Construction of a Tightened-Normal-Tightened Sampling Scheme by Variables Inspection

Alexander A Nugroho [a,*], Chien-Wei Wu [b], and Nani Kurniati [a]

[a] *Department of Industrial Management, National Taiwan University of Science and Technology, Taipei,Taiwan.*

[b] *Department of Industrial Engineering, National Tsing Hua University, Hsinchu, Taiwan.*

Abstract

Acceptance sampling is one of the important parts of the field of quality control and used primarily for incoming inspection. Acceptance sampling plans state the required sample size and the decision making rule for product sentencing. This paper proposes a tightened-normal-tightened (TNT) variables sampling scheme for product acceptance determination based on the third-generation of process capability index C_{pmk}, that considers both process yield and quality loss. The operating characteristic (OC) function of the proposed sampling scheme is derived based on the exact sampling distribution and the operating procedures is provided. The sample size required for inspection and the associated critical acceptance value are tabulated under various combinations of quality levels and risks preset by producers and consumers. The behaviors of the proposed sampling scheme with various conditions are also discussed.

Keywords: variables sampling scheme, process capability index, operating characteristic (OC) function, producer's risk, consumer's risk

[*] Corresponding author. E-mail address: M10101805@mail.ntust.edu.tw .

1. Introduction

Inspection of raw materials, semi-finished products, or finished products is one aspect of quality assurance. The inspection will conduct by take samples from the lots and use them to make decisions whether accept or reject the lots, called acceptance sampling. Acceptance sampling plans state the required sample size and the decision making rule for product sentencing. One major classification of acceptance sampling plans is by attributes and variables. Variables sampling plans will measure the quality characteristics on a numerical scale. The primary advantage of variables sampling plans is having a smaller sample size than attributes sampling plans for the same protection for both producer and customer and more beneficial when the inspection is costly and destructive.

A sampling scheme is a combination of acceptance sampling plans with switching rules for changing from one plan to another plan (Schilling and Neubauer, 2009). A sampling scheme involves only normal and tightened inspections called tightened-normal-tightened (TNT) sampling scheme. Tightened inspection will be used when the supplier's recent quality has deteriorated and normal inspection will be used when the quality is found to be very good.

Calvin (1977) firstly introduced the concept of TNT sampling scheme for attributes with different sample sizes and zero acceptance number, say $\mathrm{TNTASS}\left(n_T, n_N; 0\right)$. As for variables, Muthuraj and Senthilkumar (2006) developed TNT sampling scheme with the same sample size and different acceptance numbers $\mathrm{TNTVSS}\left(n; k_T, k_N\right)$. Balamurali and Jun (2009) developed an optimization method for TNTVSS$\left(n; k_T, k_N\right)$ by minimizing the average sample number (ASN). Senthilkumar and Muthuraj (2010) considered TNT sampling scheme with the same acceptance numbers and different sample sizes $\left(n_N < n_T\right)$, called TNTVSS$\left(n_N, n_T; k\right)$. However, all previous existing studies about variables TNT sampling scheme are designed for processes with only one-sided specification limit (which requires only lower or upper specification limit) and the quality characteristic follows a normal distribution.

Wu and Pearn (2008) introduced a variables single sampling plan based on the third-generation of process capability index C_{pmk} that covered two-sided specification limit (which require both LSL and USL). The capability index C_{pmk} is an advanced index that takes into account the process yield as well as the process loss. In this paper, we attempt to develop a variables TNT sampling scheme based on C_{pmk}. In addition, it is worth to note that the proposed sampling scheme is developed based on the exact sampling distribution rather than an approximation approach, in order to enhance the decision to be more accurate and reliable.

