} < c$, there exist two points y_1, y_2 satisfying $\Delta\lambda(y) - 2cy = 0$, indicating the optimal choice is $\max(\Pi_M(y_1), \Pi_H)$. For exponential distribution with failure rate λ, there always exists an optimal waiting time $y^* = \frac{\Delta\lambda}{2c}$. For Pareto distribution, the optimal waiting time is $y^* = \sqrt{\frac{\Delta\alpha}{2c}}$ if $y^* \geq x_m$; otherwise, the optimal choice is Π_L.

Compared with the linear waiting cost, we can see the convex waiting cost changes the optimal shipping choice decision to some extent. For example, under CFR distributions, such as the exponential distribution with rate λ in $[0, \infty)$ without discount, for linear waiting cost, the existence of an optimal mixed strategy depends on the comparison between $\Delta\lambda$ and c, either Π_H or Π_L is optimal; while for convex waiting cost, there exists an optimal waiting time $y^?$ and the mixed strategy $\Pi_M(y^*)$ is optimal. The fundamental reason here is due to the fact that, for small time, the convex waiting cost is less than the linear waiting cost; while for large time, the convex waiting cost increases faster and is larger than linear waiting cost. Therefore, under convex waiting cost, waiting for some time becomes attracting even it is not optimal under linear waiting cost.

In sum, from the above result, we can see if the arrival distribution of HVG is known, the shipping choice faced by the carrier can be easily determined statically. Whenever the carrier is available at the port of shipment, based on the waiting cost, discount factor and the profit difference, the carrier can decide to ship HVG only (even incurs waiting cost), ship LVG immediately without any waiting cost, or adopt a mixed policy, i.e., waiting for a threshold time and ship HVG if it comes, otherwise ship LVG immediately at the threshold time.

The case with unknown arrival distribution

The result in the above section indicates with known arrival distribution of HVG, the shipping choice can be easily made. The known distribution of X is an ideal situation, since in practice, the business opportunity of HVG depends on various factors, and even there exists an experimental distribution about the arrival information of HVG, the actual arrival of HVG may not be easily estimated or forecasted, i.e., the exact information about the arrival information of HVG is not easy to extract. Therefore, the carrier suffers from the risk due to the lack of information about the arrival of HVG if making the shipping choice decision statically. A dynamic decision making process is needed for the carrier to make wise shipping choice in a one-period model.

In this section, we are interested in the situation when the arrival distribution of HVG is unknown, what will be the optimal shipping strategy? Since the arrival distribution of HVG is unknown, the expected value of shipping HVG can not be evaluated. Thus, the carrier only needs to compare the expected profit of shipping LVG and the mixed strategy. There are N stages (or days) for the carrier to make the shipping choice. At the beginning of each stage $n = 1, \Lambda, N$, the carrier will ship HVG if the business opportunity comes; otherwise, he needs to decide whether shipping LVG immediately or waiting for HVG in the next stage. For simplicity, we assume the waiting cost is linear and there is no discount effect. Clearly, we need $\pi_h - cN \geq 0$ and $\pi_h - c(N+1) < 0$, i.e., $N = \left[\frac{\pi_h}{c}\right]$, where $[\cdot]$ takes the integer part from below. Therefore, beyond stage N, there is no profit to ship any goods. Therefore, the dynamic shipping choice making is a typical optimal stopping problem, i.e., what should be the optimal stopping strategy for the carrier to ship LVG after a certain stage when the arrival of HVG has not come yet.

In practice, the business opportunity of shipping HVG depends on the supply and the demand. The higher downstream demand, the more upstream supply, and the more shipping demand thus more business opportunity for the carrier. Therefore, more supply and demand about HVG implies higher probability that the business opportunity of HVG will arrive. There may be other information or events related to the arrival of HVG. In this section, we define the *information* used in the model as all the events related to the arrival of HVG, and we assume the more information in the current stage, the higher probability of the arrival of HVG occurs in the next stage. Thus, the carrier can use the information to decide whether to continue waiting for HVG or shipping LVG immediately. However, the carrier faces the tradeoff between increased waiting cost and the potential profit gain of shipping HVG. This optimal stopping decision is modeled using a dynamic programming formula. The state variable is the information about the arrival of HVG, denoted as I. We define $V_n(I_n)$ as the expected profit of shipping HVG only from period n to period N, when the current information state is I_n and HVG is not available. The dynamic programming formulation is given as

$$V_n(I_n) = E\overline{V}_{n+1}(I_{n+1}), V_{N+1}(I_{N+1}) = 0 \qquad (10)$$

where $\overline{V}_{n+1}(I_{n+1}) = p(I_n)(\pi_h - cn) + (1 - p(I_n))V_{n+1}(I_{n+1})$ and $p(I_n)$ is the probability that shipping opportunity of HVG arrives at stage $n+1$, which is increasing in I_n. The boundary value $V_{N+1}(I_{N+1}) = 0$ indicates at stage $N+1$, if HVG is still not available, the carrier will not carry any goods. The state transition function is $I_{n+1} = I_n + \varepsilon_{n+1}$, where ε_{n+1} is the information gathered between stage n and $n+1$, which is random. Therefore, the dynamic programming model is simplified as

$$V_n(I_n) = p(I_n)(\pi_h - cn) + (1 - p(I_n))\int V_{n+1}(I_n + \varepsilon)d\varepsilon \qquad (11)$$

A closed form solution of the value function is not meaningful here. Therefore we investigate the structure property of the stopping decision. Intuitively, the more information, the higher probability of the arrival of HVG in the next stage, and waiting for the arrival of HVG is more attractive. However, since the waiting cost is increasing, the attractiveness of waiting for HVG may be offset. Indeed, we have the following result in terms of the relationship between the waiting decision and the information state in each stage:

Proposition 3. *There exist decreasing information states* $I_1^* \geq I_2^* \geq \Lambda \geq I_N^*$ *when the stage is in* $1, \Lambda, N$*, such that at each stage, shipping HVG if it appears; otherwise, waiting for the next period, if* $I_n \geq I_n^*$.

