A JOINT OPTIMAL CONTROL STRATEGY FOR FARE PRICING AND SEAT INVENTORY CONTROL DECISIONS FOR AN AIRLINE WITH DEMAND LEAKAGES

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Abstract

Differentiated fare pricing is amongst widely practiced Revenue Management tactics in which an airline segments its demand into distinct fare classes. This strategy has been successfully implemented in airline industry for more than four decades. Earlier researches have shown that the benefits from differentiated pricing are evident for perfect market segmentation in which there is no leakage of the demand between the market segments. However, it is not uncommon to notice that the market segmentation is seldom prefect and regardless of a fencing strategy, demand leakage is often experienced. Thus, in realistic situation, demand behavior is expected to be uncertain and there will a demand leakage which will cannibalize demand from one market segment to another. This research addresses the issue of establishing an integrated framework to optimize the fare pricing, and seat inventory control for an airline that experiences the demand leakage in its fare classes. Models are proposed for an airline that experiences deterministic demand, stochastic demand, and stochastic demand whose distribution is unknown. These models are analyzed numerically to outline an integrated optimal control framework for fare pricing, seat inventory control decisions for an airline. Numerical experimentation show that the proposed integrated framework significantly improves the revenue gains for an airline, however, demand leakage between fare classes can substantially undermine the profitability of an airline.

Keywords: Revenue Management, Airline Industry, Demand Leakage

1 Introduction:

Revenue Management (RM) also known as Yield Management has been well recognized as an essential practice in many businesses. RM refers to the strategy and tactics used by a number of industries notably the airline industry, to manage the allocation of their capacity to different fare classes over time in order to maximize revenue (Philips, 2005). RM can be considered a special case of pricing with constrained supply. However, the two essential features of RM practice are: (i) market demand segmentation which is in the context of airline RM, it is about managing the set of fare classes, each of which often has a fixed fare price at least for a short selling period; and (ii) an airline can change the availability of fare classes over time. These two essential features are since legacy of airline RM in the 1980s. Prior to 1978, the airline industry in the United States was heavily regulated. Both schedules and fare were tightly controlled by Civil Aeronautics Board (CAB). Fare were held sufficiently high to guarantee airlines a reasonable return on their investments. In 1978, Congress passed the Airline Deregulation Act, consequently effective from 1983, all fare regulations were removed. This resulted in an advent of modern airline RM, which is among one of the the most important applications of management science and operations research (Bell, 1998). Initially RM started in airline industry in 1980s and since then it has emerged as an essential practice by numerous industries that include, travel, cargo, media, utilities and retails (Talluri and Ryzin, 2004). There has been a tremendous growth in RM research, a detailed coverage of the research in RM can be found in McGill and Ryzin (1999) and more recently in Chiang et al. (2007). An airline RM research has focused distinctly in four categories: forecasting; overbooking; quantity/inventory control (booking control); and pricing. However, there is a growing interest in joint approach to these categories, in experts' opinion an integration of pricing and quantity decision is expected to improve firms' revenues significantly (Cote et al., 2003). The RM practice is classified into Quantity-based RM and Price-based RM (Talluri and Ryzin, 2004). In quantity-based RM, the revenue of a firm is optimized by adjusting the availability of quantity for pre-determined prices. This practice is mostly observed in the aviation industry. Price-based RM is more commonly observed in the retail industries, prices are optimized for maximum profit for a fixed resource. In its simplest form, RM can be studied in the context of the Newsvendor (also referred as Newsboy) problem, having a simple yet an elegant structure. The problem is considered as a building block in stochastic inventory control. The problem serves as an excellent tool for examining how operational problems interact with marketing issues to influence decision-making process at a firm's level (Petruzzi and Dada, 1999). McGill and Ryzin (1999) showed that a single-leg flight with two fare classes RM problem is essentially equivalent to a single period inventory or newsvendor problem.

Price differentiation refers to the practice of a seller charging different prices to different customers, either for exactly the same good or for slightly different versions of the same good. Price differentiation is a powerful way for sellers to improve profitability. However, it also brings additional level of complexity in the pricing decision, and often creates a need of analytical techniques for optimal pricing and price differentiation. The price differentiation is also regarded as the ways that can enable extracting additional profit from a selling process by charging different prices. The tactics for price differentiation include charging different prices for exactly the same product, charging different price for different versions of the same product, and combination of the two. In economics literature, the price differentiation is also referred as price discrimination. Price discrimination is also subject to some controversies, but there are several less controversial strategies in price differentiation such as product versioning, regional pricing, and channel pricing. There is both an art and a science to price differentiation. The art of price differentiation is to find a way to divide the market into different segments such that higher prices are set for customer with a higher willingness to pay, and lower prices are set for customers with a lower willingness to pay. Fare price differentiation is among the principal tactics in airline RM, in which an airlines segments its market demand from only one fare class to multiple fare classes. Each fare class is differentiated with a fare price based on the willingness of passengers to pay, who are attributed by an airline to that particular fare class. Airlines often do both the price and time differentiation by offering early sales with deeply discounted tickets for customers who are willing to purchase ticket much in advance and will also be subject to penalties for any changes or cancelation; this sale is targeted to leisure/economy class passengers. Airlines also reserve their cabin capacity for late arriving business class passengers with higher willingness to pay. There are numerous examples which are observed in several other industries where a price differentiation framework leads customers to different channels using differentiated prices. For example, online versus retail store sales, in which, the firm may offer discounted prices for online sales but with less or no option of touch and feel. Whereas, physical retail stores sales are higher priced because the customers can interact with products and sales staff. It has been reported by many research studies (see Philips (2005) and Talluri and Ryzin (2004)) that the price differentiation by segmenting market brings additional profitability, however, different prices for distinct market segments would augment the movement of customers from one market segment to another. This behavior is referred as *cannibalization*. Once market is segmented there are various strategies that a firm adopt's to mitigate cannibalization and maintain the fences that differentiate the market. A fence is a device that is designed to preserve the market segmentation and limit spill over between segments, however, most fences are imperfect and do allow some amount of demand *leakage* from high priced market segment to lower priced market segments (Zhang et al., 2010). The firms can only apply improved fences by introducing restriction which would make difficult or time consuming for customers to cannibalize freely. Among such devices commonly observed are: early purchase, prolong processing time, return penalties, channel of purchase, etc.

