An Approach for Measuring Process Performance with Asymmetric Tolerances in the Presence of Measurement Errors

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Abstract: The generalized Taguchi capability index C''_{pm} has been shown superior to

other existing generalizations and to be applicable for measuring process performance with asymmetric tolerances. Existing literature related to the generalized Taguchi capability index have assumed not considering gauge measurements errors (GME). Unfortunately, evaluating process performance of such a capability index without considering GME may not accommodate the real manufacturing situations. Hence, this paper applies a novel approach, generalized confidence intervals (GCI) to evaluate process performance in the presence of GME. In order to examine the performance of the proposed approach, a series of simulations was undertaken. The simulations result claim that the proposed approach performs good enough for assessing process performance for asymmetric tolerances in the presence of measurement errors or not.

Keywords: Asymmetric tolerances, coverage rate, gauge measurement errors, interval estimation, process performance evaluation.

1. Introduction

Barely, suppliers and manufacturers entail their products not only to be high in

quality but also have a good process capability. As a high quality product with very

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low fraction of nonconformities (NC), the conventional method might be useless for measuring fraction of NC since the manufacturing sample contains no defective product items. Process Capability Indices (PCIs) as one of the tools in measuring the capability process have attract major attention in the manufacturing industries especially for tight quality needs. The first generation, C_p index has been proposed to evaluate the capability of process to meet the specification limits (see Kane, 1986). However, C_p index unable to reflect the situation where the process is off the target value, T.

In order to remedy this kind of problems, several indices have been proposed which include the deviation from the target value of a particular process (see Boyles, 1991). Since the statistical distributions of the proposed methods quite complicated and difficult, in 1988, Spiring *et al* proposed a new index called C_{pm} to asses the capability process which take into account departures from the target value. The second generation index, C_{pm} is defined as follow:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}$$

where μ denotes process mean, σ denotes process standard deviation, USL and LSL denote upper and lower specification limit.

A process is said to be asymmetric when the customer's specification is not equal to the midpoint of specification limits, i.e. $T \neq M$. In 1994, Boyles noted that such

problem can understate or overstate capability process in many cases. Even symmetric tolerances are often occurs in manufacturing industries, but still cases with asymmetric tolerances also appears in the manufacturing industries (see Chen, 1998, Pearn and Chen, 1998, and Wu and Chang, 2008). To remedy the problem, Lin et al. (1999) proposed a generalization of C_{pm} , C''_{pm} index. The result showed that C''_{pm} measures process capability more accurately than the original index C_{pm} . Unfortunately, the estimation of C''_{pm} , \hat{C}''_{pm} involving unknown parameter while using the conventional approaches. To avoid the lack of exact confidence intervals for C''_{pm} , in 1993, Weerahandi proposed an extension of conventional confidence interval. The concept of generalized pivotal quantity (GPQ) and generalized confidence interval (GCI) are developed to derive confidence interval when exact values are difficult to obtain. Usually, any variation in the measurement process has a direct effect on estimating and testing process capability. By ignoring the measurement errors, the empirical used indices are unreliable while the quality of the observed data depends on them. Consequently, gauge measurement analysis is needed in the measurement system.

Based on the complicated conventional confidence intervals and the importance of GME, this article applied the GCI approach to assess process capability based on C''_{pm} index in the presence of GME under asymmetric tolerances. The

generalization for C_{pm} is given in Section 2. In Section 3, we described about the generalized confidence intervals with GME. To assess the performance of the GCI approach, a simulation was conducted and the result is given in Section 4. Some conclusions are drawn in Section 5 as the final section.

2. The Generalized Taguchi Capability Index C_{pm}

The generalization of the second generation index, C''_{pm} provides numerical measure that takes into account the asymmetric tolerance which reflects the process capability more accurately. This index is defined as follow:

$$C''_{pm} = \frac{d^*}{3\sqrt{\sigma^2 + A^2}}$$

Where d = (USL - LSL)/2, $A = \max[d(\mu - T)/D_u, d(T - \mu)/D_l]$, $D_u = USL - T$, $D_l = T - LSL$ and $d^* = \min\{D_u, D_l\}$. The C''_{pm} index obtains its maximal value at $\mu = T$ whether the tolerances are symmetric or asymmetric. Lin *et al.* (1999) considered the natural estimator \hat{C}''_{pm} to estimate the C''_{pm} index. This estimator can be defined as follow:

$$\hat{C}''_{pm} = \frac{d^*}{3\sqrt{S_n^2 + \hat{A}^2}}$$

where $\hat{A} = \max[d(\bar{X} - T)/D_u, d(T - \bar{X})/D_l], \bar{X} = \sum_{i=1}^n X_i / n$ is unbiased estimator and Maximum Likelihood Estimation (MLE) and $S_n^2 = \sum_{i=1}^n (X_i - \bar{X})^2 / n$ is MLE estimator of σ^2 .