Proof: We prove the value function $V_n(I_n)$ is increasing (in the weak sense) in I_n using induction. At each stage n, we have $\pi_h - cn > 0$, and $V_N(I_N) = E\overline{V}_{N+1}(I_{N+1}) = p(I_N)(\pi_h - cN)$ which is increasing in I_N. Assume $V_n(I_n)$ is increasing in I_n. Thus, $\overline{V}_n(I_n)$ is increasing in I_n, since the first order derivative in I_n is $p'(I_n)\left(\pi_h - cn - \int V_{n+1}(I_n + \varepsilon)d\varepsilon\right) + (1 - p(I_n))\int V'_{n+1}(I_n + \varepsilon)d\varepsilon \geq 0$, due to the fact that shipping HVG at the current stage is always better than carrying it in the future, i.e., $\pi_h - cn - \int V_{n+1}(I_n + \varepsilon)d\varepsilon \geq 0$ and $V'_{n+1}(I_n + \varepsilon) \geq 0$ based on the inductive assumption. Thus, $V_{n-1}(I_{n-1})$ is also increasing in I_{n-1}.

Clearly, $V_n(I_n)$ is decreasing in n, i.e., for the same information, waiting more time is not beneficial, since the waiting cost is increasing in n. It is clear that $V_n(I) - V_{n+1}(I) = cp(I)$, since with the same information, $V_n(I)$ can save $cp(I)$ if HVG is available in period $n+1$.

At stage n, the profit from shipping LVG is $\pi_l - c(n-1)$. Thus, it is optimal to wait for HVG, if $V_n(I_n) \geq \pi_l - c(n-1)$. Define the critical information state as

$I_n^? = \arg_I \{I \mid V_n(I) - \pi_l + c(n-1) = 0\}$, which indicates, if $I_n \geq I_n^*$, waiting for HVG in the next stage, since $V_n(I_n) \geq \pi_l - c(n-1)$ due to the increasing property of $V_n(I_n)$; otherwise shipping LVG is optimal. We also have $V_{n+1}(I_n^*) - \pi_l + cn = V_n(I_n^*) - p(I_n^*)c - \pi_l + cn = V_n(I_n^*) - \pi_l + c(n-1) + (1 - p(I_n^*))c \geq 0$, while the critical information state at stage $n+1$ satisfies $V_{n+1}(I_{n+1}^*) - \pi_l + cn = 0$. Therefore, we have $I_{n+1}^* \leq I_n^*$ since $V_{n+1}(I)$ is increasing in I. Thus, the optimal stopping strategy has the following form: there are decreasing information states $I_1^* \geq I_2^* \geq \Lambda \geq I_N^*$, such that at stage n, carrying HVG if it arrives, otherwise, waiting for the next stage if $I_n \geq I_n^*$. ■

The conclusion can be generalized to increasing and convex waiting cost. The above result indicates at stage n, the carrier should wait additional time if the arrival of HVG is not available in current stage and the information state is beyond a critical point, i.e., the probability that HVG will arrive in the next stage is larger than a critical point. Interestingly, the critical information state which defines the attractiveness of waiting is decreasing when the stage increases. The result seems counter-intuitive, since after waiting some stages, the carrier should be more impatient. Therefore in order for him to wait longer, there should be more information and a higher probability of business opportunities of shipping HVG in the next stage. The result can be understood from a sunk-cost perspective. After the carrier has waited some stages, he has already born some waiting cost, which is the sunk cost to him. Therefore, the attractiveness of shipping LVG immediately decreases, since the carrier is reluctant to give up the waiting cost, and the attractiveness of waiting for HVG increases, which is reflected from the decreasing critical information state.

The above structural property of the stopping decision in the shipping choice problem can be used to guide the decisions on waiting in practice when the arrival distribution of HVG is unknown. On the one hand, at early stages, waiting for HVG is worthy unless the carrier can be sure that the opportunity of HVG will come in the next stage with high probability; otherwise, he should ship LVG immediately in order to save the sunk waiting cost. On the other hand, if the carrier has waited several stages, he should be more patient to wait longer for the arrival of HVG.

V. DEMAND UNCERTAINTY AND SHIPPING CHOICE

In this section, we focus on the impact of demand uncertainty of LVG on the shipping choice. When shipping LVG, the carrier actually serves as a newsvendor and he needs to decide the optimal order quantity facing the uncertain demand. The carrier may have inaccurate demand information, such as the mean and the variance of the demand distribution. Since the more accurate of the information, the more expected profit of shipping LVG, the carrier may strategically wait some time to collect and update the demand information of LVG. We try to investigate with the ability of demand information updating of LVG, what will be the optimal shipping choice.