2. Process Capability Indices

During the determination of sample size and critical acceptance value, lot fraction of defects is a required consideration. An effective alternative method to calculate the lot fraction of defects is using PCIs, which quantify the relationship between the actual process performance and the specification limits or tolerance (Wu et al, 2009). There are some indices can be applied i.e. $C_{\mathrm{pk}}, C_{\mathrm{pm}}$ and C_{pmk}. The C_{pmk} index is determined by combining the yield-based index C_{pk} and the loss-based index C_{pm}. Pearn and Kotz (2006) introduced the C_{pmk} index, the estimated index $\hat{C}_{\mathrm{pmk}}$ and the associated cumulative distribution function (CDF) as follows:

$$C_{\mathrm{pmk}} = \mathrm{Min}\left\{ \frac{\mathrm{USL} - «}{3\sqrt{±^2 + \left(« - T\right)^2}}, \frac{« - \mathrm{LSL}}{3\sqrt{±^2 + \left(« - T\right)^2}} \right\} = \frac{d - |« - M|}{3\sqrt{±^2 + \left(« - T\right)^2}} \quad (1)$$

$$\hat{C}_{\mathrm{pmk}} = \mathrm{Min}\left\{ \frac{\mathrm{USL} - \bar{X}}{3\sqrt{±^2 + \left(\bar{X} - T\right)^2}}, \frac{\bar{X} - \mathrm{LSL}}{3\sqrt{±^2 + \left(\bar{X} - T\right)^2}} \right\} = \frac{d - |\bar{X} - M|}{3\sqrt{±^2 + \left(\bar{X} - T\right)^2}} \quad (2)$$

where $d = \left(\mathrm{USL} - \mathrm{LSL}\right)/2,\ \bar{X} = \sum_{i=1}^{n} X_i / n,\ M = \left(\mathrm{USL} + \mathrm{LSL}\right)/2,\ S_n^2 = \sum_{i=1}^{n}\left(X_i - \bar{X}\right)/n$

$$F_{\hat{C}_{\mathrm{pmk}}}(y) = 1 - \int_0^{b\sqrt{n}/(1+3y)} G\left(\frac{\left(b\sqrt{n} - t\right)^2}{9y^2} - t^2 \right)\left[¤\left(t + ¶\sqrt{n}\right) + ¤\left(t - ¶\sqrt{n}\right) \right] \mathrm{d}t \quad (3)$$

for $y > 0$, where $b = d/±$, $¶ = \left(« - T\right)/±$. $G(.)$ is the CDF of the chi-square distribution, X_{n-1}^2, with $n-1$ degrees of freedom. $¤(.)$ is the PDF of the standard normal distribution, $N(0,1)$.

3. Constructing variables TNT sampling scheme

The operating procedures of the proposed TNT sampling scheme based on C_{pmk} are stated as follows: First, inspect under tightened single sampling plans with sample size n_T and critical acceptance value k. Accept the lots if $\hat{C}_{\mathrm{pmk}} \geq k$, and switch to normal inspection when t consecutive lots are accepted. Second, inspect under normal single sampling plans with sample size n_N and critical acceptance value k. Reject the lots if $\hat{C}_{\text{pmk}} < k$ and switch to tightened inspection after rejection for the additional next s lots in a row.

The TNT sampling scheme is specified by set of parameters: the sample size under tightened inspection (n_T), the sample size under normal inspection (n_N), the critical value for acceptance (k), the criteria for switching to normal and tightened inspection (s,t). It is noted that n_T must greater than n_N. As suggested by Senthilkumar and Muthuraj (2010), n_T can be expressed as $n_T = mn_N$ ($m > 1$). Then, the only parameter left need to be determined is (n_N, k).

The operating characteristic (OC) function which is usually called probability of acceptance for single inspection based on C_{pmk}, can be defined as:

$$P_a(C_{\text{pmk}}) = P(\hat{C}_{\text{pmk}} \geq k) = \int_0^{b\sqrt{n}/(1+3y)} G\left(\frac{\left(b\sqrt{n}-t\right)^2}{9y^2} - t^2\right)\left[\text{¤}\left(t+\text{¶}\sqrt{n}\right)+\text{¤}\left(t-\text{¶}\sqrt{n}\right)\right]\mathrm{d}t \qquad (4)$$