3. GCIs for C''_{pm} index with gauge measurement errors

In 1993, Weerahandi was motivated to broaden the common definition of a confidence interval such that practically useful. There seems to be no approach that is generally and practically applicable to solve the problems in such a way theoretically guaranteed to cover all cases. In fact, the quality of the data related to the process characteristics relies on the gauge capability. The numerical results indicate that the GCI approach appears satisfactory in both conditions, with or without measurements errors for assessing process capability. Let $X \sim N(\mu, \sigma^2)$ reflects relevant quality characteristic of a manufacturing process and C''_{pm} measures the real process capability of the random variable X. However, in real situations, we deal with the observed variable Y rather than variable X. Assume that X and G are stochastically independent, thus we have $Y \sim N(\mu_Y = \mu, \sigma_Y^2 = \sigma^2 + \sigma_{GME}^2)$, after substituting σ_{Y} for σ , the empirical PCI $C_{pm}^{"Y}$ can be obtained. Assume that the observed measurement with errors data is Y_i , $i = 1, 2, \dots, n$ and the existing data are from a stable process; $\hat{C}_{pm}^{"Y}$ can then be obtain using $\overline{Y} = \sum_{i=1}^{n} Y_i / n$ and $S_{nY} = \left[\sum_{i=1}^{n} (Y_i - \overline{Y})/n\right]^{1/2}$ as the estimators.

$$C_{pm}^{\prime \prime Y} = \frac{d^{*}}{3\sqrt{\sigma_{Y}^{2} + \left(\max\left\{d\left(\mu_{Y} - T\right)/D_{u}, d\left(T - \mu_{Y}\right)/D_{l}\right\}\right)^{2}}}$$

Using $Y \sim N(\mu_Y = \mu, \sigma_Y^2 = \sigma^2 + \sigma_G^2)$ with $\overline{Y} \sim N(\mu, \sigma_Y^2/n)$ and $nS_{nY}^2/\sigma_Y^2 \sim \chi_{n-1}^2$

where μ and $\sigma_{_{Y}}^2$ are unknown constant. In order to develop the GCI for μ and

 σ_{Y}^{2} , we used the same procedure for the symmetric tolerances as shown in Wu *et al* (2009) and Wu (2011). Since $\sigma_{GME}^{2} = [\lambda d/3]^{2}$ can be estimated, then the GPQ for σ^{2} , $R_{\sigma^{2}}$ can be obtained by $R_{\sigma^{2}} = \max\{R_{\sigma^{2}_{Y}} - \sigma_{GME}^{2}, \varepsilon\}$, where ε is a small positive number to preserve non-negative variance properties. Obviously, $R_{\mu_{Y}}$ and $R_{\sigma^{2}_{Y}}$ are exempt from unknown parameters, thus a GPQ of $C_{pm}^{"Y}$ is given by

$$R_{C_{pm}^{nY}} = \frac{d^{*}}{3\sqrt{R_{\sigma_{Y}^{2}} + \left(\max\left\{d\left(R_{\mu_{Y}} - T\right)/D_{u}, d\left(T - R_{\mu_{Y}}\right)/D_{l}\right\}\right)^{2}}}$$

In appearance of measurement errors, the GPQ for C''_{pm} is given by

$$R_{C_{pm}^{"}} = \frac{d^{*}}{3\sqrt{R_{\sigma^{2}} + \left(\max\left\{d\left(R_{\mu_{Y}} - T\right)/D_{u}, d\left(T - R_{\mu_{Y}}\right)/D_{l}\right\}\right)^{2}}}$$

The $100(1-\alpha)\%$ generalized lower confidence limit of $C_{pm}^{"Y}$ and $C_{pm}^{"}$ can be derived by calculate $R_{C_{pm}^{"Y}}[\alpha]$ and $R_{C_{pm}^{"}}[\alpha]$, the 100α th percentile of $R_{C_{pm}^{"Y}}$ and $R_{C_{pm}^{"}}$ which satisfies $P(R_{C_{pm}} < R_{C_{pm}}[\alpha]) = \alpha$, respectively.