We assume the arrival distribution of HVG is static and fixed as exponential distribution with rate λ (Thus, the arrival process is Poisson Process with rate λ.). Thus, due to the memoryless property of exponential distribution, the conditional probability of the business opportunity of HVG arriving in the time interval $[t, t+\delta t]$ is the same as in $[0, \delta t]$. For simplicity, we assume the waiting cost is linear and no discount effect. There are N stages from 1 to N for the carrier to make the shipping decision. The expected profit of waiting for HVG at the beginning of stage n is $\Pi_H(n) = \pi_h - \dfrac{c}{\lambda} - c(n-1)$.

During the waiting, the carrier can forecast and update the demand information of LVG and update the expected profit of shipping LVG by ordering an optimal quantity. The tradeoff is between the increased waiting cost versus the benefit from increased demand information. We assume there is no capacity constraint and the demand information updating is modeled using MMFE (martingale model of forecast evolution) developed by [2,3]. There are several papers focusing on the impact of forecast updating on the inventory problem, especially in the newsvendor context, including two-period model [4,5] and multi-stage inventory systems [6]. For the recent literature review about this topic, see [7], which considers the optimal order strategy in a multi-stage newsvendor model, where the newsvendor can dynamically make orders from a portfolio of suppliers with different lead times and procurement costs. A state-dependent base-stock policy is derived to be optimal based on MMFE in the paper.
There are two alternative models in MMFE: additive or multiplicative depending on the assumption of the forecast update distribution. We use the additive version of MMFE here. The forecast update from stage $n-1$ to n is ε_n, which is distributed as $N(0, \sigma_n^2)$. The information at stage n is $I_n = \sum_{i=2}^{n} \varepsilon_i, I_{n+1} = I_n + \varepsilon_{n+1}$. The demand distribution at stage n is $D \mid I_n : N(\mu + I_n, \sum_{i=n+1}^{N+1} \sigma_i^2)$. Denote $\sigma_n^2 = \sum_{i=n+1}^{N+1} \sigma_i^2$ in the following section.

We first focus on the value of shipping LVG with forecasting evolution. Since the carrier can only make the shipping choice once, it can be statically made at the beginning of the horizon or it can be made dynamically depending on the carrier's updated demand forecast of LVG. Therefore, we consider two different decision making styles, namely the static model and the dynamic model. We show that under MMFE, the two decision styles are equivalent.

A static model

Here, in the static model, the carrier makes the shipping choice at the beginning of the horizon whether to ship LVG at a certain stage n with a fixed order quantity or continue waiting for HVG. In stage n, after observing I_n, for the lost sale newsvendor problem with salvage value s, the carrier decides the order quantity to maximize the expected profit as

$$\pi_n(x\,|\,I_n) = E\big(p\min(D,x) - wx + s(x-D)^+\big) - C_n \qquad (12)$$

where the demand is distributed as $D\,|\,I_n : N(\mu + I_n, \sigma_n^2)$ and $C_n = c(n-1)$ is simplified for the waiting cost at the beginning of stage n. Therefore, the optimal order quantity is given as the following critical point:

$$F_n(x^*\,|\,I_n) = \frac{p-w}{p-s} = \beta, \qquad (13)$$

$$x^*(I_n) = F_n^{-1}(\beta) = \mu + I_n + z_\beta \sigma_n \qquad (14)$$

where F_n is the distribution of $D\,|\,I_n$, z_β is the inverse of standard normal distribution, $z_\beta = \Phi^{-1}(\beta)$. Therefore, the optimal expected profit at stage n is

$$\pi_n^*(I_n) = (p-w)(\mu + I_n) - (p-s)\sigma_n \phi(z_\beta) - C_n \qquad (15)$$

where $\phi(x)$ is the probability density of standard normal distribution.

Therefore, at the beginning of the horizon, the expected profit when the carrier decides to order at stage n and leaves is

$$\pi_n^* = E(\pi_n^*(I_n)) = (p-w)\mu - (p-s)\sigma_n\phi(z_\beta) - C_n \qquad (16)$$

from which, we can see clearly the tradeoff between increased waiting cost C_n and the benefit from the precision of the demand information, σ_n, which is decreasing in n.

At stage $N+1$, if the carrier has not made the shipping choice, the business opportunity of shipping is assumed to be lost, i.e., we assume $\forall I_{n+1}, \pi_{N+1}^? = 0$. Therefore, the optimal profit of shipping LVG in the static policy is

$$\Pi^s = \max_{n=1,\Lambda,N+1} \pi_n^? \qquad (17)$$

If we assume the updated forecast information is independent and identical distributed, i.e., $\sigma_n = \sigma$, we have $\sigma_n^2 = \sum_{i=n+1}^{N+1} \sigma_i^2 = (N+1-n)\sigma^2$. Thus, the expected profit of at the beginning of stage n viewed at the beginning of the horizon is

$$\pi_n^* = E(\pi_n^*(I_n)) = (p-w)\mu - (p-s)\phi(z_\beta)\sigma\sqrt{N+1-n} - c(n-1) \qquad (18)$$

Taking derivative with respect to n, we get the optimal waiting time which balances the tradeoff between increased waiting cost and the benefit from increased demand information accuracy is

$$n^* = \left[N + 1 - \left(\frac{(p-s)\phi(z_\beta)\sigma}{c} \right)^2 \right] \qquad (19)$$

where $[\cdot]$ represents the integer part of the number from below. If $n^* > 1$, the carrier can increase the expected profit from LVG with the ability of forecast updating during the waiting time. The optimal profit of shipping LVG is $\Pi^s(n^*)$.