Earlier researches have identified that maintaining appropriate fences is very essential for success of RM (see Philips (2005), Kimes (2002), Hanks et al. (2002), and Zhang et al. (2010)). However, there are still many concerns being unaddressed especially in the context of airline industry , such as: (i) How does the demand leakage impact the profitability for an airline that adopts an optimal price differentiation strategy? (ii) Given the fact that demand leakage is experienced by an airline, what can be a comprehensive control mechanism to achieve a joint optimization of fare pricing and seat inventory control decisions? and (iii) When demand distribution may not be known, what should be an optimal joint strategy for fare pricing, and seat inventory control decisions for an airline? In addition, how effective this strategy would be compared to if the demand distribution is known. In this paper, these concerns are addressed in an airline industry context.

The remainder of this paper is organized as follows: In Section 2, a brief literature review is presented. Section 3, defines the problem of an airline offering its single flight leg capacity into imperfectly segmented fare classes which causes demand leakage. Models are developed for an airline to address this issue under both the deterministic and stochastic demand situation. Section 4 presents a detailed numerical study with the proposed models, discusses the findings, and the impact of various problem related parameters onto an airline's profitability. Finally, in Section 5, the results are summarized with conclusions along with the suggestions for future research.

2 Literature Review:

A comprehensive overview of literature related to Airline RM is due to McGill and Ryzin (1999) and more recently in Chiang et al. (2007). An overview of pricing research in the context of RM is presented in Bitran and Caldentey (2003). Historically the first work related to quantity based RM was done by Littlewood (1972) with an application to airline industry. Later Belobaba (1987; 1989) extended Littlewood (1972)'s work and proposed the commercially most practiced Expected Marginal Seat Revenue (EMSR) heuristic. Among recent works on seat allocation are Brumelle and Walczak (2003) in which they considered dynamic version of revenue management problem with multiple demand. In contrast, Bertsimas and Popescu (2003) studied seat allocation problem in a flight network revenue management situation. An overview of the pricing research in the context of revenue management is done by Bitran and Caldentey (2003). Feng and Gallego (2000) studied, when it is optimal to change the price within a given allowable time dependent price paths. Each path follows general Poisson process with Markovian, time dependent and predictable intensities. An efficient algorithm is proposed to determine the optimal pricing policy. In Gallego and van

Ryzin(1994; 1997) demands are considered price dependent and the demand intensity is a function of prices for the products and time at which these prices are offered. They proposed a simple heuristics in which the demand processes are replaced by the demand expectation. The heuristic results asymptotically optimal solution for this stochastic point control problem. An optimal dynamic pricing policy is also identified that maximizes the total expected revenue over a finite planning horizon using intensity control theory.

Airline RM problem can be considered as an extended Newsvendor problem (Philips, 2005), a common objective in both problems is to maximize the profit by either setting capacity allocation rule or pricing or the combination of both. In airline RM problem usually there are more than one fare classes. Hence the problem becomes more complex. Mostly in RM seat allocation studies nesting booking limits are determined which assume low fare demand is observed before the high fare class demand. Petruzzi and Dada (1999) studied an extended Newsvendor problem where the pricing and capacity allocation is determined simultaneously. They established a number of new results with a comprehensive review of existing literature. A single period newsvendor problem is a building block in stochastic inventory control. It incorporates the fundamental techniques of stochastic decision-making and can be applied to a much broader scope. The problem is well researched and its history traces back to Edgeworth (1888)'s work in which it first appeared in the banking context. During the 1950's, war effects enabled the expansion of research in this area, leading to the formulation of this problem as the inventory control problem. Arrow et al. (1951) showed that it is critical to have optimal buffer stocks in an inventory control system. Porteus (1990) and Lee and Nahmias (1990) presented a thorough review of the newsvendor problem using a stochastic demand. In most studies, the pricing is considered as fixed parameter rather than a decision variable. Whitin (1955) was the first to discuss the pricing issues in the inventory control theory. Mills (1959) extended Whitin (1955)'s work by modeling the uncertainty of the price sensitive demand. He suggested an additive form for the study and assumed that the stochastic demand was a summation of the price-dependent risk-less demand and of the random factor. The risk-less demand is considered a deterministic function of the price. The most evident benefit of such modeling is that the random behavior of the demand is captured using standard distributions independent of pricing. Karlin and Carr (1962) presented a multiplicative form of demand. In this model, the price dependent stochastic demand is considered as the product of the riskless demand function and of the random factor. Both the additive and multiplicative models are fundamental to the pricing problem. Some subsequent contributions to the additive model are due to Ernst (1970), Young (1978), Lau and Lau (1988) and Petruzzi and Dada (1999). The contributions to the multiplicative model include Nevins (1966), Zabel (1970), Young (1978) and Petruzzi and Dada (1999). Mieghem and Dada (1999) studied the quantity and pricing of the price versus the production postponement in the competitive market. A coordination of the dynamic joint pricing and production in a supply chain is studied by Zhao and Wang (2002) using a leader/follower game. Optimal control policies are identified for the channel coordination. Bish and Wang (2004) studied the optimal resource investment decision on a two-product, price-setting firm that operates in a monopolistic market and that employs a postponed pricing scheme. The principles on the firm's optimal resource investment decision are provided. Gupta et al. (2006) developed a pricing model and heuristic solution procedures for clearing end-of-season inventory. Yao et al. (2006) revisited the standard newsvendor problem and its extension with pricing. The work generalizes the problem under the multiplicative modeling approach and shows joint concavity of the revenue function of the problem under various stochastic demand distributions. The analysis of the problem using the additive modeling approach is presented in Yao (2002).