For all submissions, the cumulative probability of acceptance for lot quality level C_{pmk}, called the eventual probability of acceptance $\text{®}_A\left(C_{\text{pmk}}\right)$ is:

$$\text{®}_A\left(C_{\text{pmk}}\right) = \frac{\left(1-\left[P_N\left(C_{\text{pmk}}\right)\right]^s\right)\left(1-\left[P_T\left(C_{\text{pmk}}\right)\right]^t\right)\left[1-P_N\left(C_{\text{pmk}}\right)\right]+P_N\left(C_{\text{pmk}}\right)\left[P_T\left(C_{\text{pmk}}\right)\right]^t\left[1-P_T\left(C_{\text{pmk}}\right)\right]\left(2-\left[P_N\left(C_{\text{pmk}}\right)\right]^s\right)}{\left(1-\left[P_N\left(C_{\text{pmk}}\right)\right]^s\right)\left(1-\left[P_T\left(C_{\text{pmk}}\right)\right]^t\right)\left[1-P_N\left(C_{\text{pmk}}\right)\right]+\left[P_T\left(C_{\text{pmk}}\right)\right]^t\left[1-P_T\left(C_{\text{pmk}}\right)\right]\left(2-\left[P_N\left(C_{\text{pmk}}\right)\right]^s\right)} \qquad (5)$$

The average sampling number (ASN) for the proposed TNT sampling scheme is:

$$\text{ASN}\left(C_{\text{pmk}}\right) = \frac{n_T\left(1-\left[P_N\left(C_{\text{pmk}}\right)\right]^s\right)\left(1-\left[P_T\left(C_{\text{pmk}}\right)\right]^t\right)\left[1-P_N\left(C_{\text{pmk}}\right)\right]+n_N\left[P_T\left(C_{\text{pmk}}\right)\right]^t\left[1-P_T\left(C_{\text{pmk}}\right)\right]\left(2-\left[P_N\left(C_{\text{pmk}}\right)\right]^s\right)}{\left(1-\left[P_N\left(C_{\text{pmk}}\right)\right]^s\right)\left(1-\left[P_T\left(C_{\text{pmk}}\right)\right]^t\right)\left[1-P_N\left(C_{\text{pmk}}\right)\right]+\left[P_T\left(C_{\text{pmk}}\right)\right]^t\left[1-P_T\left(C_{\text{pmk}}\right)\right]\left(2-\left[P_N\left(C_{\text{pmk}}\right)\right]^s\right)} \qquad (6)$$

where $P_T\left(C_{\text{pmk}}\right)$ and $P_N\left(C_{\text{pmk}}\right)$ are probability of lots expected to be accepted under tightened $\left(n_T, k\right)$ and normal $\left(n_N, k\right)$ variable single sampling plans respectively, that provide at the following two equations (7) and (8) as follows:

$$P_T\left(C_{\text{pmk}}\right)=P_a\left(C_{\text{pmk}} \mid \left(n_T, k\right)\right)=P\left(\hat{C}_{\text{pmk}}^{(T)} \geq k\right)=\int_0^{\frac{b\sqrt{n_T}}{1+3k}} G_T\left(\frac{\left(b\sqrt{n_T}-t\right)^2}{9k^2}-t^2\right)\left[\text{¤}\left(t+\text{¶}\sqrt{n_T}\right)+\text{¤}\left(t+\text{¶}\sqrt{n_T}\right)\right] \mathrm{d}t \quad (7)$$

$$P_N\left(C_{\text{pmk}}\right)=P_a\left(C_{\text{pmk}} \mid \left(n_N, k\right)\right)=P\left(\hat{C}_{\text{pmk}}^{(N)} \geq k\right)=\int_0^{\frac{b\sqrt{n_N}}{1+3k}} G_N\left(\frac{\left(b\sqrt{n_N}-t\right)^2}{9k^2}-t^2\right)\left[\text{¤}\left(t+\text{¶}\sqrt{n_N}\right)+\text{¤}\left(t+\text{¶}\sqrt{n_N}\right)\right] \mathrm{d}t \quad (8)$$

Therefore, a common approach to propose a well-designed acceptance sampling plan is to require that the OC curve pass through two designed point $\left(\text{AQL}, 1-\sim\right)$ and $\left(\text{RQL}, \quad\right)$ Wu and Pearn (2008). The parameters $\left(n_N, k\right)$ determine by satisfying the following two equations and solve it simultaneously.