At n^*, if the business opportunity has not come yet, the expected profit of waiting for HVG is $\Pi_H(n^*) = \pi_h - \frac{c}{\lambda} - c(n^? - 1)$. Therefore, in the static model, at the beginning of the horizon, the carrier can make the shipping choice based on the comparison between $\Pi^s(n^*)$ and $\Pi_H(n^*)$. If $\Pi^s(n^*) \geq \Pi_H(n^*)$, shipping LVG at n^* is optimal; if $\Pi^s(n^*) < \Pi_H(n^*)$, shipping HVG if it appears before or on $n^{'}$, such that $\Pi^s(n^{'}) \geq \Pi_H(n^{'})$; otherwise shipping LVG immediately at $n^{'}$.

A dynamic model

In this case, the shipping choice is determined in a dynamic fashion and contingent on the observed demand information I_n of LVG. This is also an optimal stopping problem. We first focus on the value of the dynamic strategy of shipping LVG. In any stage n (if the arrival of HVG has not come yet), after observing I_n, the carrier faces two options: to wait or to order LVG and leave. If he chooses to wait, nothing happens in n and he moves on to the next stage. The payoff will be the expected return in the next stage, conditional on I_n. If he chooses to order and ship LVG, then the payoff is the optimal expected profit at stage n with forecast updating. The dynamic programming formulation is given as

$$V_n(I_n) = \max\left\{\pi_n^?(I_n), EV_{n+1}(I_{n+1}\,|\,I_n)\right\} \qquad (20)$$

with the boundary condition $V_{N+1}(I_{N+1}) = 0$, where $V_n(I_n)$ stands for the optimal value given that the observed information is I_n and that the carrier has not yet made the shipping choice at stage $1,2,\Lambda\ n-1$. The optimal profit of the dynamic policy is $\Pi^d = V_1(0)$ since at the beginning of the horizon, there is no additional demand information about LVG.

Actually, the profit of shipping LVG in the above dynamic fashion is identical to the profit in the static fashion, as shown in the following result:

Proposition 4. *The profits from shipping LVG under MMFE in the static policy and the dynamic policy are the same.*

Proof: At stage $N+1$, it is clear that $V_{N+1}(I_{N+1}) = \pi_{N+1}^*(I_{N+1}) = 0$. Assume at stage n, $V_n(I_n) = \max_{t=n,n+1,\Lambda,N+1} \pi_t^*(I_n)$. At stage $n-1$, we have

$$\begin{aligned}
&V_{n-1}(I_{n-1}) \\
&= \max\left\{\pi_{n-1}^?(I_{n-1}), EV_n(I_n\,|\,I_{n-1})\right\} \\
&= \max\left\{\pi_{n-1}^?(I_{n-1}), E\left(\max_{t=n,\Lambda,N+1}\left[\pi_t^?(I_n)\,|\,I_{n-1}\right]\right)\right\} \\
&= \max\left\{\pi_{n-1}^?(I_{n-1}), \max_{t=n,\Lambda,N+1} \pi_t^?(I_n)\right\} \\
&= \max_{t=n-1,\Lambda,N+1} \pi_t^?(I_{n-1})
\end{aligned}$$

where the third equality follows from

$\forall t \geq n, E\left[\pi_t^?(I_n) \mid I_{n-1}\right] = \pi_t^?(I_{n-1})$, since $I_n = I_{n-1} + \varepsilon_n : N(I_{n-1}, \tilde{\sigma}_n^2)$.

Therefore, the expected profit from the dynamic policy is

$$\Pi^d = V_1(0) = \max_{n=1,\Lambda,N+1} \pi_n^?(0) = \max_{n=1,\Lambda,N+1} \pi_n^? = \Pi^s. \quad \blacksquare$$

The above result indicates, in terms of shipping LVG, under MMFE, there is no benefit from a dynamic policy compared with the static policy. The similar result is also found in [7] where the newsvendor has only one order chance under MMFE. This seems counter-intuitive, since contingent on the real-time demand information, the carrier would get more benefit from shipping LVG. The reason is due to the characteristics of the MMFE model, the forecast updates are independent and the expected value of the future is equal to the current state. Besides, the increased benefit due to the increased information and the increased waiting cost are fixed and deterministic in each stage. Therefore, under MMFE, the expected profit from shipping LVG dynamically or statically is identically. Thus, the shipping strategy is the same as in the static policy.

A general model

We incorporate the demand uncertainty of LVG and the arrival uncertainty of HVG in a synthesized model. We define the state as (I_n^H, I_n^L) and the dynamic model is simply formulated as

$$\Pi_n(I_n^H, I_n^L) = \max\left\{\pi_n^*(I_n^L), V_n(I_n^H)\right\} \tag{21}$$

and $\pi_n^*(I_n^L) = (p-w)(\mu + I_n^L) - (p-s)\sigma_n\phi(z_\beta) - C_n$ and $V_n(I_n^H) = p(I_n^H)(\pi_h - C_n - c) + (1 - p(I_n^H))\int V_{n+1}(I_n^H + \varepsilon)d\varepsilon$.