Earlier efforts towards integration of pricing and seat allocation in airline RM are due to Weatherford (1997) where the customer demand is assumed to be normally distributed with a mean depending on a linear function of price. Feng and Xiao (2001) also studied the integration aspect of capacity and pricing of perishable assets and presented a comprehensive model under stochastic demand situation. Yaghin et al. (2012) proposed possibilistic multiple objective pricing and lot-sizing model with multiple demand classes. In the context of airline industry, Oster and Pickrell (1988) discussed the issues of code sharing, joint fares, and competition in the regional airline industry. A bi-level mathematical programming approach is developed for joint determination of fare price and seat allocation by Cote et al. (2003). Li (2001) studied pricing non-storable perishable goods by using a purchase restriction with an application to airline fare pricing. Raza and Akgunduz (2008) proposed a game theoretic model for an integrated approach for fare pricing competition in duopoly with seat allocation. Later, Raza and Akgunduz (2010) extended their work in cooperative game setting using Nash bargain solution (Nash, 1950). More recently, there is an interest to study market segmentation and demand leakages due to imperfect segmentation mainly in the context of multi-item newsvendor problem with pricing. Zhang and Bell (2007) studied the effect of market segmentation with demand leakage between market segments on a firm's price and inventory decisions. In continuation, Zhang et al. (2010) investigated optimal fences on joint pricing and inventory decisions with demand leakages using a sequential optimization procedure. However, to author's knowledge there are no such studies which have been done to outline an integrated framework to these issues with an application to airline industry.

There are several service industries that lack historical data in order to accurately estimate the demand behavior, there is always a lack of historical data for some retailing sector such as fashion products, and with no exception to airline industry. McGill (1995) has claimed that airlines record the historical data of ticket sales but not the actual demand. In addition, Kurawarwala and Matsuo (1996) agreed with the arguments presented in Fisher and Raman (1996), and acknowledge that the lack of actual demand data is not only limited to fashion products but also to many other style goods. It is therefore reasonable to conclude that airline industry experiences among the most precarious demand behaviors, and in many of these situations historical sales information cannot be captured, therefore, the distribution free analysis primarily suggested in Scarf (1952), and later highlighted by Gallego and Moon (1993) would bring significant amount of insights for an airline's profitability.

3 Model Development:

In this section, we propose a mathematical model of RM problem for an airline offering the two fare classes to its passengers. Airline segments its market demand using a price differentiation strategy which results in different fare classes. Hence market segmentation is achieved through creation of fare classes using a fare price differentiation strategy. However, this segmentation is regarded as imperfect, and it is assumed that the passengers who belong to the full fare class would cannibalize to a discounted fare class. When an airline offers its fares into a monopolistic market, it's problem is to exercise optimal integrated control on fare class design using a price differentiation strategy, fare pricing, and seat inventory control. The models in this situation for an airline are developed assuming both the deterministic and stochastic market demands and to the case when the demand distribution is unknown to an airline. Single-leg seat inventory control is also referred as single-resource capacity control in airline industry. The objective is to best allocate capacity of a resource (seats in the cabin) among different classes of customers. Littlewood (1972) proposed his famous Littlewood's rule for two fare class which determines seat allocation (booking limits) for each fare class. The rule assumes *sequential arrival* of the customers, which means that demand for discounted fare class is observed prior to full fare class. Littlewood (1972)'s rule while assuming sequential arrival, optimally allocates the cabin capacity among fare classes by estimating the booking limits (protection level), and the resulting control is referred as *nesting control*.

Consider an airline offers two immediately adjacent fare classes at differentiated fare price in its single flight leg in monopoly, and it has a cabin capacity, c. The fare class 1 is for the passengers who are willing to pay the full fare price, p_1. The fare class 2 is for passengers with willingness to pay the discounted fare price, p_2. Without loss of generality it is assumed that $p_1 > p_2$. In a perfect market segmentation situation, a linear riskless price dependent demand $[\alpha_i - \beta_i\, p_i]^+$, where $\alpha_i,\ \beta_i > 0, \quad \forall i = \{1, 2\}$, is observed by an airline in fare class $i = \{1, 2\}$. The linear function is popular in the literature because of its simplicity and more importantly it has sufficient abilities to capture important managerial decision aspects (Choi, 1996; Chiang and Monahan, 2005; Zhang et al., 2010). Next, it is assumed that the fences (segments) observed due to this fare price differentiation strategy are imperfect, and therefore, an airline experiences $\gamma(p_1 - p_2)$ demand that is leaked from full fare class to discounted fare class, where $\gamma > 0$. γ is defined as leakage rate. The demand $\gamma(p_1 - p_2)$ from full fare price segments leaks to discounted fare class segment. If $\gamma = 0$, then airline has perfect fencing between the two fare classes, thus the leakage is zero. It is also assumed that the fences are achieved with no additional investment. As discussed previously, the linear demand curve better suits in this research, however, incorporation of the leakage into the existing linear demand curve would yield following adjusted demand functions:

$$y_1(p_1, p_2, \gamma) = \alpha_1 - \beta_1 p_1 - \gamma(p_1 - p_2) \tag{1}$$

$$y_2(p_1, p_2, \gamma) = \alpha_2 - \beta_2 p_2 + \gamma(p_1 - p_2) \tag{2}$$

For the convenience of analysis, $y_1 = y_1(p_1, p_2, \gamma)$ and $y_2(p_1, p_2, \gamma)$, are represented by y_1, and y_2 respectively. Now the scenario is extended where an airline experiences price dependent stochastic demand $D_i(p_1, p_2, \gamma, \xi_i)$ for a fare class i, $\forall\, i = \{1, 2\}$. Briefly, $D_i(p_1, p_2, \gamma, \xi_i)$ is represented by D_i, and it has two components: one is from the price dependent risk-less demand y_i and the other from is the stochastic price independent demand ξ_i. The price independent stochastic demand ξ_i has a probability distribution function f_i and a cumulative probability distribution function F_i, both are continuous,twice differentiable, inverseable, and following an increasing failure rate (see Lariviere (2006), Petruzzi and Dada (1999)). These characteristics are often found in many commonly used distributions such as Uniform, Normal, Lognormal, etc. (Bain and Engelhardt, 1992). Moreover, ξ_i is assumed to be bounded in $[\underline{\xi}_i, \overline{\xi}_i]$, the expectation of ξ_i is μ_i with the standard deviation σ_i. As can be noticed from literature review, there are two types of modeling approaches often considered to incorporate the randomness, ξ_i: i) additive; and ii) multiplicative. Recent discussions on these modeling approaches can be found in Petruzzi and Dada (1999) and Yao et al. (2006). In this paper, the additive modeling approach is used, thus price dependent stochastic demand D_i, for the fare class i is the sum of riskless demand y_i and random factor ξ_i such that:

$$D_i = y_i + \xi_i, \quad \forall\, i = \{1, 2\} \tag{3}$$

Petruzzi and Dada (1999) suggested that for an additive approach, a more convenient risk-less demand is linear function which has been already adopted for this research and discussed previously is this section. Following earlier works, this paper also assumes that when an airline experiences a price dependent stochastic demand, it observes sequential arrival of demand, and in this context the discounted fare class demand is observed prior to the full fare class demand in the sequential arrival process resulting a nesting control. The airline's optimization problem, P, would be:

$$P: \quad \pi = \underset{p_1,p_2,x_1,x_2}{\text{Max}} \; p_1 \min\{D_1, x_1 + x_2 - \min\{D_2, x_2\}\} + p_2 \min\{D_2, x_2\} \tag{4}$$

subject to:

$$x_1 + x_2 \leq c \tag{5}$$

In problem, P, an optimal expected revenue would be, $\pi^*(p_1^*, p_2^*, x_1^*, x_2^*) = \underset{p_1,p_2,x_1,x_2}{\text{Max}} \pi(p_1, p_2, x_1, x_2)$. Thus, the airline's problem is to determine an optimal integrated decision on fare prices, p_1^*, p_2^*, and also the seat inventory controls, x_1^*, and x_2^* for full fare and discounted fare classes respectively. Following earlier works from Raza and Akgunduz (2008), and Raza and Akgunduz (2010) Equation 4 is simplified, finally, an airline's constrained optimization problem, P can be written as:

$$P: \quad \pi = \underset{p_1,p_2,x_1,x_2}{\text{Max}} \; p_1\, x_1 + p_2\, x_2 + (p_1 - p_2) \int_{\underline{\xi}_2}^{x_2 - y_2} F_2(\xi_2) d\,\xi_2 - p_1 \int_{\underline{\xi}_1}^{x_1 + \int_{\underline{\xi}_2}^{x_2-y_2} F_2(\xi_2) d\,\xi_2 - y_1} F_1(\xi_1) d\,\xi_1 \tag{6}$$

subject to:

$$x_1 + x_2 \leq c \tag{7}$$

It can be clearly noticed that the problem, P is a constrained nonlinear optimization problem. The advantage of the formulation of the problem, P presented in Equations 6-7 can be extended to the more than two fare classes very conveniently. In the revenue expression presented in Equation 6 , the first two terms denote the deterministic revenues that an airlines observe by offering two fare prices, p_1, and p_2 respectively, and allocates x_2 capacity of its cabin c, to discounted fare class, and x_1 to the full fare class. The third term in the expression is the expected revenue gain that is observed by exercising the nested control. The expected demand $\int_{\underline{\xi}_2}^{x_2-y_2} F_2(\xi_2) d\,\xi_2$ is protected from the discounted fare class 2, in the sequential arrival process and would be reserved for full fare class 1. This will yield a revenue gain mainly due to fare price differential, $p_1 > p_2$. The last term is expected loss in the revenue due to the fact that the observed demand for full fare is less than the capacity allocated, that is $x_1 + \int_{\underline{\xi}_2}^{x_2-y_2} F_2(\xi_2) d\,\xi_2$.

4 Numerical Analysis:

In this section, a numerical study is presented to examine the impact of demand leakage rate and demand uncertainty on an airline's optimal strategies for joint fare pricing, seat inventory control, and fare class segmentation. In an illustrative example, the model parameters adopted from Zhang et al. (2010) but customized for an airline industry setting with some additional parameters, thus $c = 100$, $\alpha_1 = 80$, $\beta_1 = 0.2$, $\alpha_2 = 180$, $\beta_2 = 0.8$, $\mu_1 = \mu_2 = 0$. First the deterministic demand situation is considered, and then the study is extended to the case of stochastic demand.

4.1 Deterministic Demand:

In the deterministic case, it is obvious to notice that $\sigma_i = 0, \forall i = \{1, 2\}$, and therefore, demand information is perfectly known to an airline. In Table 1, the impact of demand leakage (γ) is studied. In addition, a comparative framework to calibrate an airline's fare class segmentation strategy can be realized by comparing the revenues that an airline could yield offering an equivalent single fare class with price dependent riskless demand, $260 - p$. This strategy is also referred as single fare class or 1-segment in this paper. In the case of single fare class, the problem resembles single period standard newsvendor problem with pricing which can be optimized to global optimality for most standard stochastic demand behaviors (Petruzzi and Dada, 1999; Yao, 2002). The optimal revenue to an airline for 1-segment situation is, $\pi^{*(1)} = 16000$, at an optimal fare price, $p^* = 160$. In Table 1, it can be noticed that when an airline offers two fare classes, and exercises an optimal joint control strategy that is able to yield a revenue, $\pi^{*(2)} = 17225$ which is about 8% better than the single fare class optimal revenue, given that the fare class segmentation is perfect, i.e., $\gamma = 0$. With an increase in the amount of demand leakage rate, γ from the full fare price segment to discounted fare class segment, the optimal revenue gains observed by an airline drops significantly. Thus, when the proportion of demand leakage rate is, $\gamma = 100$, the optimal revenue to an airline using the proposed integrated optimal framework is 16001.96 which is now only 0.01% superior to the corresponding unsegmented optimal revenue. In Table 1, it is also noticed that with an increase in the leakage, an optimal joint strategy for an airline would be to mitigate the demand leakage by reducing the price differentiation between the two fare classes, however, it will keep the same capacity allocation for each fare class it offers.