$$\text{®}_A\left(C_{\text{AQL}}\right) \geq 1-\sim \quad (9)$$

and

$$\text{®}_A\left(C_{\text{RQL}}\right) \leq \quad (10)$$

4. Analysis and Discussion

4.1 Behavior of OC curves with different combinations of (s,t).

The suggested $\left(s,t\right)$ values are $(1,1)$, $(1,2)$, $(2,3)$, and $(4,5)$. For each set of (s,t), we construct OC curves by equation (5). Figure 1 shows the OC curves of the proposed TNT sampling scheme under $(C_{\text{AQL}}, C_{\text{RQL}})=\left(1.50, 1.00\right)$ and $\left(\sim, \quad\right)=\left(0.05, 0.10\right)$. It can be observed that the scheme under $(s,t)=(4,5)$ can be single out from other combinations, the probability of acceptance will drop rapidly when quality level is small, therefore will provide better discriminatory power. This finding is in line with investigation found in Muthuraj and Senthilkumar (2006).

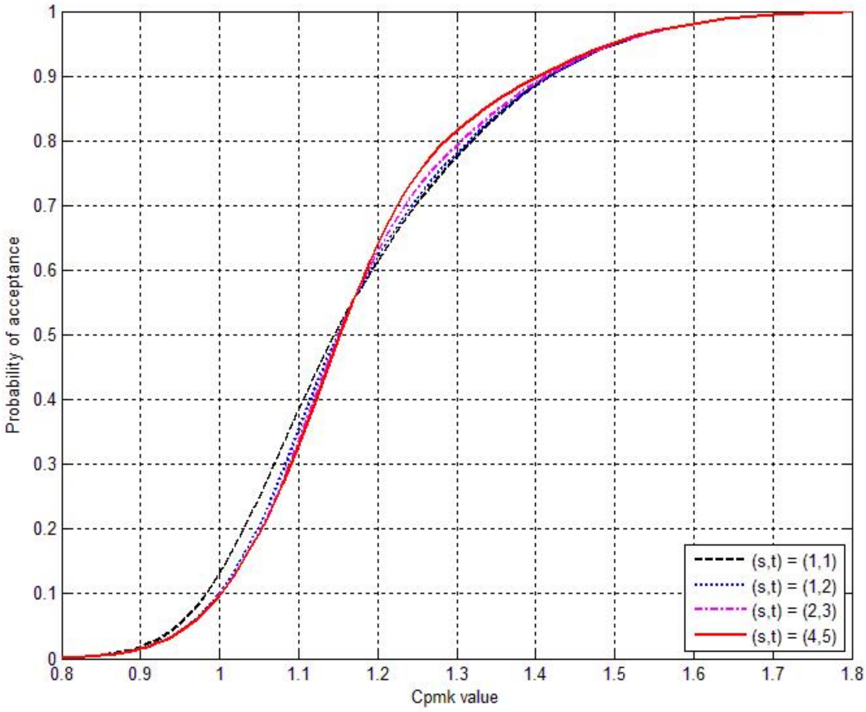

Figure 1. OC curves of TNTVSS with different combinations of (s,t).

4.2 Construction of table of plan parameters

For the convenience of applying the proposed to practitioners, we calculate and tabulate the parameters (n_N,k) of the proposed TNT sampling scheme under various quality levels $(C_{\text{AQL}},C_{\text{RQL}})$, producer's risk $(\sim)$, consumer's risk ( ), and $(s,t)=(4,5)$. Table 1 shows the plan parameters (n_N,k) for $m=1.5,\ 2,\ \text{and}\ 3$ under selected benchmarking quality levels $(C_{\text{AQL}},C_{\text{RQL}})=(1.33,1.00),\ (1.50,1.00),\ \text{and}\ (1.50,\ 1.33)$, $(\sim,\ \ )$ = (0.01, 0.05) and (0.05, 0.10).