Therefore, at stage n, the critical information state satisfies $V_n(I_n^H) = \pi_n^*(I_n^L)$, indicating for every given I_n^L, there exists a corresponding I_n^H, such that if $I^H > I_n^H$, waiting for the next stage (since HVG has not come yet), otherwise shipping LVG; for given I_n^H, there exists a corresponding I_n^L, such that if $I^L > I_n^L$, shipping LVG, otherwise waiting for the next stage. The critical value of the information state $I_n^i, i = H, L$ is increasing in the other one I_n^{-i}, since the value of each choice is increasing in the corresponding information state.

Compared with previous section, we can see the decreasing property of I_n^{H*} does not hold due to the existence of random information state I_n^L. We investigate without the impact of I_n^L, whether we have the same structure property. Therefore, we use the expected value π_n^* to substitute for $\pi_n^*(I_n^L)$, i.e., assuming $I_n^L = 0$, and we have the critical information state in stage n as $I_n^{H*} = \arg_{I^H}\{I \mid V_n(I^H) - \pi_l(n) + c(n-1) = 0\}$, where we assume $\sigma_i = \sigma$, and $\pi_l(n) = (p-w)\mu - (p-s)\phi(z_\beta)\sigma\sqrt{N+1-n}$ which is increasing in n.

Correspondingly, the critical information state in stage $n+1$ is $I_{n+1}^{H*} = \arg_{I^H}\{I \mid V_{n+1}(I^H) - \pi_l(n+1) + cn = 0\}$. We also have

$$\begin{aligned} & V_{n+1}(I_n^{H*}) - \pi_l(n+1) + cn \\ &= V_n(I_n^{H*}) - p(I_n^{H*})c - \pi_l(n+1) + cn \\ &= \left(1 - p(I_n^{H*})\right)c - \frac{(p-s)\phi(z_\beta)\sigma}{\sqrt{N+1-n} + \sqrt{N-n}} \end{aligned} \tag{22}$$

where the first part is the expected increased waiting cost in stage $n+1$, and the second part is the increased expected profit from LVG due to more accurate demand information. Thus, we can clearly see the tradeoff between increased information and increased waiting cost.

Suppose I_n^{H*} is decreasing in n. $\left(1 - p(I_n^{H*})\right)c$ is increasing in n and the second part is also increasing in n. The difference of these two forces may be negative. Under this situation, we will have $I_{n+1}^{H*} \geq I_n^{H*}$, which contradicts the decreasing hypothesis of the critical information state. Therefore, the decreasing property of I_n^{H*} still may not be hold. On the other hand, if we assume I_n^{H*} is increasing in n, $\left(1 - p(I_n^{H*})\right)c$ is increasing in n, the second part is increasing in n. If $\left(1 - p(I_1^{H?})\right)c - \frac{(p-s)\phi(z_\beta)\sigma}{\sqrt{N} + \sqrt{N-1}} \leq 0$, we will have $\forall 1 \leq n \leq N$, $\left(1 - p(I_n^{H*})\right)c - \frac{(p-s)\phi(z_\beta)\sigma}{\sqrt{N+1-n} + \sqrt{N-n}} \leq 0$, which indicates the increasing property of I_n^{H*} can be hold. In sum, the structure property of I_n^{H*} is not necessarily hold in general.

The above result indicates, with the ability of forecast updating of the demand information about LVG, under the unknown information about the arrival of HVG, the shipping choice may depend on several factors, and a contingency dependent shipping strategy is needed in practice.

VI. A WAITING STRATEGY WITH BAYESIAN UPDATING

In previous sections, we focus on the one-period model, where the carrier only makes one shipping choice. The carrier can make the shipping choice by a static fashion or a dynamic fashion with or without perfect information about the arrival distribution of HVG. The arrival distribution of HVG is either assumed to be completely known or unknown. Generally, the arrival distribution of HVG can not be fully acquired. In practice, the carrier always adopts a mixed shipping policy in order to transport HVG, since the profit from HVG is relatively large while the risk is relatively small and during the waiting, the demand information about LVG can be updated. The carrier will use the cargo ship several years and he can get more and more information from the past experience and use these information to further assist the future decision making. Therefore, in this section, we consider a multiple-period situation under the imperfect information about the arrival distribution of HVG where the carrier needs to decide the waiting time for each transport. The carrier has a prior information about the arrival distribution but with uncertain parameters. The carrier can update the arrival distribution based on the past experiences or observations which depend on his previous waiting decisions.

If the exact arrival time during each shipping can be observed, no matter how long the waiting time is, the carrier can estimate the distribution more accurately with more realizations. This situation is defined as the *observed lost opportunity case*. While if the exact arrival time can be observed only when the arrival of LVG occurs before or on the the threshold waiting time during each shipping, the waiting-time decision will be critical for the carrier to estimate the arrival distribution. This situation is defined as the *unobserved lost opportunity case*.