Table 1: Impact of γ in deterministic demand situation

γ	$\pi^{*(1)}$	x_1^*	x_1^*	p_1^*	p_2^*	$\pi^{*(2)}$
0		34	66	230.00	142.50	17225.00
0.1		34	66	203.08	149.23	16753.85
0.5		34	66	176.97	155.76	16296.97
1		34	66	169.66	157.59	16168.97
1.5		34	66	166.75	158.31	16118.07
2	16000	34	66	165.19	158.70	16090.74
5		34	66	162.17	159.46	16037.98
10		34	66	161.10	159.72	16019.29
20		34	66	160.56	159.86	16009.72
50		34	66	160.22	159.94	16003.91
100		34	66	160.11	159.97	16001.96

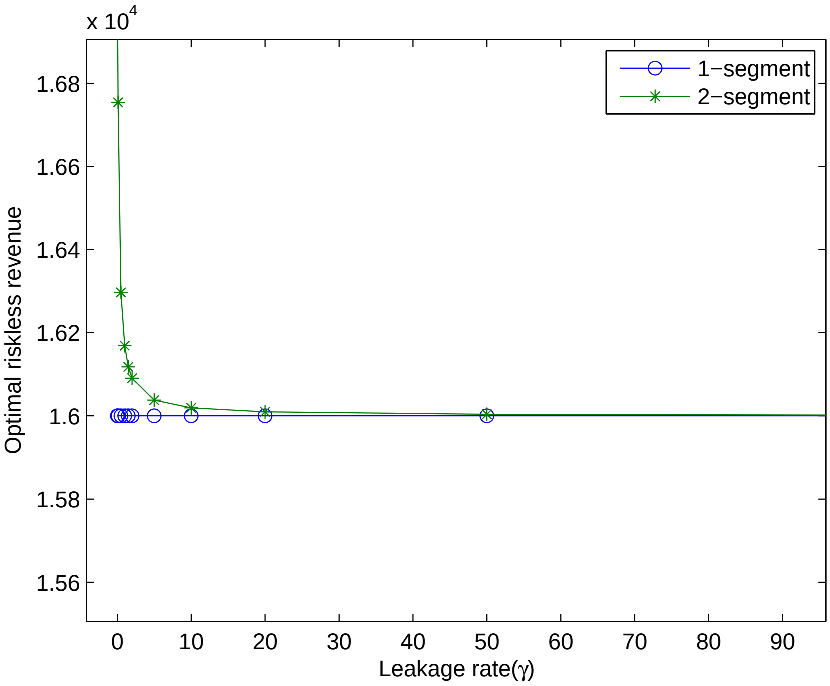

Figure 1: Impact of leakage (γ) for deterministic demand

4.2 Stochastic Demand:

In the stochastic demand situation, price dependent riskless demand parameters are same as mentioned in the previous section, also we have already assumed that for each fare class segment $\mu_i = 0, i \forall \{1, 2\}$. Furthermore, again for simplicity $\sigma_i,\ i = \{1, 2\}$ are assumed equal for each fare class segment, thus, $\sigma = \sigma_i$. $\sigma = \{1, 5, 10, 15\}$. In addition to this, consistent with Mostard et al. (2005), $\xi_i \in \left[-\sqrt{3}\sigma, \sqrt{3}\sigma\right]$. In complex problem like this one, numerical experimentation is conducted with uniform, and normal distributions only. Tables 2 and 3 report numerical experimentation when an airline faces price dependent stochastic demands which are uniformly and normally distributed respectively. Some findings are similar to that are already noticed with the deterministic demand situation, such as, at any given demand variability i.e., σ, an increase in demand leakage rate, γ drives an airline to increase seats allocation for its discounted fare class. Naturally, an airline to mitigate demand leakage reduces the price differential between the two fare classes. It reduces the full fare class price, and increases the price of the discounted fare class, but in different proportions. It can be inferred from the numerical experimentation reported in Tables 2 and 3 that the proposed sequential optimization approach is very competitive to the joint optimization, more importantly, its performance is reasonably consistent with regards to demand leakage and demand variability factors which are considered in this numerical experimentation. For a given demand variability, the performance of the sequential optimization marginally diminishes compared to joint optimization with an increase in demand leakage. Nevertheless, it is important to notice that the use of joint optimization approach is expected to return slightly higher revenue gains compared to that of the corresponding sequential optimization approach, and the performance is slightly sensitive to demand leakage and demand variability.

Next, the revenue gains of implementing the proposed integrated approach to market segmentation are compared to the corresponding single fare class segment revenues. Interestingly, when the price dependent demand in both fare classes are uniformly distributed demand, the resulting equivalent single segment demand distribution is obtained by convolution (see Bracewell (1986), Bain and Engelhardt (1992), and Boucher (2013)) of the two uniform distributions, and it follows triangular distribution such that $\xi \sim tri[-2\sqrt{3}\sigma, 2\sqrt{3}\sigma]$, where ξ is the random factor for single fare class. Additionally, the convolution of the two market segments that are normally distributed, follows a normal distribution with mean zero and standard error of $\sqrt{2}\sigma$, and bounded such that $\xi \in [-\sqrt{6}\sigma, \sqrt{6}\sigma]$. This comparative framework was earlier used in Zhang et al. (2010) in a related study on market segment in a firm's context. It is noticeable that both the demand variability and the demand leakage impact onto an airline' profitability using the proposed integrated approach. For instance, in the case of uniformly distributed demand, at $\sigma = 1$, when there is a zero demand leakage the revenue yield is 7.8% better than the corresponding single segment revenue, which drops to 0.38% only when the leakage rate is $\gamma = 5$. In the case of normally distributed demand, at $\sigma = 1$, at $\gamma = 0$, the revenue yield is 7.84% better than the corresponding optimal single segment revenue. However, at $\gamma = 5$, the revenue gain due to segmentation is only 0.45% superior to the corresponding optimal single segment revenue. Noticeably, the performance of segmentation strategy with demand leakages was found to be slightly better in the case of normally distributed demand.