4. Numerical Results

To verify the performance of the generalized lower confidence limit for capability testing using $C_{pm}^{"Y}$ index, a series of simulations is conducted. Without any loss of generality, all simulation was used the value T=0. A total of five different $\mu = -1, -0.5, 0, 0.5, 1$ and $\sigma = 0.5$ was used to represent symmetric, on target and asymmetric tolerances. Three different ratios $D_u/D_l = 1/1, 1/2, 1/3$ and different degrees of contamination of GME $\lambda = 0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3$ were considered in this paper. Thus we can have complete simulation for all conditions. For each simulation, a sample of size n = 50,100 was drawn and 10,000 values of (Z,V) were simulated. The generalized lower confidence limit was computed using 10,000 simulated values of (Z,V) by keeping the observed values of \bar{x}_{γ} and s_{γ} fixed. Every single simulation was then replicated N = 10,000 times. Hence, we were able to calculate the proportion of times the generalized lower confidence limit where less than the corresponding true value of C''_{pm} . This actual coverage rate (CR) could then be compared to the nominal confidence level $1-\alpha = 0.95$. The expected value of the generalized lower confidence limit of C''_{pm} is simply the average of these 10,000 values of the generalized lower confidence limit (L''_{pm}) . For every cases were computed under the nominal confidence level $1-\alpha = 0.95$.

The notations of generalized lower confidence limit of C''_{pm} with or without considering GME are CRx and CRy, respectively. In this case, the accuracy of the proposed approach will be considered as satisfactory when the simulation results in terms of coverage probabilities are close to the nominal value, 0.95. Since the coverage probabilities values are almost the same for every *n*, we plot the average of coverage rate under different ratio and σ , respectively. Figures 1-3 plot the averages of CR_x and CR_y versus λ under the ratios $D_u/D_l = 1/1, 1/2$ and 1/3 and σ , respectively. From these figures, it is clear that a process performance evaluation with consideration of GME result in more accurate generalized lower confidence limit. Since the calculated CRs without considering GME tends to be overestimated as λ increases and by considering GME, the variability of the CR_x is much smaller, thus we can conclude that the proposed approach results in a stable calculation and works well in the presence of GME.



1,00 - Crx - Cry **Rate** 0,99 Value of Coverage 0,98 0,97 Average 0,96 0,95 0,95 0,15 Lambda 0,00 0,05 0,10 0,20 0,25 0,30

Figure 1. Average values of CR_x and CR_y versus

 λ under $\sigma = 0.5$ and $D_u / D_l = 1/1$.

Figure 2. Average values of CR_x and CR_y versus

$$\lambda$$
 under $\sigma = 0.5$ and $D_u / D_l = 1/2$.



Figure 3. Average values of CR_x and CR_y versus

 λ under $\sigma = 0.5$ and $D_u / D_l = 1/3$.

For the generalized lower confidence limits of C''_{pm} with and without considering GME, we used the notations L_x and L_y , respectively. The values of L_x and L_y tend to increase towards the true value of C''_{pm} as *n* increases. Figures 4–9 display the generalized lower confidence limits of C''_{pm} against λ under $D_u / D_t = 1/1, 1/2$ and 1/3, respectively. It is evident that the difference between L_x and L_y becomes larger as λ increases. This condition evidence that the true process capability C''_{pm} would have been underestimated if we ignored GME while its exist.





Figure 4. Lower Confidence Bound L_x and L_y Figure 5. Lower Confidence Bound L_x and L_y versus λ under n = 50 and $D_u / D_l = 1/1$. versus λ under n = 100 and $D_u / D_l = 1/1$.





Figure 6. Lower Confidence Bound L_x and L_y

Figure 7. Lower Confidence Bound L_x and L_y

versus λ under n = 50 and $D_u / D_l = 1/2$.

versus λ under n = 100 and $D_u / D_l = 1/2$.





Figure 8. Lower Confidence Bound L_x and L_y Figure 9. Lower Confidence Bound L_x and L_y versus λ under n = 50 and $D_u / D_l = 1/3$. versus λ under n = 100 and $D_u / D_l = 1/3$.

5. Conclusions

Gauge measurement errors have become essential in manufacturing industries since they have huge impact on estimating and evaluating the capability process. The decision makers may be lead to incorect decisions if they do not take into account the presence of measurement errors on capability process. In this paper, we considered to measure capability process based on the generalized C''_{pm} index in the presence of measurement errors for asymmetric tolerances using the generalized confidence interval (GCI) approach. By conducting a series of simulations in terms of the coverage rate and the generalized lower confidence limits, we examine the performance of the proposed approach. The simulation results indicate that the proposed GCI approach seemingly quite satisfactory since it provides an accurate lower confidence limit. The calculated CRs are also very close to the nominal value in the presence of GME. Accordingly, the GCI approach is suitable for cases with asymmetric tolerances when GME are actually present.

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