In this section, we investigate the structural property of the waiting-time decisions under different situations. We assume the carrier knows the arrival distribution family with unknown parameters which is characterized by a distribution. The carrier needs to estimate the parameter based on the past arrival realizations. Some statistical models and tools have been developed to estimate the parametric distribution with censored data, such as the Kaplan-Meier (KM) estimator [8]. In inventory management, a few papers have discussed the estimation of demand distribution and the optimal order policies under censored demand data. The papers [9-11] studied dynamic inventory policy when the demand density has unknown parameters and is a member of the exponential and range families. They showed that an adaptive order-up-to policy is optimal where the order-up-to level depends on the history path through a sufficient statistic. Azoury [12] extended the work to more general demand distributions and provides conditions under which finding the optimal policy under Bayesian updating reduces to solving a stochastic dynamic programming problem with a one-dimensional state space. Lariviere and Porteus [13] examine an inventory setting with a specific demand distribution that belongs to a family called newsvendor distributions. Ding *et al*. [14] study a general demand distribution in a two-period inventory model, and the optimal order quantity is proved no less than the one in the myopic policy under censored demand data. Intuitively, the structural property of the optimal order quantity in multi-stage problem should be hold as in two-stage model. To generalize the model in Ding *et al*. [14] to a multi-stage model with censored observation, Lu *et al*. [15] prove the structural property holds using a sample path analysis. Bensoussan *et al*. [16] then provide an alternative method to proof the claim using the concept of the unnormalized probability, which simplifies the dynamic programming equation considerably.

Based on the above literature, in this section, we also use the Bayesian updating mechanism for the carrier to update the arrival distribution under the unobserved lost opportunity case. It turns out that the structural property of the optimal waiting-time decision is identical to the order quantity decisions in the censored newsvendor problem.

Observed lost opportunity case

Under the observed lost opportunity case, the carrier can get all the realizations of the arrival of HVG, even if it comes after the ship is loaded with LVG. Assume the arrival time of HVG X belongs to some distribution family, with the density function as $\psi(x\,|\,\theta)$ and the CDF as $\Psi(x\,|\,\theta)$, where the parameter $\theta\in\Theta$ is unknown. The carrier has a prior distribution of θ, denoted as $\mu_n(\theta)$ at the beginning of each period n. Given the prior distribution of $\mu_n(\theta)$ and an observation of the arrival time x_n, the posterior distribution of θ, denoted as $\mu_{n+1}(\theta\,|\,x_n)$ is given by the Bayesian rule

$$\mu_{n+1}(\theta\,|\,x_n)=\frac{\psi(x_n\,|\,\theta)\mu_n(\theta)}{\int_\Theta \psi(x_n\,|\,\tilde{\theta})\mu_n(\tilde{\theta})d\tilde{\theta}} \tag{23}$$

which will serve as the prior in period $n+1$.

The estimated arrival distribution at the beginning of period n is given as

$$\phi_n(x)=\int_\Theta \psi(x\,|\,\theta)\mu_n(\theta\,|\,x_{n-1})d\theta \tag{24}$$

For the ease of expression, we assume the domain of the arrival time is nonnegative in the interval $[\underline{x},\bar{x}]$, where $\bar{x}$ may be ∞. Therefore, the expected profit in each period n with prior μ_n and waiting-time decision y_n is

$$\Pi(y_n,\mu_n)=\int_{\underline{x}}^{y_n}\big(\pi_h-C(x)\big)\phi_n(x)dx + \int_{y_n}^{\bar{x}}\big(\pi_l-C(y_n)\big)\phi_n(x)dx \tag{25}$$

and the optimal waiting time in each period n, defined as y_n^M is assumed to be interior in the domain, i.e., the function $\Pi(y_n,\mu_n)$ is assumed to be unimodal (for example, the exponential distribution with convex waiting cost). The strategy is defined as the *myopic* shipping strategy, since it only cares about the expected profit in current period.

For the observed lost opportunity case, the dynamic programming formulation is given as

$$\begin{aligned} V_n(\mu_n) &= \max_{y_n}\{\Pi(y_n,\mu_n)+\beta \mathrm{E} V_{n+1}(\mu_{n+1})\} \\ &= \max_{y_n}\{\Pi(y_n,\mu_n)+\beta\int_{\underline{x}}^{\bar{x}} V_{n+1}\big(\mu_{n+1}(\theta\,|\,x)\big)\phi_n(x)dx\} \end{aligned} \tag{26}$$

Since the waiting time in each period is independent of the arrival realization, at period n, after observing the arrival time in the previous period x_{n-1}, the posterior is updated as $\mu_n(\theta\,|\,x_{n-1})$, and the estimated arrival distribution is updated as $\phi_n(x)$. Therefore, in each period, the problem is equivalent to the one period problem, and the optimal waiting time is y_n^M based on the updated arrival distribution.

Unobserved lost opportunity case

Under unobserved lost opportunity case, the exact arrival realization can only be observed when the arrival of HVG comes by the waiting time. Therefore, the waiting-time decision impacts the arrival observation. Based on the Bayesian updating rule, the posterior is updated as

$$\tilde{\mu}_{n+1}(\theta \mid x_n) = \begin{cases} \mu_{n+1}(\theta \mid x_n), & x_n \le y_n \\ \mu^c_{n+1}(\theta \mid y_n), & x_n > y_n \end{cases} \quad (27)$$

where $\mu^c_{n+1}(\theta \mid y_n)$ is given as the following

$$\mu^c_{n+1}(\theta \mid y_n) = \frac{\int_{y_n}^{\bar{x}} \psi(x \mid \theta)\tilde{\mu}_n(\theta)dx}{\int_\Theta \int_{y_n}^{\bar{x}} \psi(x \mid \tilde{\theta})\tilde{\mu}_n(\tilde{\theta})dxd\tilde{\theta}} \quad (28)$$