As discussed earlier, an interesting avenue of this problem can the case when the demand distribution to an airline in unknown. However, the simple parameters, mean and standard deviation of the stochastic demand are the only available information. In this situation, the distribution free approach that is fundamentally based on Scarf (1952)'s rule is utilized. Gallego and Moon (1993) suggested a framework to calibrate the effectiveness of the distribution free approach and built a performance measure, Expected Value of Additional Information (EVAI), which is determined by taking the difference of the optimal revenue when demand distribution is perfectly known and the optimal revenue achieved using the control decisions (fare pricing and seat inventory control) determined by using the distribution free approach and the distribution is known. The numerical experimentation summary in this situation is reported in Table 4. As may be notice that with low demand variability, $\sigma = 1$, the relative percentage deviation in the revenue due to not knowing the demand distribution precisely at $\gamma = 0$ was 0.185% and 0.162% in case the unknown demand may have followed the uniformly and normally distributed demands respectively. At an increasing demand leakage, $\gamma = 5$, the relative deviations are 0.222% and 0.181% for uniformly and normally distributed demand. Thus, it may be concluded that the demand leakage rate does not significantly impact the performance of the distribution free approach. At a higher demand variability, $\sigma = 15$, when demand leakage is zero, this relative deviation is 3.424% and 2.881% for uniformly and normally distributed demands respectively. Whereas, at a higher demand leakage rate, i.e., $\gamma = 5$, the relative deviations are 4.075% and 3.171% for uniformly and normally distributed demands. This is inferred that the rate of demand leakage, γ, does not significantly impact the performance of the proposed distribution free approach. However, the demand variability, impacts considerably the performance of the distribution approach to the problem, when the demand distribution is unknown.

A comprehensive comparison of the optimal revenue gains is reported in Figures 2, and 3 at two distinct demand variabilities, $\sigma = 1$, and $\sigma = 15$ are considered. These figures report the comparison of optimal revenues for the four distinct situations: 1-segment, $\pi^{*(1)}$; optimal 2-segment revenues, $\pi^{*(2)}$, for both the situations, when sequential and join optimization procedures are used; and lastly, 2- segment revenue, when the demand distributions for both fare classes are unknown and the distribution free approach is used with joint optimization procedure. It is clearly noticed that with an increase in the demand variability, an airline may loose competitive revenue gain over single fare class using a segmentation strategy as the demand leakages increase. Precisely knowing the demand distribution can be vital in contributing towards an airline's profitability, however, in the situation when the demand is unknown to an airline, the use of distribution free approach can be very competitive as long as the demand variability is not substantially high. This performance behavior of the distribution free approach can be noticed in Figure 4.

Table 2: Numerical experimentation with uniform distribution

			Sequential optimization						Joint optimization				
σ	γ	$\pi^{*(1)}$	x_1^*	x_2^*	p_1^*	p_1^*	$\pi^{*(2)}$		x_1^*	x_2^*	p_1^*	p_1^*	$\pi^{*(2)}$
	0		1.70	98.30	230.00	142.50	17125.41		32.96	67.04	230.55	143.37	17147.44
	0.1		1.68	98.32	203.08	149.23	16665.91		32.87	67.13	203.57	149.96	16680.05
1	0.5	15911.78	1.74	98.26	176.97	155.76	16220.34		29.61	70.39	177.41	156.32	16227.88
	1		1.73	98.27	169.66	157.59	16095.50		26.22	73.78	170.08	158.09	16101.45
	2		21.14	78.86	165.19	158.70	16019.21		30.74	69.26	165.60	159.17	16024.25
	5		6.60	93.40	162.17	159.46	15967.76		30.82	69.18	162.59	159.89	15972.21
	0		3.54	96.46	230.00	142.50	16727.04		28.51	71.49	232.11	146.47	16823.91
	0.1		0.85	99.15	203.08	149.23	16314.17		22.22	77.78	204.94	152.52	16374.78
5	0.5	15552.72	3.54	96.46	176.97	155.76	15913.82		25.48	74.52	178.69	158.19	15944.67
	1		3.55	96.45	169.66	157.59	15801.65		27.49	72.51	171.36	159.73	15825.40
	2		24.74	75.26	165.19	158.70	15733.11		26.92	73.08	166.89	160.65	15752.83
	5		0.85	99.15	162.17	159.46	15686.88		5.82	94.18	163.88	161.27	15704.03
	0		6.40	93.60	230.00	142.50	16229.07		1.93	98.07	232.74	149.64	16396.15
	0.1		2.80	97.20	203.08	149.23	15874.50		3.58	96.42	205.55	155.02	15975.96
10	0.5	15094.38	0.78	99.22	176.97	155.76	15530.67		20.57	79.43	179.45	159.85	15579.25
	1		16.68	83.32	169.66	157.59	15434.34		20.48	79.52	172.21	161.12	15470.46
	2		4.86	95.14	165.19	158.70	15375.47		10.76	89.24	167.82	161.88	15404.57
	5		0.29	99.71	162.17	159.46	15335.77		20.21	79.79	164.85	162.38	15360.40
	0		2.14	97.86	230.00	142.50	15731.11		1.06	98.94	232.12	152.22	15950.36
	0.1		0.42	99.58	203.08	149.23	15434.82		1.41	98.59	205.15	156.90	15564.49
15	0.5	14630.13	1.02	98.98	176.97	155.76	15147.52		13.49	86.51	179.50	160.94	15205.82
	1		1.02	98.98	169.66	157.59	15067.02		13.12	86.88	172.44	161.97	15108.61
	2		8.01	91.99	165.19	158.70	15017.83		12.88	87.12	168.16	162.57	15049.99
	5		8.02	91.98	162.17	159.46	14984.66	.	12.48	87.52	165.29	162.96	15010.81