From Table 1, the decision maker or inspector can determine the required sample size and the corresponding critical value for lot sentencing. For instance, if it was decided to set $m = 3$, $C_{\text{AQL}} = 1.50$ and $C_{\text{RQL}} = 1.00$, under ~ and risks are 0.05 and 0.10 respectively, then $n_N = 23$ and $n_T = 69$ should be sampled for normal and tightened inspection respectively with critical value $k = 1.1593$ with switching rule $(s,t) = (4,5)$. First conduct inspection under a tightened single sampling plan by taking 69 samples, accept the lot if $\hat{C}_{\text{pmk}} \geq 1.1593$. When $t = 5$ lots in a row is accepted then switch to normal inspection. Then inspect 23 samples from the lot, reject the lot if $\hat{C}_{\text{pmk}} < 1.1593$. When an additional lot is rejected in the next $s = 4$ lots after rejection, switch to tightened inspection. This plan will protect both consumer and producer to take a wrong decision i.e producer's ~ $-$risk is rejecting good lot with the probability of accepting the lot at least 95%, of rejecting a lot that has a defect level equal to the C_{AQL} = 1.50. Consumer's $-$risk is accepting bad lot with the probability to accept no more than 5%, of accepting a lot with a defect level equal to the C_{RQL} = 1.00 .

Table 1. The plan parameters of the proposed TNT sampling scheme.

$(C_{\text{AQL}}, C_{\text{RQL}})$	~		$(s,t)=(4,5)$					
			$m = 1.5$		$m = 2$		$m = 3$	
			n_N	k	n_N	k	n_N	k
(1.33, 1.00)	0.01	0.05	121	1.1200	108	1.1092	93	1.0949
	0.05	0.10	65	1.1303	57	1.1194	49	1.1044
(1.50, 1.00)	0.01	0.05	58	1.1815	51	1.1651	44	1.1434
	0.05	0.10	31	1.1986	27	1.1819	23	1.1593
(1.50, 1.33)	0.01	0.05	634	1.3921	570	1.3865	498	1.3791
	0.05	0.10	339	1.3968	301	1.3911	259	1.3835

5. Conclusions

This paper proposes variables TNT sampling scheme $(n_N, n_T; k)$ based on the process capability index C_{pmk}. The C_{pmk} index is developed by taking into account the process yield as well as process loss with two-sided specification limit. This scheme is suitable and useful when the inspection is costly and destructive because variable sampling scheme requires a smaller sample size than attribute sampling scheme. The plan parameters (n_N, k) of the proposed sampling scheme are determined by solving two nonlinear equations simultaneously under selected quality levels $\left(C_{\text{AQL}}, C_{\text{RQL}}\right)$, producer and consumer risks $\left(\sim, \quad\right)$, switching rule $\left(s, t\right)$, and m. Based on OC curve comparisons under different s and t, we find that under $\left(s, t\right) = \left(4, 5\right)$ will provide better discriminating OC curve than other combinations. The tabulated plan parameter (n_N, k) will provide convenient application of this proposed TNT sampling scheme. In addition, practitioners can determine the amount of sample size should be taken with the corresponding critical acceptance value should be required for normal and tightened inspection. Then practitioners can make a decision on product acceptance.

References

Balamurali, S., Jun, C.H., 2009. Designing of a variables two-plan system by minimizing the average sample number. Journal of Applied Statistics 36 (10), 1159-1172.

Calvin, T.W., 1977. TNT zero acceptance number sampling. American Society for Quality Control. Annual Technical Conference Transaction. Philadelphia, Pennsylvania, 1977; 35-39.

Muthuraj, D., Senthilkumar, D., 2006. Designing and construction of tightened-normal-tightened variable sampling scheme. Journal of Applied Statistics 33, 101-111.

Pearn, W.L., Kotz, S., 2006. Encyclopedia and Handbook of Process Capability Indices, Vol.12. World Scientific Publishing.

Senthilkumar, D., Muthuraj, D., 2010. Construction and selection of tightened-normal-tightened variable sampling scheme of type TNTVSS $(n_1, n_2; k)$. Journal of Applied Statistics 37 (3), 375-390.

Schilling, E.G., Neubauer, D.V., 2009. Acceptance Sampling in Quality Control, Second edition. CRC Press, New York.

Wu, C.W., Pearn, W.L., 2008. A variable sampling plan based on C_{pmk} for product acceptance determination. European Journal of Operational Research 184, 549 – 560.

Wu, C.W., Pearn, W.L., Kotz, S., 2009. An overview of theory and practice on process capability indices for quality assurance. International Journal of Production Economics 117 (2), 338– 359.