The estimated arrival distribution is given as

$$\tilde{\phi}_n(x) = \int_\Theta \psi(x \mid \theta)\tilde{\mu}_n(\theta \mid x_{n-1})d\theta \quad (29)$$

Therefore, the expected profit in each stage with prior $\tilde{\mu}_n$ and waiting-time decision y_n is

$$\begin{aligned} &\Pi(y_n, \tilde{\mu}_n) \\ &= \int_{\underline{x}}^{y_n} (\pi_h - C(x))\tilde{\phi}_n(x)dx + \int_{y_n}^{\bar{x}} (\pi_l - C(y_n))\tilde{\phi}_n(x)dx \\ &= \int_{\underline{x}}^{y_n} (\pi_h - C(x))\tilde{\phi}_n(x)dx + (\pi_l - C(y_n))\left(1 - \tilde{\Phi}_n(y_n)\right) \end{aligned} \quad (30)$$

where $\tilde{\Phi}_n(y_n) = \int_{\underline{x}}^{y_n} \tilde{\phi}_n(x)dx$.

The dynamic programming formulation under unobserved lost opportunity case is given as

$$\begin{aligned} V_n(\tilde{\mu}_n) &= \max_{y_n} \{\Pi(y_n, \tilde{\mu}_n) + \beta \mathrm{E} V_{n+1}(\tilde{\mu}_{n+1})\} \\ &= \max_{y_n} \{\Pi(y_n, \tilde{\mu}_n) + \beta \int_{\underline{x}}^{y_n} V_{n+1}\left(\mu_{n+1}(\theta \mid x)\right)\tilde{\phi}_n(x)dx \\ &\quad + \beta V_{n+1}\left(\mu^c_{n+1}(\theta \mid y_n)\right)\left(1 - \tilde{\Phi}_n(y_n)\right)\} \end{aligned} \quad (31)$$

The optimal waiting-time decision is defined as $y_n^?$. The above model is difficult to solve, since the transition function depends on previous arrival realizations. From the above formulation, we can find that there exists a critical tradeoff between the current profit and the future benefit due to the waiting-time decision. For a given prior, waiting too long will decrease the expected profit in the current period. However, a more concise information about the arrival distribution will be derived, since waiting a little longer can increase the chance of observing additional arrival realization, which will benefit the future decisions.

We investigate the structural property of the optimal waiting-time decision path. We first define the myopic policy as the decisions in each period which maximize the expected profit under the current distribution information as $y_n^M = \arg\max_{y_n} \Pi(y_n, \tilde{\mu}_n)$. We have the following structural property of the optimal waiting-time path:

Proposition 5. *Compared with the myopic waiting time, under the unobserved lost opportunity case, the optimal waiting time in each period is no less than the myopic waiting time, i.e.,* $y_n^? \ge y_n^M$.

Proof: The proof can be used a sample path analysis as similar as that in Lu et al. [15] or the unnormalized probability introduced in Bensoussan et al. [16]. Both of the papers provide an alternative proof of the structural property in the optimal order quantity in the multiple-period newsvendor problem with censored observation. Our problem of optimal waiting time under unobserved lost opportunity case is equivalent to the optimal order quantity in the newsvendor model with censored observation. Therefore, the proof is omitted here. ■

The above result indicates under the unobserved lost opportunity case, the optimal waiting time should balance the tradeoff between short-term profit and long-term benefit. It is clear that as the period n increases, the difference between the optimal waiting time and the myopic one will become smaller and smaller, since there will be enough observations of the arrival realizations and the estimated arrival distribution will become more accurate even under myopic policy.

VII. SUMMARY, MANAGERIAL INSIGHT AND FUTURE RESEARCH

Summary

A decision making process always involves a choice among multiple alternatives. To make a wise choice, decision makers need to analyze the tradeoffs in each choice. One critical tradeoff in management decision making is between the short-term interest and long-term benefit. Based on the practical situation in cargo ship transport, in this paper, we focus on the shipping choice problem faced by the carrier. The carrier can choose to either ship HVG with fixed freight given uncertain supply of goods, or ship LVG with less profit given uncertain demand of goods. The objective of the carrier is to make a wise shipping choice in order to maximize the potential profit.

Under various situations, a wise shipping choice needs to balance the tradeoff between expected profit margin and potential waiting cost. For a one-period model, we first concentrate on the shipping choice due to the supply uncertainty of HVG. Under the situation with a known arrival distribution of HVG, we demonstrate that the failure rate of the arrival distribution plays an important role. For IFR and CFR distributions, there is no optimal mixed strategies, and the carrier either waiting for HVG or shipping LVG immediately. However, for DFR distributions, there may exist an optimal mixed strategy with higher expected profit than pure strategies. Under the situation with unknown arrival distribution of HVG, the shipping choice problem is converted to an optimal stopping problem, i.e., whether continuing to wait for the arrival of HVG (if it does not come yet) or stopping and shipping LVG immediately. A dynamic model is developed to show that a threshold policy should be adopted in terms of the arrival probability of HVG, in the sense that the carrier should stop waiting if the arrival probability is below a critical value in each stage. Due to increased waiting cost, the critical value is decreasing in waiting time.

The demand uncertainty of LVG can also impact the shipping choice. The carrier can strategically wait to update the demand information about LVG and the arrival information about HVG before selecting the shipping choice. The carrier updates the demand information based on the MMFE model. Under exponential arrival distribution of HVG, the shipping choice made statically is equivalent to that in a dynamic model. With demand forecast updating, the carrier can benefit from the strategic waiting decision. If the arrival distribution of HVG is unknown, a contingency based shipping strategy may be optimal during the increased expected profit of shipping LVG.