Table 3: Numerical experimentation with normal distribution

			Sequential optimization					Joint optimization				
σ	γ	$\pi^{*(1)}$	x_1^*	x_2^*	p_1^*	p_2^*	$\pi^{*(2)}$	x_1^*	x_2^*	p_1^*	p_1^*	$\pi^{*(2)}$
	0		32.27	67.73	230.00	142.50	17147.24	32.77	67.23	230.32	143.13	17162.38
	0.1		32.27	67.73	203.08	149.23	16685.19	32.71	67.29	203.37	149.75	16694.85
1	0.5	15914.02	32.27	67.73	176.97	155.76	16237.14	32.63	67.37	177.25	156.15	16242.27
	1		32.27	67.73	169.66	157.59	16111.61	32.61	67.39	169.93	157.93	16115.65
	2		32.27	67.73	165.19	158.70	16034.90	32.60	67.40	165.46	159.02	16038.32
	5		32.27	67.73	162.17	159.46	15983.16	32.59	67.41	162.45	159.75	15986.18
	0		25.34	74.66	230.00	142.50	16836.21	27.68	72.32	231.22	145.42	16904.98
	0.1		25.34	74.66	203.08	149.23	16410.57	27.38	72.62	204.22	151.63	16453.68
5	0.5	15564.17	25.34	74.66	176.97	155.76	15997.82	27.04	72.96	178.12	157.51	16019.94
	1		25.34	74.66	169.66	157.59	15882.18	26.93	73.07	170.83	159.12	15899.29
	2		25.34	74.66	165.19	158.70	15811.52	26.86	73.14	166.38	160.10	15825.78
	5		25.34	74.66	162.17	159.46	15763.85	26.81	73.19	163.38	160.76	15776.31
	0		16.68	83.32	230.00	142.50	16447.43	20.99	79.01	231.58	147.89	16570.50
	0.1		16.68	83.32	203.08	149.23	16067.29	20.42	79.58	204.66	153.57	16142.82
10	0.5	15117.57	16.68	83.32	176.97	155.76	15698.68	19.78	80.22	178.74	158.83	15735.70
	1		16.68	83.32	169.66	157.59	15595.40	19.58	80.42	171.53	160.25	15623.31
	2		16.68	83.32	165.19	158.70	15532.29	19.46	80.54	167.14	161.10	15555.05
	5		16.68	83.32	162.17	159.46	15489.73	19.37	80.63	164.19	161.66	15509.19
	0		8.01	91.99	230.00	142.50	16058.64	14.00	86.00	231.21	149.99	16225.47
	0.1		8.01	91.99	203.08	149.23	15724.01	13.20	86.80	204.51	155.16	15824.37
15	0.5	14665.02	8.01	91.99	176.97	155.76	15399.53	12.31	87.69	178.95	159.82	15446.46
	1		8.01	91.99	169.66	157.59	15308.62	12.03	87.97	171.87	161.05	15342.95
	2		8.01	91.99	165.19	158.70	15253.06	11.85	88.15	167.57	161.78	15280.27
	5		8.01	91.99	162.17	159.46	15215.60	11.73	88.27	164.68	162.26	15238.26

Table 4: Numerical experimentation with distribution free approach

		Distribution free optimal control				EVAI	
σ	γ	x_1^*	x_2^*	p_1^*	p_2^*	Uniform	Normal
1	0	34.47	65.53	230.27	143.25	31.64	27.72
	0.1	34.39	65.61	203.40	149.90	32.97	28.17
	0.5	34.32	65.68	177.37	156.32	34.40	28.59
	1	34.29	65.71	170.09	158.11	34.84	28.70
	2	34.28	65.72	165.64	159.21	35.13	28.76
	5	34.27	65.73	162.63	159.94	35.33	28.81
5	0	36.12	63.88	230.38	145.48	165.09	141.95
	0.1	35.77	64.23	203.81	151.83	172.41	144.64
	0.5	35.40	64.60	178.20	157.85	179.87	146.75
	1	35.29	64.71	171.06	159.51	182.08	147.24
	2	35.22	64.78	166.71	160.51	183.46	147.51
	5	35.17	64.83	163.77	161.18	184.41	147.68
10	0	37.85	62.15	228.93	146.99	342.83	293.43
	0.1	37.18	62.82	202.86	153.01	358.60	299.10
	0.5	36.47	63.53	177.93	158.56	373.81	302.91
	1	36.25	63.75	171.03	160.06	378.08	303.61
	2	36.11	63.89	166.83	160.95	380.68	303.95
	5	36.02	63.98	164.01	161.55	382.44	304.14
15	0	39.32	60.68	226.21	147.49	528.11	454.30
	0.1	38.33	61.67	200.72	153.21	552.90	462.68
	0.5	37.29	62.71	176.61	158.32	575.64	467.45
	1	36.98	63.02	169.98	159.66	581.70	468.07
	2	36.79	63.21	165.97	160.45	585.32	468.27
	5	36.65	63.35	163.28	160.98 .	587.72	468.33

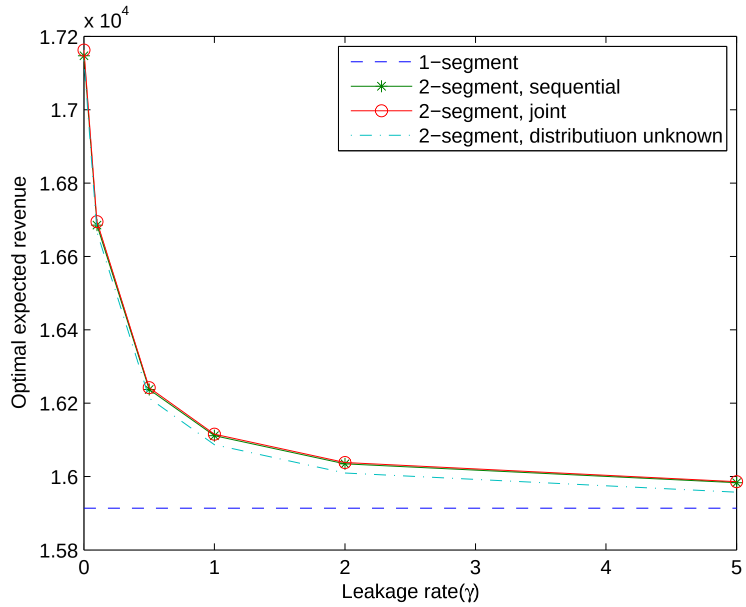

Figure 2: Impact of leakage rate (γ) for normally distributed demand with $\sigma = 1$