If the carrier has multiple shipping choices to make, under the situation with imperfect arrival distribution, he can update the arrival distribution based on the previous experiences. We therefore concentrate on the optimal waiting-time decisions in a multiple-period model based on the Bayesian updating mechanism. Under observed lost opportunity case, the carrier is simply adopt the optimal waiting time in single period model based on the updated arrival distribution, i.e., the myopic waiting strategy. Under unobserved lost opportunity case, the carrier can observe the arrival realization only when the arrival realization occurs on or before the waiting time. It turns out that the optimal waiting time in this case is no less than one in the myopic policy.

Managerial insight

From the shipping choice decision making in bulk transport, we can see the tradeoffs in each choice under different situations. Under known arrival distribution of HVG and demand distribution of LVG, the tradeoff is between the profit margin and the potential waiting cost when the carrier decides to ship HVG. The carrier may adopt a mixed strategy to balance the tradeoff. However, the result indicates, there may not exist an optimal mixed strategy under some conditions. Under unknown arrival distribution of HVG, the carrier can make the shipping decision dynamically. The tradeoff associated with the waiting decision is the increased waiting cost and the updated information about the arrival of HVG. Therefore, the carrier should make the shipping decision to balance these two forces. With the ability of forecast updating about the demand of LVG, the carrier can achieve the benefit from accurate information. However, the waiting decision should balance the tradeoff between the increased waiting cost and the improved demand information. If the carrier ships multiple times, under the unknown arrival distribution parameter, the arrival distribution can be updated based on previous experience. However, if the lost shipping opportunity of HVG is unobservable, the carrier needs to decide the waiting time to balance the tradeoff between short-term profit and the long-term benefit. In sum, the shipping choice and associated waiting-time decision should be determined to make a balance between the inherent critical tradeoffs between short-term interest and long-term profit.

Future research

It would be interesting to investigate the interplay between manager's risk attitude and the long-term vs. short-term tradeoffs. Time preference attitude, namely the extent of patience, of the decision maker is another important factor in such decision-making context. Empirical evidences of the above-mentioned factors in other managerial decision-making contexts such as investment, human capital development, and productivity can be further researched.

REFERENCES

[1] Zipkin, P.H. (2000). *Foundations of inventory management*, McGraw-Hill.

[2] Graves, S.C., Kletter, D.B., Hetzel, W.B. (1998). 'A dynamic model for requirements planning with application to supply chain optimization', *Operations Research*, Vol. 46, No. 3, pp. 35-49.

[3] Heath, D.C., Jackson, P.L. (1994). 'Modeling the evolution of demand forecasts with application', *IIE Transactions*, Vol. 26, No. 3, pp. 17-30.

[4] Fisher, M.L., Raman, A. (1996). 'Reducing the cost of demand uncertainty through accurate response to early sales', *Operations Research*, Vol. 44, No. 1, pp. 87-99.

[5] Huang, H., Sethi, S.P., Yan, H. (2005). 'Purchase contract management with demand forecast updates', *IIE Transactions*, Vol. 37, No. 8, pp. 775-785.

[6] Lu, X., Song, J.S., Regan, A. (2006). 'Inventory planning with forecast updates: Approximate solutions and cost error bounds', *Operations Research*, Vol. 54, No. 6, pp. 1079-1097.

[7] Wang, T., Atalay, A., Mumin, K. (2010). 'A Multi-Ordering Newsvendor Model with Dynamic Forecast Evolution', Working paper, National University of Singapore.

[8] Kaplan, E.L., Meier, P. (1958). 'Nonparametric estimation from incomplete observations', *Journal of the American Statistics Association*, Vol. 53, No. 282, pp. 457-481.

[9] Karlin, S. (1960). 'Dynamic inventory policy with varying stochastic demands', *Management Science*, Vol. 6, No. 3, pp. 231-258.

[10] Scarf, H. (1960). 'Some remarks on Bayes solutions to the inventory problem', *Naval Research Logistics Quarterly*, No. 7, 591-596.

[11] Iglehart, D.L. (1964). 'The dynamic inventory problem with unknown demand distribution', *Management Science*, Vol. 10, No. 3, pp. 429-440.

[12] Azoury, K.S. (1985). 'Bayes solution to dynamic inventory models under unknown demand distribution', *Management Science*, Vol. 31, No. 9, pp. 1150-1160.

[13] Lariviere, M.A., Porteus, E.L. (1999). 'Stalking information: Bayesian inventory management with unobserved lost sales', *Management Science*, Vol. 45, No. 3, pp. 346-363.

[14] Ding, X.,Puterman, M.L., Bisi, A. (2002). 'The censored newsvendor and the optimal acquisition of information', *Operations Research*, Vol. 50, No. 3, pp. 517-527.

[15] Lu, X., Song, J.S., Zhu, K. (2008). 'Analysis of perishable inventory systems with censored demand data', *Operations Reseach*, Vol. 56, No. 4, pp. 1034-1038.

[16] Bensoussan, A., Cakanyildirim, M., Sethi, S.P. (2009). 'A Note on "The Censored Newsvendor and the Optimal Acquisition of Information"', *Operations Reseach*, Vol. 57, No. 3, pp. 791-794.