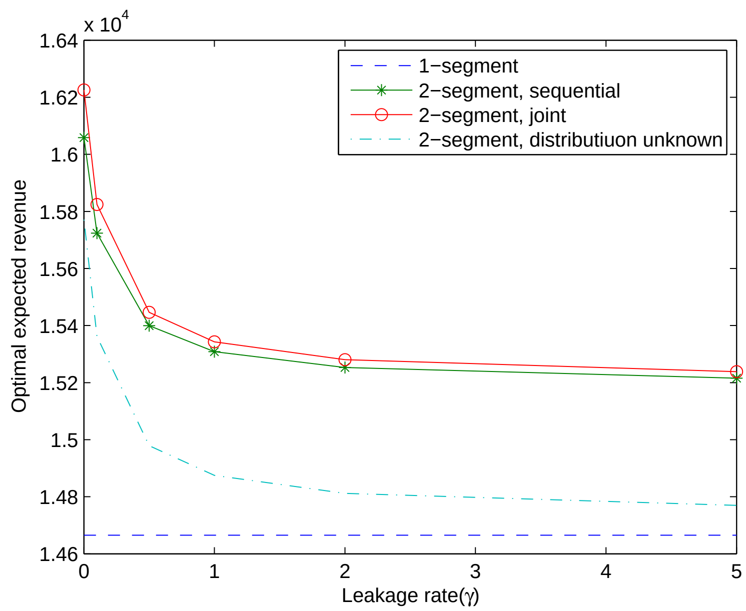

Figure 3: Impact of leakage rate (γ) for normally distributed demand with $\sigma = 15$

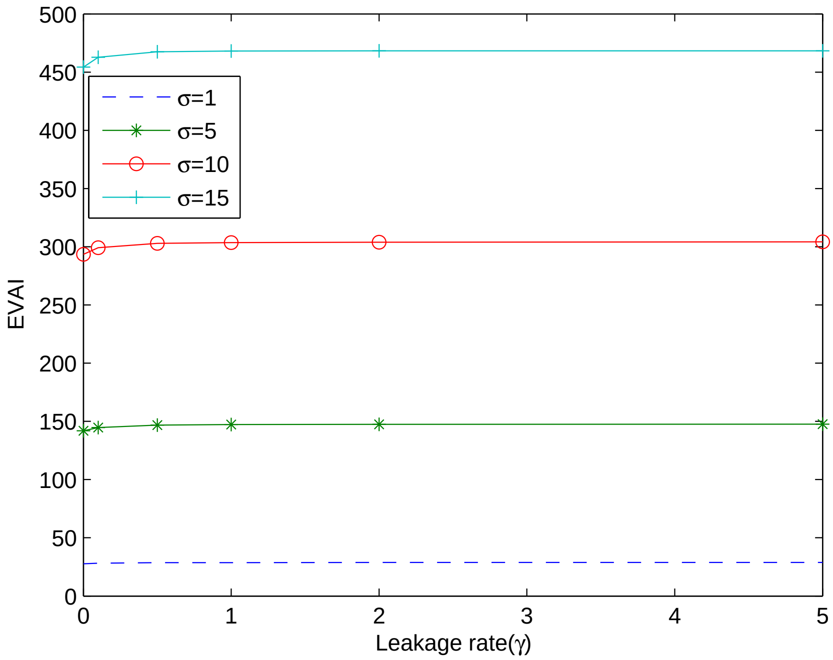

Figure 4: Impact of leakage rate (γ) and σ on using the distribution free approach

5 Conclusion and future research suggestions:

In this paper an integrated approach to optimal fare pricing, and seat inventory control is presented for an airline which is experiencing demand leakage. The fence that segments the market demand is considered imperfect. Due to imperfect market segmentation, the airlines experiences the demand leakage from full fare price market segment to discounted fare market segment. The research develops models for RM for an airline in situations when the airlines experiences price dependent deterministic, and stochastic demand. The models are analyzed to determine an integrated optimal control to fare pricing, and seat inventory control decisions. In addition, this research also explores the situation when the distribution of the price dependent stochastic demand is unknown, a joint optimal control on fare pricing, and seat inventory control is computed likewise the deterministic and stochastic demand models. The numerical experimentation reported in this paper reveals the following salient outcomes:

1. When an airline adopts an integrated optimal strategy to fare pricing, and seat inventory control, it yields superior revenues compared to the situation when an airlines does not opt to segment the market into fare classes, however, this competitive edge deteriorates significantly by demand leakage and demand uncertainty. In a numerical study, this finding is consistently noticed in situations when an airline faces three distinct demand situations: deterministic price dependent demand; stochastic price dependent demand; and when demand distribution is unknown.

2. Increasing amount of leakage rate γ, diminishes the improvement in revenue gains that an airlines can obtain over selling its seats into a single fare class.

3. The use of distribution approach based on Scarf (1952)'s rule can yield competitive revenue gains for the situation when the demand distribution is unknown to an airline. However, its performance deteriorates with an increase in demand variability. Unlike demand variability, an increase in demand leakage rate does not significantly impact the performance of the distribution free approach.

The future work directions include to investigate the optimal strategies and investments that an airlines may adopt to be immune from demand leakage effects. The present analysis has considered the firm in monopoly only, an interesting avenue, therefore would be to consider game theoretic approach to this problem in a duopoly or oligopoly.

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